Leptonic Decays of Charged Pseudoscalar Mesons

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Abstract

We review the physics of purely leptonic decays of $\pi^{\pm}$, $K^{\pm}$, $D^{\pm}$, $D_s^{\pm}$, and $B^{\pm}$ pseudoscalar mesons. The measured decay rates are related to the product of the relevant weak-interaction-based CKM matrix element of the constituent quarks and a strong interaction parameter related to the overlap of the quark and anti-quark wave-functions in the meson, called the decay constant f_P. The interplay between theory and experiment is different for each particle. Theoretical predictions of f_B that are needed in the B sector can be tested by measuring f_{D^+} and $f_{D_s^+}$ in the charm sector. Currently, these tests are unsatisfactory. The lighter π and K mesons provide stringent comparisons between experiment and theory due to the accuracy of both the measurements and the theoretical predictions. An abridged version of this review was prepared for the Particle Data Group's 2010 edition [1].

I. INTRODUCTION

Charged mesons formed from a quark and anti-quark can decay to a charged lepton pair when these objects annihilate via a virtual W boson. Fig. 1 illustrates this process for the purely leptonic decay of a D^+ meson.
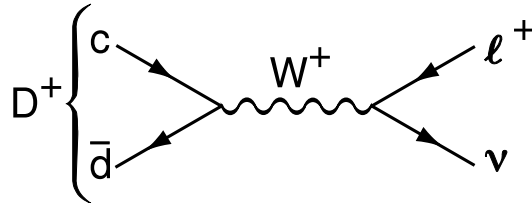

FIG. 1: The annihilation process for pure D^+ leptonic decays in the Standard Model.

Similar quark-antiquark annihilations via a virtual W^+ to the $\ell^+\nu$ final states occur for the π^+, K^+, D_s^+, and B^+ mesons. (Charge-conjugate particles and decays are implied.) Let P be any of these pseudoscalar mesons. To lowest order, the decay width is

$$\Gamma(P \to \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 \, m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 |V_{q_1 q_2}|^2 \ . \tag{1}$$

Here M_P is the P mass, m_ℓ is the ℓ mass, $V_{q_1 q_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the constituent quarks $q_1 \bar{q}_2$ in P, and G_F is the Fermi coupling constant. The parameter f_P is the decay constant, and is related to the wave-function overlap of the quark and antiquark.

The decay $P^\pm$ starts with a spin-0 meson, and ends up with a left-handed neutrino or right-handed antineutrino. By angular momentum conservation, the $\ell^\pm$ must then also be left-handed or right-handed, respectively. In the $m_\ell = 0$ limit, the decay is forbidden, and can only occur as a result of the finite ℓ mass. This helicity suppression is the origin of the m_ℓ^2 dependence of the decay width.

There is a complication in measuring purely leptonic decay rates. The process $P \to \ell\nu\gamma$ is not simply a radiative correction, although radiative corrections contribute. The P can make a transition to a virtual P^*, emitting a real photon, and the P^* decays into $\ell\nu$, avoiding helicity suppression. The importance of this amplitude depends on the decaying particle and the detection technique. The $\ell\nu\gamma$ rate for a heavy particle such as B decaying into a light particle such as a muon can be larger than the width without photon emission [2]. On the other hand, for decays into a $\tau^\pm$, the helicity suppression is mostly broken and these effects appear to be small.

Measurements of purely leptonic decay branching fractions and lifetimes allow an experimental determination of the product $|V_{q_1 q_2}| f_P$. If the CKM element is well known from other measurements, then f_P can be well measured. If, on the other hand, the CKM element is not well measured, having theoretical input on f_P can allow a determination of the CKM element. The importance of measuring $\Gamma(P \to \ell\nu)$ depends on the particle being considered. For the B system, f_B is crucial for using measurements of B^0-$\overline{B}^0$ mixing to extract information on the fundamental CKM parameters. Knowledge of f_{B_s} is also needed, but it cannot be directly measured as the B_s is neutral, so the violation of the SU(3) relation $f_{B_s} = f_B$

must be estimated theoretically. This difficulty does not occur for D mesons as both the D^+ and D_s^+ are charged, allowing the direct measurement of SU(3) breaking and a direct comparison with theory.

