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Limiting soft particle emission in e^+e^-, hadronic and nuclear collisions

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Abstract

In e^+e^- collisions the particle spectra at low momenta reflect the properties of the underlying "soft" QCD gluon bremsstrahlung: the particle density, in the limit $p \to 0$, becomes independent of the incoming energy $\sqrt{s}$ and directly proportional to the colour factors C_A, C_F for primary gluons or quarks respectively. We find that experimental data from the pp and nuclear reactions reveal the same behaviour: in the limit $p_T \to 0$ the invariant particle spectra become independent of the collision energy, and their intensities in e^+e^-, pp and nuclear reactions are compatible with the expected colour factors C_F: C_A: $(N_{part}/2)C_A$ for N_{part} nucleons, participating in the interaction. Coherent soft gluon bremsstrahlung is, therefore, suggested to be the dominant QCD mechanism for the soft particle production in all these reactions. These "soft" particles probe the very early stage of hadron formation in the collision. Future measurements at the LHC will provide crucial tests on the contributions from possible incoherent multi-component processes.

1 Introduction

One of the interesting phenomena in high energy collisions is the production of *soft* particles with low momenta. It is usually described in the existing Monte Carlo models by a non-perturbative mechanism which transforms the partonic final state into a corresponding final state of hadrons. Best understood is the e^+e^- process where the primarily produced $q\bar{q}$ pair evolves into a partonic cascade and finally into hadrons. A similar description applies to the hadronic collisions with the high p_T jets, where the partonic cascades are initiated by the *active* partons and the parton *spectators*; also an incoherent superposition of such processes could become important at high energies in the TeV region. In nuclear collisions the collective phenomena play an important role as well.

This paper is focused on the universal feature of the soft ($p \to 0$) particle production in all these collision processes which is based on the universality of the soft gluon bremsstrahlung with the intensity depending on the directions and colour charges of the participating partons but not on the collision energy. From this point of view the perturbative analysis of the quark gluon cascade (see, for example, [1, 2] is naturally combined with the idea of a close similarity between the parton and hadron spectra ("Local Parton Hadron Duality" - LPHD [3]). Such a scenario is deeply rooted in the space-time picture of the parton-cascade development and hadronization, see [1, 4]. This approach has been developed and tested in details for the hadronic final states in e^+e^- annihilation verifying, in particular, the energy independence of the soft limit and the expected dependence on the primary colour charges (quark or gluon jets), but it should also be applicable to other hard processes including ep and pp ($p\bar{p}$) collisions with high p_T particle production, where the soft particles belong to the "underlying event" [5] - [7]. A distinct dependence on the primary colour charges makes this approach quite different from the simple process-independent universality, for example, of a thermodynamical origin.

Here, we are extending this approach to the description of the so-called "minimum bias" events in the hadronic and nuclear collisions where we expect similar effects generated by the primary soft gluon bremsstrahlung. Note, first, that it is plausible to expect that at high energies the bulk of the soft particles originates in a semihard process from the fragmentation of the so-called minijets, which are produced by the perturbative mechanism similar to the e^+e^- case. Next, recall that the perturbative dynamics results in the limiting behaviour of the soft particle densities, since the radiation of soft gluons from several quarks and gluons develops in a coherent way: different sources of secondaries should act as a single source with an effective colour 'charge' equal to the *vector* sum of all the colour 'charges', since a gluon with a large wave-length cannot resolve the smaller details (individual elements of the 'colour antenna'). It is worth emphasizing also that the low momentum hadrons are formed at the initial stage, before the formation of fast particles [1, 4]. Thus, the low-p hadrons emitted at small rapidities in the centre-of-mass frame (*c.m.s.*) bear no information about the complicated structure of the whole event, in particular, about the secondaries in the fragmentation regions and the properties of the initial hadrons.

It is natural to expect the analogous interference phenomena in the light-hadron (pion) emission, when the particle Compton wave-length exceeds the size of the interaction region. Thus, we should observe the limiting behaviour in the soft particle densities in the proton-proton, proton-nuclear and nuclear-nuclear collisions similar to that observed in the e^+e^- annihilation. All these processes can be characterized by the limiting density I_0 of the soft particle spectra

[5] at vanishing rapidity y and p_T [1]

$$I_0 = \frac{1}{\sigma_{norm}} E \frac{d\sigma}{d^3p}\bigg|_{y\to 0, p_T\to 0} . \tag{1}$$

In the case of e^+e^- annihilation the normalization cross section is σ_{tot}, while for the pp and nuclear collisions the rates are often normalized by $\sigma_{in}= \sigma_{tot} - \sigma_{el}$. However, for the studies of soft particle production it is more appropriate to use for normalization another quantity, namely the so-called non-diffractive cross section, σ_{ND}, defined as $\sigma_{ND}= \sigma_{in} - \sigma_D$. Here σ_D describes the corresponding contribution from the single diffractive and double diffractive dissociation. Strictly speaking, σ_D depends on the collision energy (see, for example, a compilation in Ref.[8] and the analysis in [9]). However, this energy variation is within the uncertainties of the current data, and it is plausible to evaluate the non-diffractive cross section σ_{ND} as being 10% lower than σ_{in}. Note also, that here and in what follows the quantity I_0 is defined as the density averaged over the charged [2] particles $[\frac{1}{2}(h^+ + h^-)]$. After recalling the relevant observations for the e^+e^- collisions we will turn to the study of the limiting behaviour of the density (1) in the pp and AA collisions. We assume that these processes are initiated by the gluon exchanges, but more general configurations are possible. While such a picture with t-channel gluon exchanges appeared in various applications in the past (for early references, see [10]), we aim here at the quantitative consequences of the universal soft-gluon bremsstrahlung mechanism in the strong interaction processes.

2 Lessons from the soft particle spectra in e^+e^- collisions

Let us start from recalling the behaviour of the momentum spectrum of particles in e^+e^- annihilation in the soft limit as the benchmark reaction. In this process it is convenient to measure the inclusive momentum spectrum since this does not require a determination of the jet axis which is needed for the spectra defined in terms of y and p_T.

