

Dalitz Plot Analysis of $B^- \rightarrow D^+ \pi^- \pi^-$

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We present a Dalitz plot analysis of $B^- \rightarrow D^+ \pi^- \pi^-$ decays, based on a sample of about 383 million $\Upsilon(4S) \rightarrow B\bar{B}$ decays collected by the *BABAR* detector at the PEP-II asymmetric-energy *B* Factory at SLAC. The analysis has been published previously in [1]. We measure the inclusive branching fraction of the three-body decay to be $\mathcal{B}(B^- \rightarrow D^+ \pi^- \pi^-) = (1.08 \pm 0.03(\text{stat}) \pm 0.05(\text{syst})) \times 10^{-3}$. We observe the established D_2^{*0} and confirm the existence of D_0^{*0} in their decays to $D^+ \pi^-$, where the D_2^{*0} and D_0^{*0} are the 2^+ and 0^+ $c\bar{u}$ P-wave states, respectively. We measure the masses and widths of D_2^{*0} and D_0^{*0} to be: $m_{D_2^*} = (2460.4 \pm 1.2 \pm 1.2 \pm 1.9) \text{ MeV}/c^2$, $\Gamma_{D_2^*} = (41.8 \pm 2.5 \pm 2.1 \pm 2.0) \text{ MeV}$, $m_{D_0^*} = (2297 \pm 8 \pm 5 \pm 19) \text{ MeV}/c^2$, $\Gamma_{D_0^*} = (273 \pm 12 \pm 17 \pm 45) \text{ MeV}$. The stated errors reflect the statistical and systematic uncertainties, and the uncertainty related to the assumed composition of signal events and the theoretical model.

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I. INTRODUCTION

Orbitally excited states of the *D* meson, denoted as D_J , provide a unique opportunity to test Heavy Quark Effective Theory (HQET). There are expected to be four P-wave states of positive parity with the quantum numbers $0^+(j=1/2)$, $1^+(j=1/2)$, $1^+(j=3/2)$ and $2^+(j=3/2)$, which are labeled as D_0^* , D_1 , D_1' and D_2^* , respectively, where j is the sum of the spin of the light quark and the angular momentum L . Conservation of parity and angular momentum restricts the final states and partial waves that are allowed in the decays of the various D_J mesons. The resonances that decay through a D-wave are expected to be narrow ($\sim 30 \text{ MeV}/c^2$) and the resonances that decay through an S-wave are expected to be wide (a few hundred MeV/c^2). The D_2^* can only decay via a D-wave and the D_0^* can only decay via an S-wave. The D_1 and D_1' may decay via S-wave and D-wave. Fig. 1 shows the spectroscopy of the D-meson excitations and expected transitions. The Belle Collaboration has reported the first observation of the broad D_0^* and D_1' mesons in *B* decays [3]. However, the Particle Data Group [4] considers that the J and P quantum numbers of the D_0^* and D_1' states still need confirmation.

In this analysis, we fully reconstruct the decays of $B^- \rightarrow D^+ \pi^- \pi^-$ final states [2] and measure the inclusive branching fraction. Then we perform a Dalitz plot analysis to measure the exclusive branching fractions for $B^- \rightarrow D_0^0 \pi^-$ and to study the properties of the D_J mesons. The decay $B^- \rightarrow D^+ \pi^- \pi^-$ is expected to be dominated by the intermediate states $D_2^* \pi^-$ and $D_0^* \pi^-$ and has a possible contribution from the $B^- \rightarrow D^+ \pi^- \pi^-$ non-resonant (NR) decay. Also, the $D^*(2007)^0$ (labeled as D_v^*) may contribute as a virtual intermediate state, as well as the B^* (labeled as B_v^*), produced in a virtual process $B^- \rightarrow B^* \pi^-$. The data used in this analysis were collected with the *BABAR* detector at the PEP-II asymmetric e^+e^- storage rings. The *BABAR* detector is described in [1] and in the references therein. The sample consists of 347.23 fb^{-1} corresponding to 382.9 ± 4.2 million $B\bar{B}$ pairs taken on the peak of the $\Upsilon(4S)$ resonance.