For B^- and D_s^+ decays, the existence of a charged Higgs boson (or any other charged object beyond the Standard Model) would modify the decay rates; however, this would not necessarily be true for the D^+ [3, 4]. More generally, the ratio of $\tau\nu$ to $\mu\nu$ decays can serve as one probe of lepton universality [3, 5].

As $|V_{ud}|$ has been quite accurately measured in super-allowed β decays [6], with a value of 0.97425(22) [7], measurements of $\Gamma(\pi^+ \to \mu^+\nu)$ yield a value for f_π. Similarly, $|V_{us}|$ has been well measured in semileptonic kaon decays, so a value for f_K from $\Gamma(K^- \to \mu^-\bar{\nu})$ can be compared to theoretical calculations. Lattice gauge theory calculations, however, have been claimed to be very accurate in determining f_K, and these have been used to predict $|V_{us}|$ [8].

II. CHARMED MESONS

We review current measurements, starting with the charm system. The CLEO collaboration has performed the only measurement of the branching fraction for $D^+ \to \mu^+\nu$ [9]. CLEO uses e^+e^- collisions at the $\psi(3770)$ resonant energy where D^-D^+ pairs are copiously produced. They fully reconstruct one of the D's, find a candidate muon track of opposite sign to the tag, and then use kinematical constraints to infer the existence of a missing neutrino and hence the $\mu\nu$ decay of the other D. They find $\mathcal{B}(D^+ \to \mu^+\nu) = (3.82\pm0.32\pm0.09)\times10^{-4}$. We use the well-measured D^+ lifetime of 1.040(7) ps, and assuming $|V_{cd}|$ equals $|V_{us}| = 0.2246(12)$ [7] minus higher order correction terms [10], we find $|V_{cd}| = 0.2245(12)$. The CLEO branching fraction result then translates into a value of

$$f_{D^+} = (206.7 \pm 8.5 \pm 2.5) \text{ MeV} .$$

This result includes a 1% correction (lowering) of the rate due to the presence of the radiative $\mu^+\nu\gamma$ final state based on the estimate by Dobrescu and Kronfeld [11].

Before we compare this result with theoretical predictions, we discuss the D_s^+. Measurements of f_{D_s} have been made by several groups and are listed in Table I [12–15]. We exclude several measurements [16–21] with normalizations to other D_s^+ decay modes (mentioned in the 2008 version of this Review [1]), of which the most recent [21] obtains $\mathcal{B}(D_s^+ \to \mu^+\nu) = (6.67 \pm 0.83 \pm 0.26 \pm 0.66) \times 10^{-3}$ and $f_{D_s^+} = (281 \pm 17 \pm 7 \pm 14)$ MeV based on $\mathcal{B}(D_s^+ \to \phi\pi^+) = (4.71 \pm 0.46)\%$. These early measurements determined the ratio of the leptonic decay to some hadronic decay, usually $\Gamma(D_s^+ \to \ell^+\nu)/\Gamma(D_s^+ \to \phi\pi^+)$. This introduces a large additional source of error since the denominator is not well known or even well defined [22].

The CLEO and Belle $\mu^+\nu$ results rely on fully reconstructing all the final state particles except for the neutrino and using a missing-mass technique to infer the existence of the neutrino. CLEO uses $e^+e^- \to D_sD_s^*$ collisions at 4170 MeV, while Belle uses $e^+e^- \to DKn\pi D_s^*$ collisions at energies near the $\Upsilon(4S)$. Selecting the $\tau^+ \to \pi^+\bar{\nu}$ and $\tau^+ \to \rho^+\bar{\nu}$ decay modes uses both calculation of the missing-mass and the fact that there should be no extra energy in the event beyond that deposited by the measured tagged D_s and the τ^+ decay products. The $\tau^+ \to e^+\nu\bar{\nu}$ mode uses only extra energy.

TABLE I: Experimental results for $\mathcal{B}(D_s^+ \to \mu^+\nu)$, $\mathcal{B}(D_s^+ \to \tau^+\nu)$, and $f_{D_s^+}$. Numbers for $f_{D_s^+}$ have been extracted using updated values for masses and $|V_{cs}|$ (see text); radiative corrections have been included. Common systematic errors in the CLEO results have been taken into account.