The existing data can be well described within the Modified Leading Logarithmic Approximation (MLLA) for the QCD parton cascade together with LPHD for the description of hadrons (see, for instance [1, 2]). For the very low momenta a model dependent prescription is applied allowing to account for the mass effects. The perturbative expansion of the momentum spectrum of partons can be derived by an iteration of the Born term in the respective evolution equation. The first term in MLLA has the form [5, 4]

$$\frac{dn_g^{A,F}}{d\bar{y}dk_T^2} \simeq C_{A,F}\frac{\alpha_s(k_T)}{k_T^2} \tag{2}$$

while the higher α_s corrections, which depend on the primary energy, vanish as $\ln(\ln(k_T/\Lambda)/\ln(Q_0/\Lambda))$ when the transverse momentum of the gluon, k_T, aproaches its minimum (cut-off)

[1] We refer here to the one-side limit $y \to 0$, not to the double-side limit $|y| \to 0$

[2] In some cases, when we refer to the charged pions, the value of I_0 is 7% lower.

value Q_0. Here $n_g^{A,F}$ is the multiplicity of gluons emitted by the configuration with the gluon (A) or quark (F) colour charge; $\bar{y} = \ln(1/x)$, momentum fraction carried by the gluon is denoted as x and QCD scale - as Λ. In this limit the Born term remains and it is independent of primary energy and directly proportional to the colour factors. These features stay unchanged if the spectra are slightly modified at low momentum within LPHD to include particles with mass and one arrives at the perturbative expectation for the limiting behaviour [5, 6]

$$p \to 0: \qquad E\frac{dn}{d^3p} \to const. \tag{3}$$

In this limit the gluon has a large wavelength and only resolves the primarily produced partons, a radiation process represented by the Born term.

The available data show, that at the lowest accessible particle momenta, around $p = 0.2$ GeV, the invariant density $E\frac{dn}{d^3p}$ for pions rises only by about 20% over the *c.m.s.* energy range $\sqrt{s} = 3.0 \div 160$ GeV, and with increasing momentum p this rise becomes stronger, see [5] - [7]. The MLLA formulae provide a rather good fit of the total energy and momentum dependence of the observed one-particle spectrum, and clearly demonstrate the approach towards energy independence of the spectrum as in the limit (3). Therefore, the observed rise of the particle density at mid-rapidities and the rise of the overall multiplicity with energy increasing is caused by the high p_T particles. Note that in the soft limit $p \to 0$ one obtains

$$E\frac{dn}{d^3p}\bigg|_{p\to 0} = E\frac{dn}{d^3p}\bigg|_{y\to 0, p_T\to 0}, \tag{4}$$

since the invariant particle distribution approaches a constant value.

The second important lesson from the e^+e^- data concerns the dependence of the soft particle density on the primary colour charges, that is, on the difference between the primary quark-antiquark and the gluon-gluon colour antennae-dipoles. This was confirmed by the study of 3-jet ($q\bar{q}g$) events at different inter-jet angles. In the limit when the gluon is parallel to a quark (antiquark) the soft radiation pattern is the same as that in the case of a $q\bar{q}$ dipole, since the soft gluon cannot resolve the parallel quark and gluon. On the other hand, if the $q\bar{q}$ system recoils against the gluon, it is seen by the emitted soft gluon as a colour octet source. Then the soft radiation density in these two limiting cases should differ by the ratio of colour factors

$$p \to 0: \qquad E\frac{dn^{gg}}{d^3p} \bigg/ E\frac{dn^{q\bar{q}}}{d^3p} = C_A/C_F \tag{5}$$

with $C_A/C_F = 9/4$, in agreement with (2). Experimentally, one cannot reach these extreme limits but we can derive predictions on how the soft radiation density perpendicular to the event plane in the 3-jet event varies with the inter-jet angles between the two extremes [5]. These expectations are well confirmed by the DELPHI data [11] which successfully reproduce the ratio (5). Note that the corresponding ratio for the global quark and gluon jet multiplicities strongly deviates from (5) because of the influence of the particles with higher p_T and the importance of higher orders in the perturbative MLLA calculation.

Neglecting the effects of order of $1/N_C^2$, the primary $q\bar{q}g$ antenna can be represented by the two $q\bar{q}$ dipoles, where the gluon is replaced by the parallel q and $\bar{q}$. In this approximation the ratio (5) becomes $C_A/C_F \to 2$, corresponding to the number of radiating dipoles. The radiation of soft gluons perpendicular to the dipole varies with the opening angle Θ as $E\frac{dn}{d^3p} \sim (1-\cos\Theta)$. The experimental results demonstrate that the soft particle density follows the lowest order QCD expectations for the given configuration of colour charges, see for a review [4].

3 Soft particle spectra in high energy pp ($p\bar{p}$) collisions

3.1 Minimal model for the minimum bias events

In analogy to the e^+e^- process, let us start from the minimal partonic process, which can be responsible for the very soft gluon bremsstrahlung. We assume here that the underlying physics of the minimum bias events is based on the collisions of two partons within the protons. The partonic process of lowest perturbative order corresponds to one gluon exchange, which leads to a dominantly small scattering angle and a non-vanishing cross section at high energies. In the case of elastic scattering between the two incoming quarks the exchange of the t-channel gluon rearranges the incoming colours and creates the outgoing colour charges which leads to the radiation of the soft gluons from the effective colour octet dipole. For a large number of colours in the above-mentioned approximation this radiation can be described as being generated by a superposition of the *two* aligned dipoles. In the case of small-angle scattering the same soft gluon radiation pattern appears also with the incoming quarks replaced by gluons, as can be explicitly seen from the radiation patterns given in [12].

Therefore, the limiting soft radiation densities in the pp and e^+e^- collisions should differ by a factor of

$$p \to 0 : \qquad I_0^{pp}/I_0^{e^+e^-} \approx C_A/C_F \tag{6}$$

at all energies, as in the ratio in Eq. (5) for the spectra induced by the gg and $q\bar{q}$ dipoles.

For a similar reason the particle multiplicity ratio for the two processes is expected to be equal to the same quantity C_A/C_F [13]. In our case, we anticipate that this QCD expectation could be valid only in a specific soft limiting case, while as we discuss, the p_T-behaviour of the spectra in both processes are different and energy dependent, therefore the total multiplicities should differ as well.

