Prediction of new charmed and bottom exotic pentaquarks

Dmitri Diakonov[1,2]

[1] *Petersburg Nuclear Physics Institute,*

Gatchina 188300, St. Petersburg, Russia

[2] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto, Japan*

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Abstract

Baryons of the type $Qqqq\bar{q}$ (where $Q = c, b$ and $q = u, d, s$ quarks) forming anti-decapenta ($\overline{15}$)-plets with spin-parity $\frac{1}{2}^+$ are predicted on simple theoretical considerations. The lightest members of these multiplets are explicitly exotic doublets $cuud\bar{s}$, $cudd\bar{s}$ with mass about 2420 MeV, and $buud\bar{s}$, $budd\bar{s}$ with mass about 5750 MeV, only 130 MeV heavier than Λ_c and Λ_b, respectively, and thus stable against strong decays. Although the production rate is probably very low, these remarkable pentaquarks can be looked for at LHC, Fermilab, B-factories, RHIC and elsewhere: their signatures are briefly discussed.

Keywords: large N_c, mean field, baryon resonances, exotic hadrons, charmed baryons, bottom baryons

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I. INTRODUCTION

In this paper, arguments are suggested in favor of the existence of exotic pentaquarks which may well prove to be the second lightest charmed (bottom) baryons, after Λ_c and Λ_b. Since they are light, the new baryons decay only weakly. They may have escaped direct observation in the past because the production rate is expected to be quite low.

The arguments are based on considering baryons at large number of colors N_c. While in the real world N_c is only three, we do not expect qualitative difference in the baryon spectrum with the large-N_c limit. The bonus is that at large N_c baryon physics simplifies considerably, which enables one to take into full account the important relativistic and field-theoretic effects that are often ignored.

The relativistic approach to baryons is key to the prediction. In implies that baryons are not just three (or N_c) quarks but contain additional quark-antiquark pairs, as it is well known experimentally. Baryon resonances may be formed not only from quark excitations as in the customary non-relativistic quark models, but also from particle-hole excitations and "Gamov–Teller" transitions. At large N_c these effects become transparent and tractable. At $N_c=3$ it is a mess called "strong interactions". The hope is that if one develops a clear picture at large N_c, its imprint will be visible at $N_c=3$.

The approach can be illustrated by the chiral quark soliton model [1] or by the chiral bag model [2] but actually the arguments of this paper are much more general. Dynamics is not considered here, which today would require adopting a model. A concrete model would say what is the "intrinsic" relativistic quark spectrum in baryons. It may get it approximately correct, or altogether wrong. Instead of calculating the intrinsic spectrum from a model, I extract it from the known baryon spectrum by interpreting baryon resonances as collective excitations about the ground state and about the one-quark and particle-hole transitions.

In Section II the key question what is the symmetry of the ground-state baryon is addressed. Arguments are presented that it is not the expected maximal possible symmetry. In particular, $SU(3)$ flavor symmetry is spontaneously broken even in the limit of zero current quark masses. The two critical consequences are: (i) the intrinsic spectrum of s quarks in a baryon is totally different from that of u, d quarks, and (ii) the observable baryon spectrum has characteristic "rotational bands" following from quantizing the rotations of the baryon as a whole in flavor and ordinary spaces.

In Sections III, IV the lowest light baryon resonances both with positive and negative parities are interpreted from the above viewpoint. As a byproduct, the light exotic pentaquark Θ^+ is theoretically confirmed at about 1520 MeV and shown to be a consequence of the existence of three well-known resonances: $N(1440, 1/2^+)$, $\Lambda(1405, 1/2^-)$ and $N(1535, 1/2^-)$.

In Sections V, VI heavy baryons are discussed using the intrinsic quark spectrum established from light baryons – at large N_c the intrinsic spectra are the same in light and heavy baryons, up to $1/N_c$ corrections. Numerical checks show that the appropriate relations between light and heavy baryons are reasonably satisfied even at $N_c = 3$.

Section VII is central as an *anti-decapenta* ($\overline{15}$)-plet of heavy exotic charmed (bottom) pentaquarks is predicted there. It is explained why the lightest members of that multiplet are relatively light and hence stable under strong decays. These pentaquarks are distinct from *anti*-charmed (bottom) pentaquarks suggested previously, that are about 500 MeV heavier.

In Sections VIII, IX certain properties of the predicted pentaquarks are discussed, in particular possibilities to observe them experimentally.

The Appendix deals with the mathematical description of the "rotational bands" about various intrinsic quark excitations.

II. MEAN FIELD IN BARYONS

Recently a classification of baryon resonances was suggested, according to what they would look like if the number of colors N_c was large [3, 4]. Long experience tells us that the large-N_c world does not differ much from the real world with $N_c = 3$, except for several very special cases, and in many circumstances the $1/N_c$ corrections are under control [5].