Experiment	Mode	$\mathcal{B}$	$f_{D_s^+}$ (MeV)
CLEO-c [12]	$\mu^+\nu$	$(5.65 \pm 0.45 \pm 0.17) \times 10^{-3}$	$257.6 \pm 10.3 \pm 4.3$
Belle [13]	$\mu^+\nu$	$(6.38 \pm 0.76 \pm 0.57) \times 10^{-3}$	$274 \pm 16 \pm 12$
Average	$\mu^+\nu$	$(5.80 \pm 0.43) \times 10^{-3}$	261.5 ± 9.7
CLEO-c [12]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$(6.42 \pm 0.81 \pm 0.18) \times 10^{-2}$	$278.0 \pm 17.5 \pm 3.8$
CLEO-c [14]	$\tau^+\nu$ $(\rho^+\overline{\nu})$	$(5.52 \pm 0.57 \pm 0.21) \times 10^{-2}$	$257.8 \pm 13.3 \pm 5.2$
CLEO-c [15]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$(5.30 \pm 0.47 \pm 0.22) \times 10^{-2}$	$252.6 \pm 11.2 \pm 5.6$
Average	$\tau^+\nu$	$(5.58 \pm 0.35) \times 10^{-2}$	$259.7 \pm 7.8 \pm 3.4$
Average	$\mu^+\nu + \tau^+\nu$		260.7 ± 6.5

We extract the decay constant from the measured branching ratios using the D_s^+ mass of 1.96849(34) GeV, the τ^+ mass of 1.77684(17) GeV, and a lifetime of 0.500(7) ps. We use the first order correction $|V_{cs}| = |V_{ud}| - |V_{cb}|^2/2$ [10] ; taking $|V_{ud}| = 0.97425(22)$ [6], and $|V_{cb}| = 0.04$ from an average of exclusive and inclusive semileptonic B decay results as discussed in Ref. [23], we find $|V_{cs}| = 0.97345(22)$. Our experimental average,

$$f_{D_s} = (260.7 \pm 6.5) \text{ MeV},$$

uses only those results that are absolutely normalized [22]. We have included the radiative correction of 1% in the $\mu^+\nu$ rates listed in the Table [11] (the $\tau^+\nu$ rates need not be corrected). Other theoretical calculations show that the $\gamma\mu^+\nu$ rate is a factor of 40–100 below the $\mu^+\nu$ rate for charm [24].

Two ratios are of particular interest. The ratio of decay constants for the $\tau^+\nu : \mu^+\nu$ modes is $f_{D_s}(\tau^+\nu)/f_{D_s}(\mu^+\nu) = 0.99 \pm 0.05$, and the ratio of D_s^+ to D^+ decay constants is $f_{D_s}/f_{D^+} = 1.27 \pm 0.06$.

Table II compares the experimental f_{D_s} with theoretical calculations [25–35]. While most theories give values lower than the f_{D_s} measurement, the errors are sufficiently large, in most cases, to declare success. An unquenched lattice calculation [25], however, differs by 2.8 standard deviations [36]. Remarkably it agrees with f_{D^+} and consequently disagrees in the ratio $f_{D_s^+}/f_{D^+}$, with less significance as the error in f_{D^+} is substantial.

The Fermilab-MILC result has been updated; the preliminary values for f_{D^+} and f_{D_s} were raised by 10 MeV and 11 MeV, respectively [26]. These changes bring the predictions for both numbers within errors of experiment.

Upper limits on f_{D^+} and f_{D_s} of 230 and 270 MeV, respectively, have been determined using two-point correlation functions by Khodjamirian [37]. The D^+ result is safely below this limit, while the average D_s result is also, but older results [1] not used in our average are often above the limit.

Akeroyd and Chen [38] pointed out that leptonic decay widths are modified in two-Higgs-doublet models (2HDM). Specifically, for the D^+ and D_s^+, Eq. 1 is modified by a factor r_q multiplying the right-hand side [39]:

TABLE II: Theoretical predictions of $f_{D_s^+}$, f_{D^+}, and $f_{D_s^+}/f_{D^+}$. QL indicates a quenched-lattice calculation, while PQL indicates a partially-quenched lattice calculation. (Only selected results having errors are included.)