One can also consider more complex situations corresponding to the multiple gluon exchange. However, if there is a multiple gluon exchange between the pair of quarks, then the total colour of the exchanged system could be only an octet or a singlet; since the singlet exchange rather corresponds to a diffractive process, its contribution to the central production is negligible, and we come back to the previous case. If the multiple gluon exchange involves both quarks and spectator diquarks, but does not destroy the diquark, (which acts as a local object), then the colour of the 'quark-diquark' system can be either an octet or a singlet, and, thus, again after the multiple gluon exchange this generates in the t-channel only the 'octet' colour flow. Finally, recall that in the case of more complicated diagrams the dominant Leading Logarithmic contribution in both the DGLAP and BFKL evolutions also corresponds to the colour octet exchange.

In the early discussions [14] several specific models were proposed, which predicted the ratio of the central rapidity particle densities in the pp and e^+e^- collisions. In these models the final state in pp-interactions is constructed from the two chains of particles connecting q and qq while there is one chain between q and $\bar{q}$ in e^+e^- collisions, but the ratio of these densities in both processes depends on the amount of overlap between both of the chains and can vary between one and two. Later on, the semi-hard gluon bremsstrahlung has been included in the models and this leads to a rapidity plateau rising with energy, and a simple relation between both processes fades away. An application of the Lund model to hadronic scattering at the energies below ISR was discussed in [15], where the same one-string description was used for both, e^+e^- and pp collisions.

At higher energies this scenario should be modified in order to incorporate the multiple interactions (MI) of partons (that is simultaneous interactions of two or more pairs of partons) and minijet formation, which would lead to a certain energy growth of I_0 [3], for recent development based on the dipole cascades and multiple collisions, see [16].

Current Monte Carlo models include the MI option, which in terms of a simple eikonal model for soft pp-scattering, where one interaction corresponds to the exchange of a cut *bare* Pomeron, is described by the contribution of the few cut Pomerons. Particle density produced in the central region by one cut Pomeron is energy independent [4]. Recall, that the cross section of hard subprocess is $d\hat{\sigma}/d^2q_t \propto 1/q_t^4$. However, in the Gribov-Regge theory the mean number of MIs, that is the number of cut Pomerons, increases as s^Δ, where the bare Pomeron intercept is $\alpha_P(0) = 1 + \Delta$ (in terms of the gluon density $xg(x, q_t) \sim x^{-\lambda}$ we identify $\Delta = \lambda$) [5]. This is the well known AGK result [18] – single particle inclusive cross section is described just by one Pomeron exchange, and, thanks to the AGK cancellation, there is no absorptive corrections. Therefore, we arrive at $d\sigma/d^3p \propto s^\Delta$, while the growth of inelastic cross section is reduced by the absorptive effects (asymptotically instead of $\sigma_{in} \propto s^\Delta$ we should reach the Froisart limit $\sigma_{in} \propto \ln^2 s$). Phenomenologically, the energy behaviour of σ_{in} is close to s^ϵ with $\epsilon \sim 0.08 \div 0.1$. Thus, the particle density $I_0 = (1/\sigma_{in})E d\sigma/d^3p \propto s^{\Delta-\epsilon}$, and this growth with energy reflects the increasing number of MIs, which act in the present Monte Carlo models incoherently. An account of the coherence effect should diminish the multiplicity of low p_T-secondaries produced by MI. An explicit computation [19] using Perugia P0 tune [20] of the Pythia MC model, shows

[3] We thank Gosta Gustafson for a discussion of the Lund model results.

[4] In a simple soft scattering model particle density is energy-independent. For shower MCs, where each MI is modeled by the perturbative QCD contribution of the form of $\int dx_1 dx_2 g(x_1, q_t) g(x_2, q_t) d\hat{\sigma}/d^2q_t|_{q_t > q_{min}}$, the approximately constant behaviour results from a compensation between the growth of the gluon density $xg(x, q_t) \sim x^{-\lambda}$ with decreasing momentum fraction $x \sim q_t/\sqrt{s}$ and an increase of a cutoff $q_{min} \sim 1/s^{0.08 \div 0.13}$.

[5] In the recent model for soft hadronic interactions [9] $\Delta \sim 0.3$, which is in agreement with the resummed NLL BFKL result, for example [17].

that in the *c.m.s.* energy interval $\sqrt{s} = 23 \div 14000$ GeV the invariant density in the pp collisions $E\frac{dn}{d^3p}$ in the range $p_T = 0 \div 0.5$ GeV and $y = 0 \div 0.5$ is rising by a factor of about 2. Other, 'more aggressive', tunes may result in even higher rise, up to a factor of 3 at most [6]. As we discussed above, without incorporation of coherence, such increase should hold on up to the asymptotic energies.

3.2 Energy dependence of the limiting soft spectrum

For pp interactions in case of the minimal model discussed above the soft radiation pattern should again be energy independent analogously to the e^+e^- case, but with the intensity about a factor of two higher. We are analyzing below the p_T-spectra in pp collisions in order to determine the quantity I_0 for $p_T \to 0$ at the central rapidity $y = 0$, as defined by relation (1). The previous analysis of the invariant cross section $\frac{E}{\sigma_{in}}\frac{d\sigma}{d^3p}$ in the p_T-range $0.3 \div 1.0$ GeV indicated a moderate rise with energy increasing from the ISR to the S$p\bar{p}$S colliders, see Ref. [7]. Later on, further data from CDF at the Tevatron at higher energies up to 1800 GeV [21] and from RHIC became available, which required a fresh look at this problem.

The inclusive spectra were measured down to the transverse momenta of $0.1 \div 0.4$ GeV. Some experimental groups performed fits using certain simple parametrizations of the spectra from which the quantity I_0 can be easily determined. The British Scandinavian Collaboration (BS) [22] fitted the invariant cross sections at different energies between $\sqrt{s} = 23$ to 63 GeV as

$$E\frac{d\sigma}{d^3p} = A\exp\left(Bp_T + Cp_T^2 + Dy^2\right), \quad (7)$$

while the UA1 [23] and CDF [21] groups used the parametrization

$$E\frac{d\sigma}{d^3p} = A(1 + p_T/p_0)^{-n}. \quad (8)$$

¿From these fits the quantity (1) can be found as $I_0 = A/\sigma_{in}$. The BS collaboration [22] confirmed the exponential behaviour down to the pion momenta as low as $p_T \sim 0.1$ GeV.

Correspondingly, we derived the quantity I_0 from the STAR data [24] using the exponential extrapolation of their charged particle spectra. To obtain the cross section, σ_{in}, we take the total and elastic cross section data for the pp (BS) and $p\bar{p}$ scattering (UA1, CDF), as collected by the Particle Data Group [25]. The results are shown in Table 1 after an interpolation. In order to determine the quantity I_0 for the STAR entry in this Table, we renormalized their minimum bias data to the σ_{in} as well.