II. EVENT SELECTION

Five charged particles are selected to reconstruct $B^- \rightarrow D^+ \pi^- \pi^-$ decays with $D^+ \rightarrow K^- \pi^+ \pi^+$. At the $\Upsilon(4S)$ resonance, *B* mesons can be characterized by two nearly independent kinematic variables, the beam energy substituted mass m_{ES} and the energy difference ΔE :

$$m_{\text{ES}} = \sqrt{(s/2 + \vec{p}_0 \cdot \vec{p}_B)^2/E_0^2 - p_B^2}, \quad \Delta E = E_B^* - \sqrt{s}/2, \quad (1)$$

where E and p are energy and momentum, the subscripts 0 and B refer to the e^+e^- -beam system and the B candidate respectively; s is the square of the center-of-mass energy and the asterisk labels the CM frame. For $B^- \rightarrow D^+ \pi^- \pi^-$ signal decays, the m_{ES} distribution is well described by a Gaussian resolution function with a width of $2.6 \text{ MeV}/c^2$ centered at the B^- mass, while the ΔE distribution can be represented by a sum of two Gaussian functions with a common mean near zero and different widths with a combined RMS of 20 MeV . The ΔE distribution is shown in Fig. 1.

Continuum events are the dominant background. We suppress this background by restricting two topological variables: the magnitude of the cosine of the thrust angle, $\cos \theta_{th}$, defined as the angle between the thrust axis of the

selected B candidate and the thrust axis of the rest of the event; and the ratio of the second to zeroth Fox-Wolfram moment [5], R_2 . Small values of R_2 indicate a more spherical event shape (typical for $B\bar{B}$ events) while larger values indicate a 2-jet event topology (typical for $q\bar{q}$ events). We also place restrictions on m_{ES} and ΔE . Then we fit the ΔE distribution to determine the fractions of signal and background events in the selected data sample. The result of the fit is shown in Fig. 1, it yields 3496 ± 74 signal events.

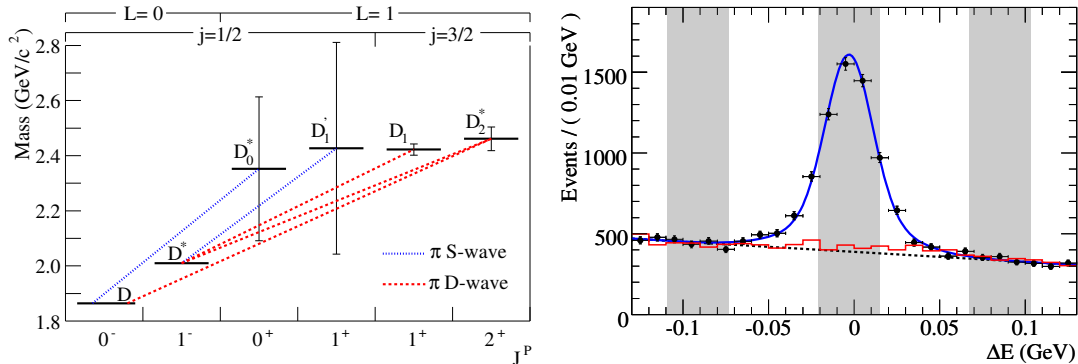


FIG. 1: Left: Mass spectrum [4] of $c\bar{u}$ states. The vertical bars show the widths. The dotted and dashed lines between the levels show the dominant pion transitions. Although it is not indicated in the figure, the two 1^+ states may be mixtures of $j = 1/2$ and $j = 3/2$, and D_1^* may decay via a D-wave and D_1 may decay via an S-wave. Right: ΔE distribution. The points with error bars are data, the red curve is the full fit, the blue dashed curve is the background, the three shaded regions, correspond to the ΔE left side- and signal bands. The histogram shown background expected from MC.