Model	$f_{D_s^+}$(MeV)	f_{D^+}(MeV)	$f_{D_s^+}/f_{D^+}$
Experiment (our averages)	260.7 ± 6.5	206.0 ± 8.9	1.27 ± 0.06
Lattice(HPQCD+UKQCD) [25]	241 ± 3	208 ± 4	1.162 ± 0.009
Lattice (FNAL+MILC+HPQCD) [26]	260 ± 10	217 ± 10	1.20 ± 0.02
PQL [27]	244 ± 8	197 ± 9	1.24 ± 0.03
QL (QCDSF) [28]	$220 \pm 6 \pm 5 \pm 11$	$206 \pm 6 \pm 3 \pm 22$	$1.07 \pm 0.02 \pm 0.02$
QL (Taiwan) [29]	$266 \pm 10 \pm 18$	$235 \pm 8 \pm 14$	$1.13 \pm 0.03 \pm 0.05$
QL (UKQCD) [30]	$236 \pm 8^{+17}_{-14}$	$210 \pm 10^{+17}_{-16}$	$1.13 \pm 0.02^{+0.04}_{-0.02}$
QL [31]	$231 \pm 12^{+6}_{-1}$	$211 \pm 14^{+2}_{-12}$	1.10 ± 0.02
QCD Sum Rules [32]	205 ± 22	177 ± 21	$1.16 \pm 0.01 \pm 0.03$
QCD Sum Rules [33]	235 ± 24	203 ± 20	1.15 ± 0.04
Field Correlators [34]	260 ± 10	210 ± 10	1.24 ± 0.03
Light Front [35]	268.3 ± 19.1	206 (fixed)	1.30 ± 0.04

$$ r_q = \left[1 + \left(\frac{1}{m_c + m_q} \right) \left(\frac{M_{D_q}}{M_{H^+}} \right)^2 \left(m_c - \frac{m_q \tan^2 \beta}{1 + \epsilon_0 \tan \beta} \right) \right]^2 , $$

where m_{H^+} is the charged Higgs mass, M_{D_q} is the mass of the D meson (containing the light quark q), m_c is the charm quark mass, m_q is the light-quark mass, and $\tan \beta$ is the ratio of the vacuum expectation values of the two Higgs doublets. In models where the fermion mass arises from coupling to more than one vacuum expectation value ϵ_0 can be non-zero, perhaps as large as 0.01. For the D^+, $m_d \ll m_c$, and the change due to the H^+ is very small. For the D_s^+, however, the effect can be substantial.

A major concern is the need for the Standrd Model (SM) value of $f_{D_s^+}$. We can take that from a theoretical model. Our most aggressive choice is that of the unquenched lattice calculation [25], because they claim the smallest error. Since the charged Higgs would lower the rate compared to the SM, in principle, experiment gives a lower limit on the charged Higgs mass. However, the value for the predicted decay constant using this model is 2.8 standard deviations *below* the measurement, implying that (a) either the model of Ref. [25] is not representative; (b) no value of m_{H^+} in the two-Higgs doublet model will satisfy the constraint at 99% confidence level; or (c) there is new physics, different from the 2HDM, that interferes constructively with the SM amplitude such as in the R-parity-violating model of Akeroyd and Recksiegel [40]. Also in the context of R parity violation, Kundu and Nandi relate this discrepancy with preliminary indications of a large phase in $B_s - \overline{B_s}$ mixing and explain both with a specific supersymmetry model [41].

Dobrescu and Kronfeld [11] emphasize that the discrepancy between the theoretical lattice calculation and the CLEO data is substantial and "is worth interpreting in terms of new physics" (at least prior to the change in Fermilab-MILC result and the updated experimental values). They give three possible examples of new physics models that might be responsible.