The data for I_0 in the last column of Table 1, shown in Fig. 1, indicate an initial rise in the BS data with increasing energy, but it does not hold at higher energies. Remarkably, the data

[6] We are very grateful to Torbjorn Sjostrand for the discussion of this and other issues related to results in Pythia model.

Table 1: Soft limit $I_0 = A/\sigma_{in}$ from the exponential fits to the single charged-particle $[(h^+ + h^-)/2]$ spectra in the pp ($p\bar{p}$) collisions; for the BS and STAR data we sum over the $\pi^{\pm}, K^{\pm}, p^{\pm}$ distributions.

Exp	$\sqrt{s}$ [GeV]	$p_{T,min}$ [GeV]	A [mb/GeV^2]	σ_{tot} [mb]	σ_{el} [mb]	σ_{in} [mb]	$I_0 = A/\sigma_{in}$ [GeV$^{-2)}$]
BS	23	0.1	191 ± 7	39.4	6.8	32.6 ± 0.5	5.9 ± 0.3
BS	45	0.3	238 ± 7	41.9	7.5	34.4 ± 0.7	6.9 ± 0.3
BS	63	0.1	307 ± 20	43.0	7.8	35.2 ± 0.6	8.7 ± 0.7
STAR	200	0.2					7.5 ± 0.8
UA1	200	0.25	286 ± 17	52	9.2	43 ± 4	6.6 ± 0.7
UA1	500	0.25	408 ± 24	62	13	49 ± 2	8.3 ± 0.6
CDF	630	0.4	300 ± 20	63	13	50 ± 2	6.0 ± 0.5
UA1	900	0.25	382 ± 20	68	15	53 ± 4	7.2 ± 0.7
CDF	1800	0.4	450 ± 10	74	17	57 ± 3	7.9 ± 0.5

in the large energy range of $40 \div 1800$ GeV show a rather energy-independent behaviour, and they fluctuate around some common mean value by about $\pm 15\%$:

$$\text{inelastic } pp/p\bar{p} \text{ collisions} : I_0 \approx (7 \pm 1)\ \text{GeV}^{-2} \tag{9}$$

and for minimum bias events with the above estimate for diffractive cross sections

$$\text{non} - \text{diffractive } pp/p\bar{p} \text{ collisions} : I_0 \approx (8 \pm 1)\ \text{GeV}^{-2} \tag{10}$$

As discussed below, the data from the nuclear collisions at very low $p_T \sim 0.03$ GeV are not described by the exponential behaviour, but rather follow the thermal model distribution (see Eq. (17)). We checked that both functional forms allow for the description with a comparable quality of the STAR [24] and BS [22] pp data at $p_T < 0.5$ GeV, but the thermal formula fit leads to the value of I_0, which is 25% lower, namely $I_0 \approx (6 \pm 1)$ GeV^{-2}, instead of (10).

This observation of energy independence is similar to that in e^+e^- annihilation, discussed above, and it is consistent with the expectation corresponding to a single coherent bremsstrahlung process. On the other hand, for an incoherent superposition of many processes one would rather expect I_0 values, which rise with energy increasing since the number of such processes would typically increase.

Finally, we comment on the first publication of charged particle p_T spectra in non-diffractive minimum bias events from the LHC at $\sqrt{s} = 0.9$ and 2.36 TeV by the CMS collaboration [26]. One can observe the convergence of the spectra for $p_T \to 0$. An exponential extrapolation of the data to this limit yields $I_0 \approx 9$ GeV^{-2}, which is consistent with (10). Further studies at higher energies at LHC will provide a critical test of the contributions of additional incoherent multiple interaction components.

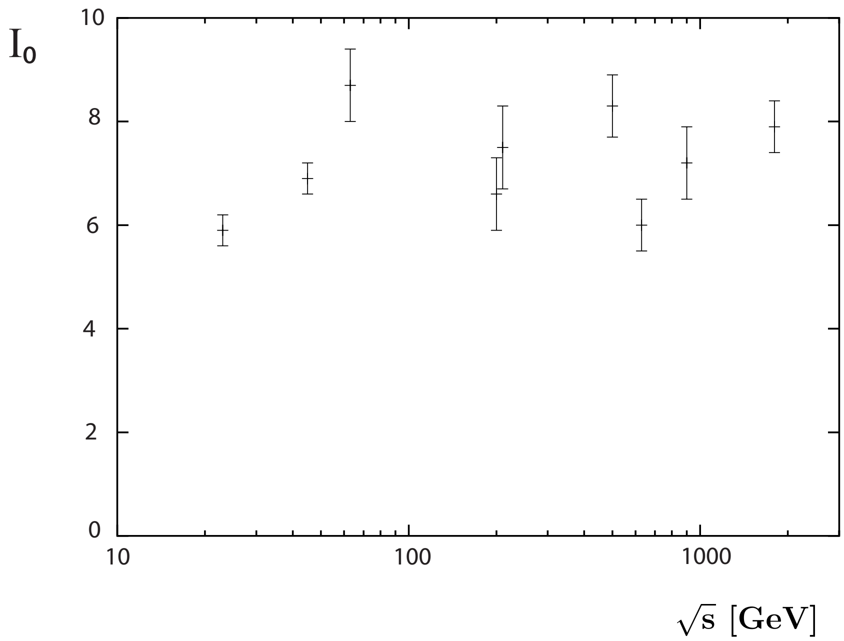

Figure 1: Soft limit I_0 of the invariant density $E\frac{dn}{d^3p}$ of charged particles $[(h^+ + h^-)/2]$ in pp collisions as a function of c.m.s. energy $\sqrt{s}$.

3.3 Comparison with the e^+e^- data

It is also instructive to compare the observation (10) with the corresponding quantity for the e^+e^- annihilation. As we already discussed, since in the pp collisions the diffractive events lead only to a small contribution to the central rapidity region, the diffractive piece should be subtracted in this comparison. We estimate the limiting value $I_0^{e^+e^-}$ in three different ways:

1. The TPC/2γ collaboration [27] compared the results for inclusive $\pi/K/p$ particle spectra, measured in the e^+e^- annihilation at 29 GeV, with the corresponding spectra from the non-diffractive pp collisions at 53 GeV, obtained by the BS collaboration [22]. The higher pp collision energy was chosen in order to take into account the lower effective energies of the parton-parton collisions.