To distinguish signal and background in the Dalitz plot studies, we divide the candidates into three subsamples: the ΔE signal region, and two ΔE sidebands, all defined in Fig. 1. A background MC sample of resonant and continuum events is shown as the histogram in Fig. 1. There is a small amount of peaking, a fit yields 82 ± 41 peaking events. The background subtracted number of signal events is $N_{\text{sig}} = 3414 \pm 85$, resulting in the background fraction of $(30.4 \pm 1.1)\%$.

III. DALITZ PLOT ANALYSIS

In this analysis we choose the two $D\pi$ invariant mass-squared combinations $x = m^2(D^+\pi_1^-)$ and $y = m^2(D^+\pi_2^-)$ as the independent variables, where the two like-sign pions are randomly assigned to x and y . This has no effect on our analysis since the likelihood function (described below) is explicitly symmetrized with respect to interchange of the two identical particles. We describe the distribution of candidate events in the Dalitz plot in terms of a probability density function (PDF). The PDF is the sum of signal and background components and has the form:

$$\text{PDF}(x, y) = f_{\text{bg}} \frac{B(x, y)}{\int_{\text{DP}} B(x, y) dx dy} + (1 - f_{\text{bg}}) \frac{[S(x, y) \otimes \mathcal{R}] \epsilon(x, y)}{\int_{\text{DP}} [S(x, y) \otimes \mathcal{R}] \epsilon(x, y) dx dy}, \quad (2)$$

where $S(x, y) \otimes \mathcal{R}$ is the signal term convolved with the signal resolution function, $B(x, y)$ is the background term, f_{bg} is the fraction of background events, and ϵ is the reconstruction efficiency. An unbinned maximum likelihood fit to the Dalitz plot is performed in order to maximize the value of $\mathcal{L} = \prod_{i=1}^{N_{\text{event}}} \text{PDF}(x_i, y_i)$ with respect to the parameters used to describe S , where x_i and y_i are the values of x and y for event i , respectively. It is difficult to find a proper binning at the kinematic boundaries in the x - y -plane of the Dalitz plot. For this reason, we choose to estimate the goodness-of-fit χ^2 in the $\cos\theta$ and m_{min}^2 plane, which is a rectangular representation of the Dalitz plot. The helicity angle θ is the angle between the momentum of the pion from the B decay and that of the pion of the $D\pi$ system in the $D\pi$ restframe; m_{min}^2 is the lesser of x and y . This analysis uses an isobar model formulation in which the signal decays are described by a coherent sum of a number of two-body ($D\pi$ system + bachelor pion) amplitudes. The orbital angular momentum between the $D\pi$ system and the bachelor pion is denoted as L . The total decay matrix element \mathcal{M} is then given by:

$$\mathcal{M} = \sum_{L=(0,1,2)} \rho_L e^{i\Phi_L} [N_L(x, y) + N_L(y, x)] + \sum_k \rho_k e^{i\Phi_k} [A_k(x, y) + A_k(y, x)], \quad (3)$$