These include two leptoquark models, and a specific two-Higgs doublet model which leads to constructive interference with the Standard Model where one doublet gives the c and u quark masses and the lepton masses, but not the d, s, b, or t masses. Gninenko and Gorbunov argue that the neutrino in the D_s decay mixes with a sterile neutrino, which enhances the rate and also explains the excess number of low energy electron-like events in the MiniBooNE data [42].

Akeroyd and Mahmoudi [39] point out that new physics can affect the $\mu^+\nu$ and $\tau^+\nu$ final states differently and thus should be studied separately. They present constraints for the charged Higgs mass in a specific SUSY model, the Non-Universal Higgs Mass, using the branching ratios values for $D_s^+ \to \tau^+\nu$ and $\mu^+\nu$ separately. These constraints in some regions are better than any other. The model of Gninenko and Gorbunov is an example of a model where $\mu^+\nu$ and $\tau^+\nu$ should be treated separately in that a sterile neutrino is likely to couple very differently to ν_μ and ν_τ. Other theoretical papers that are pertinent to this discussion concern leptoquark models [43], R-parity-violating models [44, 45], 2HDM [46], anomalous W-boson charm quark couplings [47], unparticle physics [48], and constraints on 2HDM models [49, 50].

To sum up, the situation is not clear. To set limits on new physics we need an accurate calculation of f_{Ds} and more precise measurements would also be useful.

III.   THE B MESON

The Belle and BaBar collaborations have found evidence for $B^- \to \tau^-\bar{\nu}$ decay in $e^+e^- \to B^-B^+$ collisions at the $\Upsilon(4S)$ energy. The analysis relies on reconstructing a hadronic or semi-leptonic B decay tag, finding a τ candidate in the remaining track and or photon candidates, and examining the extra energy in the event which should be close to zero for a real τ decay opposite a B tag. The results are listed in Table III.

TABLE III: Experimental results for $\mathcal{B}(B^- \to \tau^-\overline{\nu})$. We have computed an average for the two Belle measurements assuming that the systematic errors are fully correlated.

Experiment	Tag	$\mathcal{B} \times 10^{-4}$
Belle [51]	Hadronic	$(1.79^{+0.56}_{-0.49}\,^{+0.46}_{-0.51})$
Belle [52]	Semileptonic	$(1.65^{+0.38}_{-0.37}\,^{+0.35}_{-0.34})$
Belle	Our Average	$(1.70^{+0.47}_{-0.46})$
BaBar [53]	Hadronic	$(1.8^{+0.9}_{-0.8} \pm 0.4)$
BaBar [54]	Semileptonic	$(1.7 \pm 0.8 \pm 0.4)$
BaBar [54]	Average	$(1.8^{+1.0}_{-0.9})$
	Our Average	$(1.72^{+0.43}_{-0.42})$

There are large backgrounds under the signals in all cases. The systematic errors are also quite large, on the order of 20%. Thus, the significances are not that large. Belle quotes 3.5σ and 3.8σ for their hadronic and semileptonic tags, respectively, while BaBar quotes 2.8σ for their combined result. We note that the four central values are remarkably close to the average considering the large errors on on all the measurements. More accuracy would be

useful to investigate the effects of new physics. Here the effect of a charged Higgs is different as it can either increase or decrease the expected SM branching ratio. The factor r in the 2HDM that multiplies the right side of Eq. 1 is given in terms of the B meson mass, M_B, by [3, 39]

$$r = \left(1 - \frac{\tan^2 \beta}{1 - \epsilon_0 \tan \beta} \frac{M_B^2}{m_{H^+}^2}\right)^2 . \tag{2}$$

We can derive limits in the 2HDM $\tan \beta$–m_{H^+} plane. Again, we need to know the SM prediction of this decay rate. We ascertain this value using Eq. 1. Here theory provides a value of $f_B = (193 \pm 11)$ MeV [55]. We also need a value for $|V_{ub}|$. Here significant differences arise between using inclusive charmless semileptonic decays and the exclusive decay $B \rightarrow \pi \ell^+ \nu$ [56]. We find that the inclusive decays give rise to a value of $|V_{ub}| = (4.21 \pm 0.25) \times 10^{-3}$, while the $\pi \ell^+ \nu$ measurements yield $|V_{ub}| = (3.50 \pm 0.35) \times 10^{-3}$. Taking an average over inclusive and exclusive determinations, and enlarging the error using the PDG prescription because the results differ, we find $|V_{ub}| = (3.97 \pm 0.55) \times 10^{-3}$, where the error is dominantly theoretical. We thus arrive at the SM prediction for the $\tau^- \bar{\nu}$ branching fraction of $(1.04 \pm 0.31) \times 10^{-4}$.