The spectra measured in both reactions have comparable magnitude in the transverse momentum range $0.25 < p_T < 0.5$ GeV, where the rapidity y and p_T in e^+e^- annihilation events are defined with respect to the sphericity axis. As discussed above, the pp data [22] follow the exponential behaviour for small $p_T < 1$ GeV down to $p_T \sim 0.1$ GeV, and one can extrapolate the data for pions (π^+ or π^- reproduced in [27])[7] to $I_0 \approx 8$ GeV^{-2} in agreement with (10). On the other hand, the p_T-spectra in e^+e^- annihilation [27] deviate from the exponential behaviour, decreasing more weakly for higher p_T and flattening towards the small p_T-values with an extrapolated limit $I_0 \approx (3 \pm 0.3)$ GeV^{-2}. This way we arrive for the pions at the ratio

[7] The data plotted in Fig. 2 of [27] should be divided by 2π in order to obtain the quantity $\frac{E}{\sigma}\frac{d\sigma}{d^3p}$.

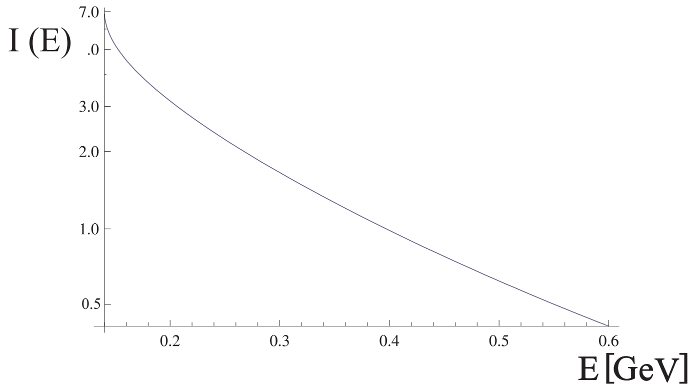

Figure 2: The density $I(E) = \frac{E}{4\pi p^2}\frac{dn}{dp}$ of pions $[(\pi^+ + \pi^-)/2]$ as a function of the *c.m.s.* pion energy E, as derived from the fit to the BS pp data and normalized to the non-diffractive cross section at 53 GeV.

$I_0^{pp}/I_0^{e^+e^-} \approx 2.7$. For the kaons an exponential extrapolation appears to be applicable for both processes at $p_T < 1$ GeV, but with different slopes, and the corresponding values are $I_0 \approx 0.48$ GeV^{-2} and $I_0 \approx 0.24$ GeV^{-2} so that $I_0^{pp}/I_0^{e^+e^-} \approx 2$ in the case of kaons.

2. Since no other group presented their results in terms of the y, p_T variables in the e^+e^- experiments, we consider here also the extrapolation in the standard variables, momentum p or the energy E, which are directly measurable without any reference to a jet axis. Then in the e^+e^- collisions one considers the distribution $\frac{E}{4\pi p^2}\frac{dn}{dp}$. Such spectrum can be also determined in the pp collisions, and we calculate its shape using the parametrisation for the invariant spectrum of the pions $[(\pi^+ + \pi^-)/2]$ at 53 GeV $E\frac{d\sigma}{d^3p} = \frac{1}{\pi}\frac{d\sigma}{dydp_T^2} \equiv I(y, p_T)\ \sigma$, as given by the BS collaboration [22], see Eq. (7), with the parameters $A = 212$ mb/GeV^2, $B = -7.3$ GeV^{-1}, $C = 1.2$ GeV^{-2} and $D = -0.13$ and with normalization by the non-diffractive cross section as in [27]. We calculate the corresponding distribution over the *c.m.s.* momentum p, using $dy = dp_z/E$ as

$$\frac{dn}{dp} = 2\int_0^p dp_z \int_0^{p^2} dp_T^2 \frac{dn}{dp_z dp_T^2}\delta(p - \sqrt{p_z^2 + p_T^2}) \tag{11}$$

$$\frac{E}{4\pi p^2}\frac{dn}{dp} = \int_0^p \frac{dp_z}{p} I(y(p, p_z), p_T(p, p_z)) \tag{12}$$

where $y = \frac{1}{2}\ln\frac{E(p)+p_z}{E(p)-p_z}$ and $p_T = \sqrt{p^2 - p_z^2}$. This spectrum is shown in Fig. 2 as a function of *c.m.s.* pion energy E. One can see the peak structure at low energy E [8]. Pion distributions over the *c.m.s.* energy, E, from the e^+e^- annihilation are collected in Fig. 3a of [5]. The available data, especially ARGUS results [28] at $\sqrt{s} \sim 10$ GeV, extending towards the small momenta $p \sim 0.05$ GeV, do not show this kind of peak. They rather show a flatter exponential distribution over E with the limit $I_0 \approx 3$ GeV^{-2} for $E \to m_\pi$, about half the pp value. [9] The

[8] The Gaussian parametrization of the distribution over the rapidity y seems to be a bit low at $y > 1$ (see Fig. 24 of [22]), but this affects the spectrum shown in Fig. 2 by less than $\sim 5\%$.

[9] The data plotted in [5] refer to $[\pi^+ + \pi^-]$, and we have to multiply the results by a factor 1/2.

e^+e^- data at higher energies E are rising with $\sqrt{s}$ and will cross eventually the spectrum for pp collisions in Fig. 2. Note that the dependence on the jet-axis definition disappears for $p \to 0$.

3. The TASSO collaboration [29] has presented fits to the invariant spectrum with exponential form in E

$$\frac{E}{4\pi p^2}\frac{d\sigma}{dp} = \sum_m A_m \exp(-B_m E), \tag{13}$$

with 2 or 3 terms, which we can use to obtain I_0 from the limit $E \to m_\pi$. For normalization we take the total cross section as $\sigma_{tot} = \sigma_{\mu\mu} R$ with $R \approx 4$ representing the data collection in [25] in this energy region.