At large N_c, the N_c quarks constituting a baryon can be considered in a mean (non-fluctuating) mesonic field which does not change as $N_c \to \infty$. Consequently, all quark levels in the mean field are stable in N_c. All negative-energy levels should be filled in by N_c quarks in the antisymmetric state in color, corresponding to the zero baryon number state. Filling in the lowest positive-energy level makes a baryon. Exciting higher quark levels or making particle-hole excitations produces baryon resonances. The baryon mass is $\mathcal{O}(N_c)$, and the excitation energy is $\mathcal{O}(1)$. When one excites one quark the change of the mean field is $\mathcal{O}(1/N_c)$ that can be neglected to the first approximation.

The key issue is what is the symmetry of the mean field. In the chiral limit when the strange quark mass m_s is set to zero, u, d, s quarks are on the same footing, and one may think that the mean field is maximally symmetric, that is flavor symmetric and spherically symmetric or, in the mathematical language, invariant under $SU(3)_{\rm flav} \times SO(3)_{\rm space}$ rotations. Although natural, this assumption is wrong. For unknown dynamical reasons the maximal possible symmetry of the mean field in baryons is broken spontaneously down to the $SU(2)_{\rm iso+space}$ symmetry: the mean field is invariant only under simultaneous isospin and compensating space rotations.

The case is analogous to heavy nuclei: For some reasons many of the large-A nuclei, although not all, are not spherically-symmetric (as would seem natural) but have an ellipsoid form. It means that spherical symmetry, or invariance under $SO(3)_{\rm space}$ rotations, is partially spontaneously broken in the ground state. The mean field in such nuclei is not invariant under arbitrary rotation but only with respect to rotation about the symmetry axis of the ellipsoid. In principle, symmetry could be broken completely, *e.g.* a heavy nucleus could be a three-axes ellipsoid, or the mean field in baryons could have no symmetries at all. However in practice this does not happen: the symmetry is broken but not completely.

The question what is the symmetry of the mean field can be answered theoretically if full dynamics is well understood: one has to try all possible symmetry patterns and check which of them leads to the lowest energy of the ground state. It is a quantitative question. In the absence of a reliable dynamical theory one can, however, use phenomenological, circumstantial evidence in favor of this or that symmetry. For example, if symmetry is spontaneously broken one expects low-lying excitations, the (pseudo) Goldstone modes, their number being equal to the number of broken symmetry generators.

If the broken symmetry group is compact (like $SU(3)_{\rm flav}$ and $SO(3)_{\rm space}$) the energy of the Goldstone excitations is quantized. One expects then rotation bands about the ground state and about each one-particle and particle-hole $\mathcal{O}(1)$ excitation, split as $1/I$ where I is the moment of inertia. For heavy nuclei, I scales as $I \sim mr^2 \sim A^{\frac{5}{3}}$ whereas for the baryon it scales as $I \sim N_c$ (since the baryon radius does not rise with N_c). In most of heavy nuclei ($A \gg 1$) one clearly sees rotational excitations whose splitting is much less than the $\mathcal{O}(1)$ one-particle and particle-hole excitations [6]. This is a clear evidence that such nuclei are not spherically-symmetric, otherwise there would have been no rotational bands at all.

In real-world baryons there is no spectacular separation of scales since $N_c = 3$ is not

a very large number. However some $SU(3)$ multiplets have definitely smaller splitting between themselves than others. This is an indication that certain baryon multiplets can be interpreted as rotational states whereas others are one-particle or particle-hole excitations. It implies, then, that the would-be $SU(3)_{\rm flav} \times SO(3)_{\rm space}$ symmetry of the mean field is broken; the question is what is the pattern.

If the mean field is only $SU(2)_{\rm iso+space}$ invariant, the quantization of the rotations needed to restore the original $SU(3)_{\rm flav} \times SO(3)_{\rm space}$ symmetry for a ground-state baryon leads precisely to the baryon multiplets $(\mathbf{8}, \frac{1}{2}^+)$ and $(\mathbf{10}, \frac{3}{2}^+)$ observed in Nature. It is an argument in favor of this particular pattern of symmetry breaking. To specify the N_c behavior of the splitting between the centers of these multiplets, one needs to generalize them to certain prototype $SU(3)$ multiplets at arbitrary N_c, that reduce to the octet and decuplet at $N_c = 3$ [7, 8]: the splitting turns out to be $3/2I_1 = \mathcal{O}(1/N_c)$, see the Appendix. Numerically this splitting is 1382-1152=230 MeV, such that $1/I_1 = 153\,{\rm MeV}$. The number is indeed considerably less than the splitting from the center of the next nearest $(\mathbf{8}, \frac{1}{2}^+)$ multiplet involving the Roper resonance, 1630-1152=478 MeV. In the present interpretation, the first splitting is $\mathcal{O}(1/N_c)$ as due to the rotation of a baryon as a whole, whereas the second is $\mathcal{O}(1)$ and is due to a one-quark excitation in the mean field [3].