where the first term represents the S-wave ($L = 0$), P-wave ($L = 1$) and D-wave ($L = 2$) non-resonant contributions, the second term stands for the resonant contributions, the parameters ρ_k and Φ_k are the magnitudes and phases of the k^{th} resonance, while ρ_L and Φ_L correspond to the magnitudes and phases of the non-resonant contributions with angular momentum L . The functions $N_L(x, y)$ and $A_k(x, y)$ are the amplitudes of non-resonant and resonant terms, respectively. The resonant amplitudes $A_k(x, y)$ are expressed as $A_k(x, y) = R_k(m) F_L(p' r') F_L(q r) T_L(p, q, \cos \theta)$, where $R_k(m)$ is the k^{th} resonance lineshape, $F_L(p' r')$ and $F_L(q r)$ are the Blatt-Weisskopf barrier factors [6], and $T_L(p, q, \cos \theta)$ gives the angular distribution. The parameter $m(= \sqrt{x})$ is the invariant mass of the $D\pi$ system. The parameters p' , p , q and θ are functions of x and y . The non-resonant amplitudes $N_L(x, y)$ are similar to $A_k(x, y)$ but do not contain resonant mass terms. The Blatt-Weisskopf barrier factors depend on a single parameter, r' or r , the radius of the barrier, which we take to be $1.6 (\text{GeV}/c)^{-1}$, similarly to Ref. [3]. The functional forms of the F_L are given in Ref. [1]. For virtual D_v^* decays, $D_v^* \rightarrow D^+ \pi^-$, and virtual B_v^* production in $B^- \rightarrow B_v^* \pi^-$, we use an exponential form factor in place of the Blatt-Weisskopf barrier factor, as discussed in Ref. [3]: $F(z) = \exp(-(z - z'))$, where $z' = r p_v$ for $D_v^* \rightarrow D^+ \pi^-$ and $z' = r' p_v$ for $B^- \rightarrow B_v^* \pi^-$. Here, we set $p_v = 0.038 \text{ GeV}/c$, which gives the best fit. The resonance mass term $R_k(m)$ describes the intermediate resonance. All resonances in this analysis are parameterized with relativistic Breit-Wigner functions:

$$R_k(m) = \frac{1}{(m_0^2 - m^2) - i m_0 \Gamma(m)}, \quad \Gamma(m) = \Gamma_0 \left(\frac{q}{q_0} \right)^{2L+1} \left(\frac{m_0}{m} \right) F_L^2(qr), \quad (4)$$

where m_0 and Γ_0 are the values of the resonance pole mass and decay width, respectively. The terms $T_L(p, q, \cos \theta)$ describe the angular distribution of final state particles and are based on the Zemach tensor formalism [7]. The definitions are given in [1]. The signal function is then given by $S(x, y) = |\mathcal{M}|^2$.

In this analysis, the masses of D_v^* and B_v^* are taken from the world averages [4] while their widths are fixed at 0.1 MeV; the magnitude ρ_k and phase Φ_k of the D_2^* amplitude are fixed to 1 and 0, respectively, while the masses and widths of the D_J resonances and the other magnitudes and phases are free parameters to be determined in the fit. The effect of varying the masses of D_v^* and B_v^* between 0.001 and 0.3 MeV is negligible compared to the other model-dependent systematic uncertainties.

The fit fraction for the k^{th} decay mode is defined as the integral of the resonance decay amplitudes divided by the coherent matrix element squared for the complete Dalitz plot:

$$f_k = \frac{\int_{\text{DP}} |\rho_k (A_k(x, y) + A_k(y, x))|^2 dx dy}{\int_{\text{DP}} |\mathcal{M}|^2 dx dy}. \quad (5)$$

The detector has a finite resolution. For the narrow resonance D_2^* with the expected width of about 40 MeV, the signal resolution needs to be taken into account. We study the resolution on MC simulated events. We find the resolution to be independent of $\cos \theta$ for truth-matched events, and we describe it by a sum of two Gaussian functions with a common mean. The signal resolution for an invariant mass of the $D\pi$ combination around the D_2^* region is about $3 \text{ MeV}/c^2$. There is a small self crossfeed (SCF) component, which varies from 0.5% to 4.0% with $\cos \theta$. For this component, also the resolution varies with $\cos \theta$, which we take into account. We also check the estimated biases in the fitted parameters due to uncertainties in the signal resolution functions are small.

The signal term is modified in order to take into account the particle detection efficiency. Since different regions of the Dalitz plot correspond to different event topologies, the efficiency is not expected to be uniform over the Dalitz plot. We determine the efficiency function, $\epsilon(x, y)$, by fitting twice a large sample of $B^- \rightarrow D^+ \pi^- \pi^-$ MC: before and after the final selection was applied. The properly normalized ratio of the fit functions gives $\epsilon(x, y)$. The efficiency is flat in the center of the Dalitz plot, and drops close to its boundaries.