Results based on the TASSO fits and our estimates from the experiments at lower e^+e^- energies with small momentum cut-off are summarized in Table 2. They show the consistent values for I_0. At higher energies no fits allowing to perform the extrapolation are available. However, as already noted in Sect. 2, the energy dependence at $p = 0.2$ GeV is weak over the energy range $\sqrt{s} \approx 3 \div 160$ GeV, and the deviations from the model incorporating QCD coherence are below $\sim$15%. Therefore, we evaluate I_0 from Table 2 as

$$e^+e^- \text{ annihilation} : I_0^{e^+e^-} \approx (3.3 \pm 0.5)\ \text{GeV}^{-2}. \tag{14}$$

The error in this result should include the uncertainty caused by the extrapolation (13) over the energy E, which also allows to fit the data at very low momenta $p \sim 0.05$ GeV.

Table 2: Soft limit I_0 in the e^+e^- collisions of the fits (13) (leading term A_1, B_1) from TASSO [29] to the single pion energy spectra $[(\pi^+ + \pi^-)/2]$ and the estimates, found using data from ARGUS [28] and TPC/2γ [27].

Exp	$\sqrt{s}$ [GeV]	p_{min} [GeV]	A_1 [nb/GeV^2]	B_1 [GeV^{-1}]	σ_{tot} [nb]	I_0 [GeV$^{-2)}$]
ARGUS	10	$p > 0.05$				3.0 ± 0.5
TASSO	14	$p > 0.3$	23.9 ± 3.9	5.25 ± 0.43	1.77	3.5 ± 0.6
TASSO	22	$p > 0.3$	8.0 ± 1.2	4.70 ± 0.32	0.72	3.4 ± 0.5
TASSO	34	$p > 0.3$	3.7 ± 0.6	4.97 ± 0.45	0.30	3.4 ± 0.5
TPC/2γ	29	$p_T \sim 0.05$				3.0 ± 0.3

It is also interesting to compare with DIS results in the current fragmentation region in the Breit frame, which should correspond to the quark fragmentation similar to e^+e^- annihilation. The H1 Collaboration at HERA [30] performed a study of this type at Q^2 varying from 12 to 100 GeV^2. Indeed, they found that the particle density at low momenta is nearly constant. The soft limit is obtained by extrapolation of the data in energy towards $E = Q_0 = 270$ MeV, which is used as an effective mass of charged particles. Extrapolating the data points at the lowest energy above $E = 0.3$ GeV to $E = Q_0$ at $< Q >= 19.6$ GeV we arrive at $I_0 \approx 4 \pm 0.5$

GeV^{-2}, which is compatible with Eq. (14). It would be interesting to analyse the recent more detailed data from ZEUS [31] in a similar way.

Finally, we can compare with the pp collision data, using the result (10) for the non-diffractive events. Then we obtain

$$I_0^{pp}/I_0^{e^+e^-} \approx (1.8 \pm 0.4) \div (2.4 \pm 0.5), \tag{15}$$

where the two numbers correspond to the thermal or exponential parametrization respectively.

This agrees well with the expectation from the different primary quark and gluon sources in (6). The approximate energy independence of the quantity I_0 in both collision processes, as well as the obtained ratio of the I_0 values in the pp and e^+e^- reactions, which is about 2, are remarkable. This is a serious argument in favour of the relevance of the elementary bremsstrahlung process, also for the soft pp interactions.

4 Nucleus-Nucleus interaction

4.1 Spectra at low transverse momenta

In the case of AA scattering one would naively expect, that the soft particle density is equal to that in the pp-collisions times the mean number of nucleon-nucleon collisions, N_{coll}. Such an estimate should be valid for the point-like interactions, for example, for the events with the large p_T particle production, if we neglect the energy losses in the nuclear medium. However, at low momentum transfer, p_T, the coherence effects should reduce the particle production, in particular, due to the destructive interference between the different amplitudes within the space domain of the size of $\sim 1/p_T$. Thus, we can expect the limiting behaviour when the soft particle wave-length, $1/p_T$, becomes comparable with the coherence range, at most with the nuclear radius r_A (for the central AA-collisions) or with the size of the region, where two nuclei overlap, that is, for instance, for the $AuAu$ collisions $p_T < 1/r_A \sim 30$ MeV. Inspection of the RHIC data [32] - [35] shows, that, indeed, the ratio

$$R_{AB}^{N_{coll}} = \frac{1}{N_{coll}} \frac{dN_{AB}/dp_T}{dN_{pp}/dp_T}, \tag{16}$$

decreases with decreasing transverse momentum p_T of the secondaries. The ratio $R_{AB}^{N_{coll}}$ (16) allows to compare the particle density for the nucleus-nucleus collisions with that in the proton-proton interactions times the mean number of nucleon-nucleon collisions N_{coll}, which is calculated using the Glauber model. Another way used to present the RHIC data is to replace the number of collisions N_{coll} by the number of nucleons participating in the interaction, N_{part}. Since in the pp-collisions just two protons scatter, the normalization factor is $N_{part}/2$ rather than N_{coll}.

Similar to the limiting distributions, corresponding to the soft bremsstrahlung in the e^+e^- and pp processes, we, first of all, expect the energy-independent behaviour of the soft particle densities. The existing data confirm, that at low transverse momenta, $p_T < 0.3$ GeV, the energy dependence of the ratios R_{AB} is, indeed, quite flat. At so low p_T's the ratios R_{AB}, measured by PHOBOS [35] at $\sqrt{s} = 62.4$ and 200 GeV, largely coincide for all studied centrality ranges (see Fig. 8 and Fig. 32 in [35] for the normalizations with N_{coll} and N_{part} respectively). At lower energies $\sqrt{s_{NN}} = 5$ and 17 GeV the results were obtained by PHENIX [33] (see Fig. 47), and a comparison is possible for $p_T > 0.4$ GeV. A convergence of the p_T spectra towards low p_T at the two energies can be seen, although not yet a coincidence in the observed p_T-range.

Let us now turn to the limiting density of the soft particle production. The data at the lowest $p_T \approx 0.03$ GeV were collected by PHOBOS [35] (Fig. 3), the data with $p_T > 0.2$ GeV were obtained by PHENIX [33] (Fig. 11) and by STAR [24] (Fig. 46, Tab. 33) in the Au+Au collisions at 200 GeV. If we extrapolate the invariant pion spectra from PHENIX and STAR by the exponential form at $p_T < 0.5$ GeV we arrive at $I_0^{AuAu} \approx 1270$ GeV^{-2} to be compared with $I_0^{pp} \approx 7.8$ GeV^{-2}, found with the same procedure for the pp collisions [24]. However, the PHOBOS data at very low p_T indicate a flattening of the distribution according to a functional form following from a thermal model

$$E\frac{dn}{d^3p} = \frac{A}{\exp(m_T/T) - 1} \tag{17}$$

with $m_T = \sqrt{m^2 + p_T^2}$ and $T = 0.229$ GeV.