Taking the ratio of the experimental value to the predicted branching ratio at its 90% c.l. *upper* limit and using Eq. 2 with ϵ_0 set to zero, we find that we can limit $M_{H^+} / \tan \beta > 3.3$ GeV. The 90% c.l. *lower* limit also permits us to exclude the region 3.8 GeV $< M_{H^+} / \tan \beta < 18.0$ GeV [57]. Considering the large uncertainties on V_{ub} and the branching ratio measurements, this should be taken more as indication of what the data can eventually tell us when and if the situation improves.

IV.   CHARGED PIONS AND KAONS

We now discuss the determination of charged pion and kaon decay constants. The sum of branching fractions for $\pi^- \rightarrow \mu^- \bar{\nu}$ and $\pi^- \rightarrow \mu^- \bar{\nu} \gamma$ is 99.98770(4)%. The two modes are difficult to separate experimentally, so we use this sum, with Eq. 1 modified to include photon emission and radiative corrections [58]. The branching fraction together with the lifetime 26.033(5) ns gives

$$f_{\pi^-} = (130.41 \pm 0.03 \pm 0.20) \text{ MeV} .$$

The first error is due to the error on $|V_{ud}|$, 0.97425(22) [6]; the second is due to the higher-order corrections, and is much larger.

Similarly, the sum of branching fractions for $K^- \rightarrow \mu^- \bar{\nu}$ and $K^- \rightarrow \mu^- \bar{\nu} \gamma$ is 63.55(11)%, and the lifetime is 12.3840(193) ns [59]. Measurements of semileptonic kaon decays provide a value for the product $f_+(0)|V_{us}|$, where $f_+(0)$ is the form-factor at zero four-momentum transfer between the initial state kaon and the final state pion. We use a value for $f_+(0)|V_{us}|$ of 0.21664(48) [59]. The $f_+(0)$ must be determined theoretically. We follow Blucher and Marciano [7] in using the lattice calculation $f_+(0) = 0.9644 \pm 0.0049$ [60], since it appears to be more precise than the classic Leutwyler-Roos calculation $f_+(0) = 0.961 \pm 0.008$ [61]. The result is $|V_{us}| = 0.2246 \pm 0.0012$, which is consistent with the hyperon decay value of 0.2250 ± 0.0027 [62]. We derive

$$f_{K^-} = (156.1 \pm 0.2 \pm 0.8 \pm 0.2) \text{ MeV} .$$

The first error is due to the error on Γ; the second is due to the CKM factor $|V_{us}|$, and the third is due to the higher-order corrections. The largest source of error in these corrections depends on the QCD part, which is based on one calculation in the large N_c framework. We have doubled the quoted error here; this would probably be unnecessary if other calculations were to come to similar conclusions. A large part of the additional uncertainty vanishes in the ratio of the K^- and π^- decay constants, which is

$$f_{K^-}/f_{\pi^-} = 1.197 \pm 0.002 \pm 0.006 \pm 0.001 \ .$$

The first error is due to the measured decay rates; the second is due to the uncertainties on the CKM factors; the third is due to the uncertainties in the radiative correction ratio.

These measurements have been used in conjunction with calculations of f_K/f_π in order to find a value for $|V_{us}|/|V_{ud}|$. Two recent lattice predictions of f_K/f_π are 1.189 ± 0.007 [25] and $1.192 \pm 0.007 \pm 0.006$ [63]. Together with the precisely measured $|V_{ud}|$, this gives an independent measure of $|V_{us}|$ [8, 59].

[1] This article is an update of the one prepared for the PDG 2008 edition; see J. L. Rosner and S. Stone in C. Amsler *et al.* (Particle Data Group), Physics Letters B **667**, 1 (2008), `arXiv:0802.1043 [hep-ex]`.

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