The background distribution is modeled using an analytic function describing MC background events. Since we find the Dalitz plot distributions of ΔE sideband events in data and in MC to be consistent within their statistics, we are confident that the MC simulation can accurately represent the background in the signal region.

IV. PHYSICS RESULTS

The total $B^- \rightarrow D^+ \pi^- \pi^-$ branching fraction is calculated using the relation: $\mathcal{B} = N_{\text{signal}} / (\bar{\epsilon} \cdot \mathcal{B}(D^+) \cdot 2N(B^+ B^-))$, where $N_{\text{signal}} = 3414 \pm 85$, $\bar{\epsilon}$ is the average efficiency, $\mathcal{B}(D^+) \equiv \mathcal{B}(D^+ \rightarrow K^- \pi^+ \pi^+) = (9.22 \pm 0.21) \%$, and the total number of $B^+ B^-$ events $N(B^+ B^-) = (197.2 \pm 3.1) \times 10^6$. The measured total branching fraction is $\mathcal{B}(B^- \rightarrow D^+ \pi^- \pi^-) = (1.08 \pm 0.03(\text{stat})) \times 10^{-3}$.

The Dalitz plot distribution for data is included in [1]. Since the composition of events in the Dalitz plot and their distributions are not known a priori, we have tried a variety of different assumptions. In particular, we test the

inclusion of various components, such as the virtual D_v^* and B_v^* as well as S-, P-, and D-wave modeling of the non-resonant component, in addition to the expected components of D_2^* , D_0^* , and background. We choose as the nominal fit model the one with the D_2^* , D_0^* , D_v^* , B_v^* , and P-wave non-resonant components considered. It produces the best fit quality with the smallest number of components. The P-wave non-resonant component is an addition to the fit model used in the previous measurement from Belle [3]. The detailed fit results are: $m_{D_2^*} = 2460.4 \pm 1.2$, $\Gamma_{D_2^*} = 41.8 \pm 2.5$, $m_{D_0^*} = 2297 \pm 8$, $\Gamma_{D_0^*} = 273 \pm 12$, $f_{D_2^*} = 32.2 \pm 1.3$, $\Phi_{D_2^*} = 0.0$ fixed, $f_{D_0^*} = 62.8 \pm 2.5$, $\Phi_{D_0^*} = -2.07 \pm 0.06$, $f_{D_v^*} = 10.1 \pm 1.4$, $\Phi_{D_v^*} = 3.00 \pm 0.12$, $f_{B_v^*} = 4.6 \pm 2.6$, $\Phi_{B_v^*} = 2.89 \pm 0.21$, $f_{\text{P-NR}} = 5.4 \pm 2.4$, $\Phi_{\text{P-NR}} = -0.89 \pm 0.18$, $f_{\text{bg}} = 30.4$ fixed, where masses are in units of MeV/c^2 , widths in MeV, fractions in %, and angles in radians. All errors are statistical only. The total χ^2 over degrees of freedom is 220/153. The details of the other fit models in question are detailed in Ref. [1]. Ref. [8] argues for an addition of a $D\pi$ S-wave state near the $D\pi$ system threshold to the model of the $D\pi\pi$ final state. We have performed according tests [1], which all yielded worse fit qualities than the nominal fit.