Fitting the PHOBOS and STAR $AuAu$ data for pions at $p_T < 0.5$ GeV using this parametrization, we obtain for the mean pion density at $p_T = 0$ in the average

$$\text{central } AuAu \text{ collisions (pions)}: \qquad I_0 \simeq (950 \pm 100)\ \text{GeV}^{-2}, \tag{18}$$

which is about 30% lower than the result of exponential parametrization. Assuming a similar fit for the low p_T pp STAR data, we arrive at $I_0^{pp} \simeq 5.9$ GeV^{-2} with $T = 0.182$ GeV, therefore, the limiting density for the nuclear collisions is more than 100 times larger than the corresponding value in the pp interactions. On the other hand, in this case the Glauber model calculation [35] gives $N_{coll} = 1040$ and $N_{part}/2 = 172$ ($\pm 15\%$). Clearly, the soft particle density is about 10 times lower than that expected in the case of independent (incoherent) collisions. Indeed, from the extrapolation of the p_T spectra in the $AuAu$ and pp collisions by either an exponential or the thermal function (17), as presented above, we find the ratio

$$I_0^{AA}/I_0^{pp} \approx 160 \pm 17, \tag{19}$$

which agrees with the calculated $N_{part}/2$, and, therefore,

$$p_T \to 0: \qquad R_{AA}^{N_{part}} \to 1 \quad \text{and} \quad I_0^{AuAu} \approx \frac{N_{part}}{2}\, I_0^{pp}. \tag{20}$$

Here $R_{AA}^{N_{part}}$ is defined as in (16), but with $N_{part}/2$ as a normalizing quantity.

A detail experimental study was performed by PHOBOS [35] (Fig. 32) for different ranges of centrality, that is, the number of participants, at the two energies 62.4 GeV and 200 GeV. Remarkably, the ratio $R_{AA}^{N_{part}}$ approaches about unity in the soft limit for all selections of centralities or the N_{part} parameter. STAR [37] measured the ratio R_{AA}^{coll} with high precision for $p_T > 0.5$ GeV. It is found that this quantity decreases below $p_T < 2$ GeV and the extrapolation below the measured values of p_T suggests the limiting behaviour (20) as well.

That is, the data are consistent with the model where each pair of participating nucleons produces its own number of secondaries independently of the *number of collisions* each nucleon is participated in. This is related to the idea, that the overall characteristics of bulk particle production depend only on the number of 'wounded nucleons' [36] and not on the number of rescatterings.

4.2 Antenna pattern in nuclear collisions

Within the bremsstrahlung scenario this result suggests that there is a coherent particle production over the range of nucleon size, but different nucleons are separated in space-time so that their contributions remain incoherent [10]. This is in agreement with the observation that no specific structure reveals itself in the nuclear collisions at very small $p_T \sim 30$ MeV corresponding to the nuclear size (see, for example, Fig. 3 in [35] with pions in this range).

In spite of the multiple re-interactions of the same nucleon in the nucleus which proceed through the corresponding number of gluon exchanges in the minimal model for the pp collisions considered above, we still expect the limiting behaviour. The low p_T particles emitted in this process interact coherently with all t-channel gluons and, actually, probe the overall colour flow originated by this (t-channel) system of gluons. On the other hand, as we discussed in Sect. 3.1, even a large number of gluons produces dominantly the same octet colour flow as in the pp inelastic collision (first, the octet exchange gives the dominant Leading Logarithmic contribution, both in the DGLAP and in the BFKL kinematics, next - interacting with the individual quark or with the colourless $q\bar{q}$- or 'quark-diquark'-system we can transfer only the colour octet quantum number). Examples of diagrams contributing in this minimal model to the production of the low p_T particles in the pp and pA collisions are shown in Fig. 3. In the AA collisions re-interactions of nucleons may occur as in the pA collisions. In these depicted processes the colour flow and the associated soft particle production are determined by the number of participating nucleons irrespectively of the number of rescatterings, in agreement with the phenomenological result (20).

4.3 Universality of particle ratios at low p_T

Having in mind this universal mechanism for soft particle production we may ask whether there are any consequences for the relative rates of different hadron yields in the same limit. If the

[10] The coherence is destroyed by the presence of different recoil nucleons.

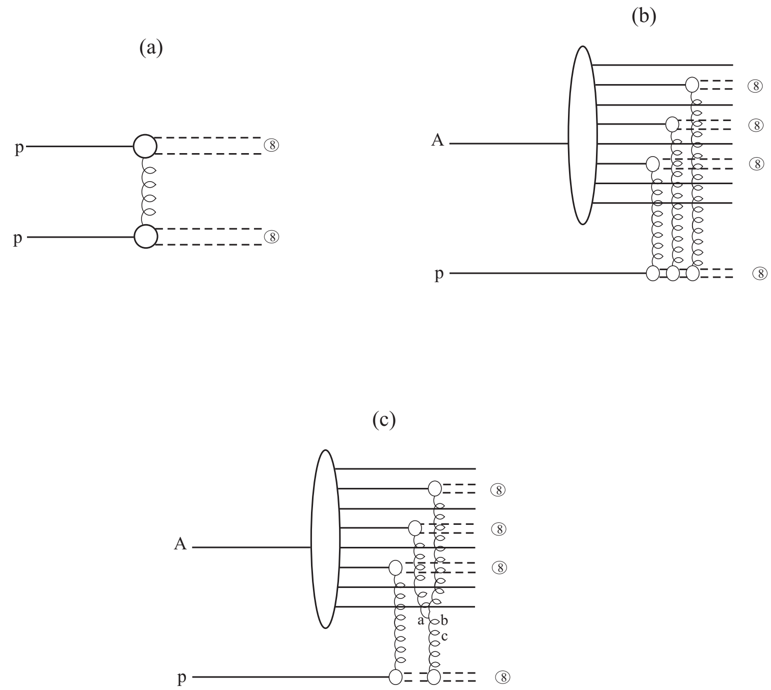

Figure 3: Diagrams, contributing to the pp and pA collisions in the minimal model for soft particle production: (a) In the pp collisions the exchanged gluon interacts with the colour triplet constituents q or qq in the proton to form an outgoing colour octet system; (b,c) In the pA collisions the proton can rescatter inside the nucleus and then forms a colour octet system again. This implies that the multiple gluon exchange acts as a single gluon exchange in the particle production. In this example: $N_{coll} = 3,\ N_{part}/2 = 2$.

particles at low momenta in the hadronic and nuclear collisions are related to the universal bremsstrahlung from the (incoherent) superposition of primary colour octet charges, then, in this limit, the particle composition should be the same in the different processes. To the extent, that the dominant underlying mechanism in the pp collisions is the gluon exchange between the quark constituents of the proton with the initial and final bremsstrahlung, the e^+e^- and hadronic data on the particle ratios should approach each other as well.

A similarity of particle ratios K/π and $\bar{p}/\pi$ in the e^+e^- and pp reactions at $p_T < 0.5$ GeV has been indeed noted already some time ago by the TPC collaboration [27]. In this measurement the transverse momentum p_T for the particles in e^+e^- collisions was defined with respect to the sphericity axis.

The p_T dependence of the particle ratios for several hadronic collisions have been compared by PHENIX [38]. While at the large $p_T > 2$ GeV the ratios p/π and K/π tend to approach large values ~ 1 in the central $AuAu$ collisions, these ratios are reduced for non-central and minimum bias pp collisions. Remarkably, these ratios converge for the different processes towards lower $p_T < 1$ GeV. In Fig. 4 we collect data in the low p_T region on the ratio K^-/π^- from the e^+e^-, pp and AA interactions. As one can see, the particle ratios, indeed, approach each other

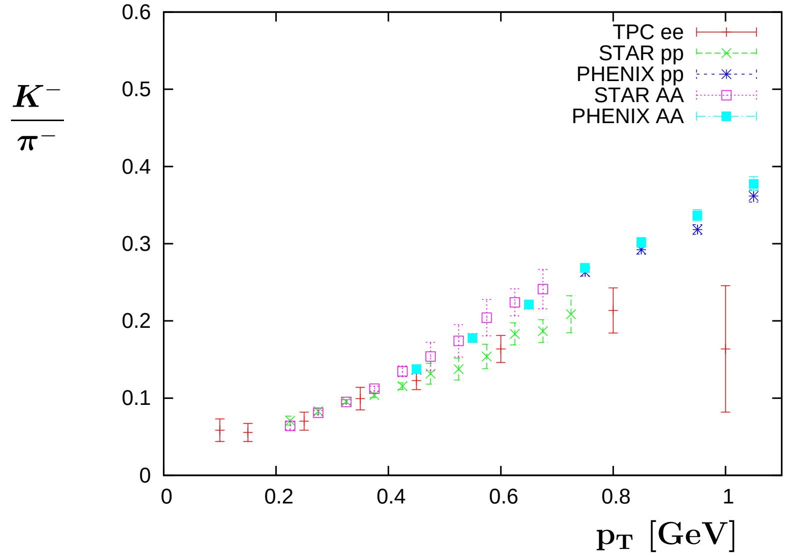

Figure 4: Convergence of particle ratios K^-/π^- towards small p_T for various processes: e^+e^- annihilation (TPC [27] data, p_T with respect to sphericity axis), pp (minimum bias) and central (0-5%) $AuAu$ collisions (STAR [24] and PHENIX [38] Collaborations).

towards low $p_T < 0.4$ GeV pointing towards a dominance of multiple $q\bar{q}$ dipole radiation in all processes.

5 Conclusions

An issue of universality of particle production in various collision processes predated QCD. The perturbative dynamics based on the QCD gluon bremsstrahlung suggests a particular type of universality, which, however, allows only for the predictions for the soft limit $p_T \to 0$. This is the case, when the lowest order Born term with its energy independence and elementary colour factors dominates, whereas for the larger p_T's the (non-universal) higher order contributions take over. The predictions from this approach are confirmed with high accuracy in the e^+e^- measurements, where the energy independence of the soft particle yield is observed, and its intensity in quark and gluon jets is in agreement with the expectation from corresponding colour factors C_F and C_A.

In this paper we discuss the production of low momentum particles in hadronic collisions. Surprisingly, the soft particle production density in the pp collisions is practically constant over the large energy range up to 1800 GeV. Moreover, the energy independence holds also in the nuclear collisions at RHIC energies. The colour factors related to the primary interaction process reveal themselves also in the comparison between the e^+e^- and pp collisions, according to a minimal model with gluon exchange in the case of hadronic processes. In the nuclear collisions the perturbative expectations, based on the coherent bremsstrahlung in a multi-gluon exchange process lead to the energy independence of the soft particle yield. Moreover, the magnitude should scale with the number of participant nucleons, N_{part}.

As a result, the yield of the low p_T particles can be directly related to the soft QCD gluon bremsstrahlung from the colour charges, which are created in the primary hard or semi-hard interaction, and is proportional to C_F, C_A and $\frac{N_{part}}{2}C_A$ for e^+e^-, pp and AA collisions respectively. This concept of universality is also supported by the observed convergence of the $\pi : K : p$ particle ratios in all these processes in the $p_T \to 0$ limit.

From the first sight, this seems to contradict the idea of thermalisation in the particle production. However, we recall, that according to the space-time picture of particle production, the central soft hadrons in the nucleon-nucleon *c.m.s.* system are those, which are formed first, and these soft particles with their large wavelength probe only the colour charges participating in the primary local interaction at a transverse size of ~ 1 f. The fast particles are formed later on, and leave the interaction region without re-scattering on the slow hadrons which stay in the interaction region. Therefore, the slow particles may be not in a thermo-equilibrium with the whole hadronic system. At the very early stages following the primary interaction no re-interactions of produced hadrons from the different nucleons will take place, while the re-interactions of one nucleon with other nucleons in the nucleus should not be accounted for because of the coherent nature of soft particle emission. Then, the universality of this soft particle production from the primary colour sources becomes a plausible phenomenon, since it basically reflects the universality of soft particle production from the individual isolated quark-antiquark dipoles.

It will be very interesting to extend such measurements on the limiting soft particle production in pp and AA collisions to the higher energies at the LHC and to see whether any new incoherent sources appear. Such measurements could set a critical benchmark for the models of multiparticle production in hadronic interactions.

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