The nominal fit model results in the following branching fractions: $\mathcal{B}(B^- \rightarrow D_2^* \pi^-) \times \mathcal{B}(D_2^* \rightarrow D^+ \pi^-) = (3.5 \pm 0.2) \times 10^{-4}$ and $\mathcal{B}(B^- \rightarrow D_0^* \pi^-) \times \mathcal{B}(D_0^* \rightarrow D^+ \pi^-) = (6.8 \pm 0.3) \times 10^{-4}$, where the errors are statistical only. Fig. 2a-c show the $m_{\text{min}}^2(D\pi)$, $m_{\text{max}}^2(D\pi)$, and $m^2(\pi\pi)$ projections, respectively, while Fig. 3a and 3b show the $\cos\theta$ distributions for the D_0^* and D_2^* mass regions, respectively. The distributions show good agreement between the data and the fit. The angular distribution in the D_2^* mass region is clearly visible and is consistent with the expected D-wave distribution of $|\cos^2\theta - 1/3|^2$ for a spin-2 state. In addition, the D_0^* signal and the reflection of D_2^* can be easily distinguished in the $m_{\text{min}}^2(D\pi)$ and $m_{\text{max}}^2(D\pi)$ projection, respectively. The lower edge of $m_{\text{min}}^2(D\pi)$ is better described with the D_v^* component included than without.

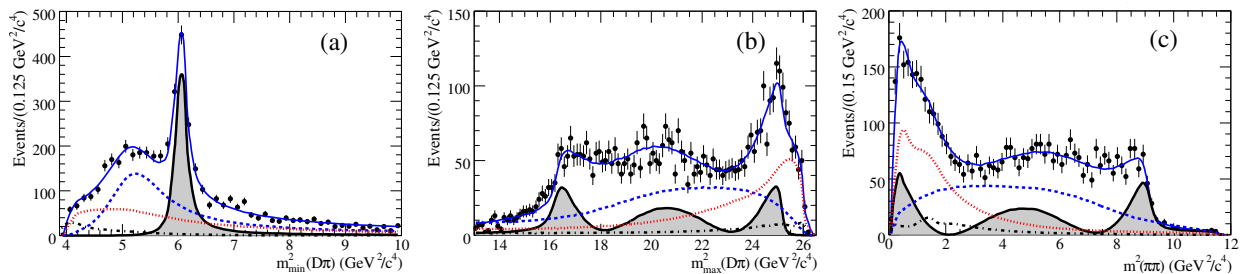


FIG. 2: Result of the nominal fit to the data: projections on (a) $m_{\text{min}}^2(D\pi)$, (b) $m_{\text{max}}^2(D\pi)$, and (c) $m^2(\pi\pi)$. The points are the data, the solid curves represent the nominal fit. The shaded areas show the D_2^* contribution, the dashed curves show the D_0^* signal, the dash-dotted curves show the D_v^* and B_v^* signals, and the dotted curves show the background.

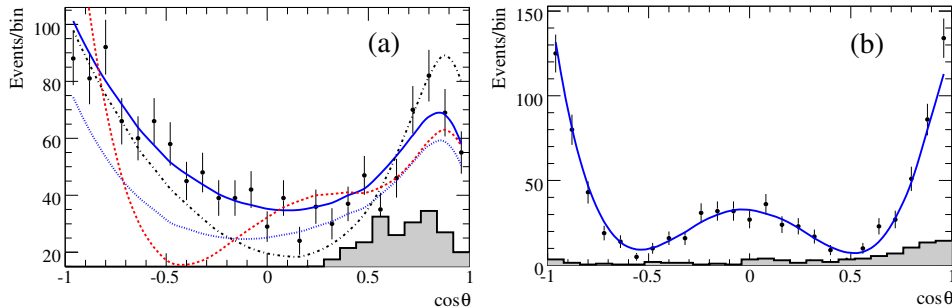


FIG. 3: Result of the nominal fit to the data: the $\cos\theta$ distributions for (a) $4.5 < m^2(D\pi) < 5.5 \text{ GeV}^2/c^4$ region and (b) $5.9 < m^2(D\pi) < 6.2 \text{ GeV}^2/c^4$ region. The points with error bars are data, the solid curves represent the nominal fit. The dashed, dash-dotted and dotted curves in (a) show the fit of hypotheses 2-4 in Table I, respectively. The shaded histograms show the $\cos\theta$ distributions from ΔE sidebands in data.

Table I shows the NLL and χ^2/NDF values for the nominal fit and for the fits with the broad resonance D_0^* excluded or with the J^P of the broad resonance replaced by other quantum numbers. In all cases, the NLL and χ^2/NDF values are significantly worse than that of the nominal fit. Fig. 3a illustrates the helicity distributions in the D_0^* mass region from hypothesis 2-4; clearly the nominal fit gives the best description of the data. We conclude that a broad spin-0 state D_0^* is required in the fit to the data. The same conclusion is obtained when performing the same tests using the alternative non-nominal fit models.

Hypothesis	Model	NLL	χ^2/NDF
	nominal fit	22970	220/153
1	D_2^* , D_v^* , B_v^* , P-NR	23761	1171/143
2	D_2^* , D_v^* , B_v^* , P-NR, (2^+)	23699	991/144
3	D_2^* , D_v^* , B_v^* , P-NR, (1^-)	23427	638/135
4	D_2^* , D_v^* , B_v^* , P-NR, S-NR	23339	652/157

TABLE I: Comparison of the models with different resonance composition. The labels, S-NR and P-NR, denote the S- and P-wave non-resonant contributions.

The systematic uncertainties under consideration are detailed in Ref. [1]. The systematic effects considered include the number of B^+B^- events, tracking efficiencies, particle identification, uncertainty on the background shapes, external D^+ branching fraction, and fit bias.

V. SUMMARY

In conclusion, we measure the total branching fraction of the $B^- \rightarrow D^+\pi^-\pi^-$ decay to be $\mathcal{B}(B^- \rightarrow D^+\pi^-\pi^-) = (1.08 \pm 0.03(\text{stat}) \pm 0.05(\text{syst})) \times 10^{-3}$. Analysis of the $B^- \rightarrow D^+\pi^-\pi^-$ Dalitz plot using the isobar model confirms the existence of a narrow D_2^* and a broad D_0^* resonance as predicted by HQET. The mass and width of D_2^* are determined to be $m_{D_2^*} = (2460.4 \pm 1.2(\text{stat}) \pm 1.2(\text{syst}) \pm 1.9(\text{mod})) \text{ MeV}/c^2$, $\Gamma_{D_2^*} = (41.8 \pm 2.5(\text{stat}) \pm 2.1(\text{syst}) \pm 2.0(\text{mod})) \text{ MeV}$, while of the D_0^* they are: $m_{D_0^*} = (2297 \pm 8(\text{stat}) \pm 5(\text{syst}) \pm 19(\text{mod})) \text{ MeV}/c^2$, $\Gamma_{D_0^*} = (273 \pm 12(\text{stat}) \pm 17(\text{syst}) \pm 45(\text{mod})) \text{ MeV}$, where the third uncertainty is related to the assumed composition of signal events and the Blatt-Weisskopf barrier factors. The measured masses and widths of both states are consistent with the world averages [4] and the predictions of some theoretical models (see references in [1]). We have also obtained exclusive branching fractions for D_2^* and D_0^* production: $\mathcal{B}(B^- \rightarrow D_2^*\pi^-) \times \mathcal{B}(D_2^* \rightarrow D^+\pi^-) = (3.5 \pm 0.2(\text{stat}) \pm 0.2(\text{syst}) \pm 0.4(\text{mod})) \times 10^{-4}$, $\mathcal{B}(B^- \rightarrow D_0^*\pi^-) \times \mathcal{B}(D_0^* \rightarrow D^+\pi^-) = (6.8 \pm 0.3(\text{stat}) \pm 0.4(\text{syst}) \pm 2.0(\text{mod})) \times 10^{-4}$. Our results of the masses, widths and branching fractions are consistent with but more precise than previous measurements performed by Belle [3]. The relative phase of the scalar and tensor amplitude is measured to be $\Phi_{D_0^*} = -2.07 \pm 0.06(\text{stat}) \pm 0.09(\text{syst}) \pm 0.18(\text{mod}) \text{ rad}$.

VI. ACKNOWLEDGEMENTS

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