I note in passing that in the non-relativistic quark model the splitting between the lowest octet and decuplet is interpreted as due to hyperfine interaction [9]. It also behaves as $\alpha_s^2 N_c \sim 1/N_c$, however to fit the splitting numerically one needs to take $\alpha_s \approx 2$ whereas fits of deep inelastic scattering data and other phenomena tend to freeze α_s in the infrared at the value about 0.5. Such value would give a tiny hyperfine splitting, hinting that it may be irrelevant. The collective quantization interpretation is, numerically, more realistic. Indeed, an estimate of the baryon moment of inertia is $I = mr^2 \approx (1\,{\rm GeV}) \cdot (0.5\,{\rm fm})^2$ yielding $1/I \approx 160\,{\rm MeV}$ as needed.

Another argument in favor of the $SU(2)_{\rm iso+space}$ symmetry of the mean field comes from the fact that baryons are strongly coupled to the pseudoscalar mesons ($g_{\pi NN} \approx 13$). It means that there is a strong pseudoscalar field inside baryons; at large N_c it is a classical mean field. There is no way of writing down an *Ansatz* for the pseudoscalar field that would be odd with respect to space inversion and simultaneously compatible with the $SU(3)_{\rm flav} \times SO(3)_{\rm space}$ symmetry. The minimal extension of spherical symmetry is to write the "hedgehog" *Ansatz*

"marrying" the isotopic and space axes [10]:

$$\pi^a(\mathbf{x}) = \begin{cases} n^a\, F(r),\ n^a = \frac{x^a}{r}, & a = 1,2,3, \\ 0, & a = 4,5,6,7,8. \end{cases} \tag{1}$$

This *Ansatz* breaks the $SU(3)_{\rm flav}$ symmetry. If $m_s = m_u = m_d$, all flavor axes are equivalent, therefore writing the pseudoscalar field in this form means nothing but naming the $SU(3)$ axes. Analogously, the spontaneous magnetization in a ferromagnet can assume any direction, and we can *name* the direction of magnetization as, say, the z direction. If there is an external magnetic field, even infinitesimal, it sets a preferred direction such that the spontaneous magnetization will be along it. A nonzero magnetic field is analogous to the case of $m_s > m_u = m_d$ when the strange direction is privileged. However, the *Ansatz* (1) implies a spontaneous (as contrasted to explicit) symmetry breaking, since s quarks are treated in a totally different way than the u, d ones, even if m_s differs infinitesimally from $m_u = m_d$.

Moreover, the *Ansatz* (1) breaks the symmetry under independent space $SO(3)_{\rm space}$ and isospin $SU(2)_{\rm iso}$ rotations, and only a simultaneous rotation in both spaces leaves (1) invariant. Therefore, the *Ansatz* (1) breaks spontaneously the original $SU(3)_{\rm flav} \times SO(3)_{\rm space}$ symmetry down to the $SU(2)_{\rm iso+space}$ symmetry. This is precisely what is needed to obtain the correct baryon spectrum, where some excitations are large ($\mathcal{O}(1)$) and some are small ($\mathcal{O}(1/N_c)$). We note that the splittings *inside* $SU(3)$ multiplets can be determined as a perturbation in m_s [13].

The full list of other possible mesonic fields in baryons (scalar, vector, axial, tensor), compatible with the $SU(2)_{\rm iso+space}$ symmetry is given in Ref. [4].

III. BARYONS MADE OF u, d, s QUARKS

Given the $SU(2)_{\rm iso+space}$ symmetry of the mean field, the Dirac Hamiltonian for quarks actually splits into two: one for s quarks and the other for u, d quarks. It should be stressed that the energy levels for u, d quarks on the one hand and for s quarks on the other are completely different, even in the chiral limit $m_s \to 0$.

The energy levels for s quarks are classified by half-integer J^P where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ is the angular momentum, and are $(2J+1)$-fold degenerate. The energy levels for u, d quarks

are classified by integer K^P where $\mathbf{K} = \mathbf{T} + \mathbf{J}$ is the 'grand spin' (T is isospin), and are $(2K+1)$-fold degenerate.

All energy levels, both positive and negative, are probably discrete owing to confinement. Indeed, a continuous spectrum would correspond to a situation when quarks are free at large distances from the center, which contradicts confinement. One can model confinement *e.g.* by forcing the effective quark masses to grow linearly at infinity.

According to the Dirac theory, all *negative*-energy levels, both for s and u, d quarks, have to be fully occupied, corresponding to the vacuum. It means that there must be exactly N_c quarks antisymmetric in color occupying all degenerate levels with J_3 from $-J$ to J, or K_3 from $-K$ to K; they form closed shells. Filling in the lowest level with $E > 0$ by N_c quarks makes a baryon [1, 3], see Fig. 1. A similar picture arises in the chiral bag model [2].

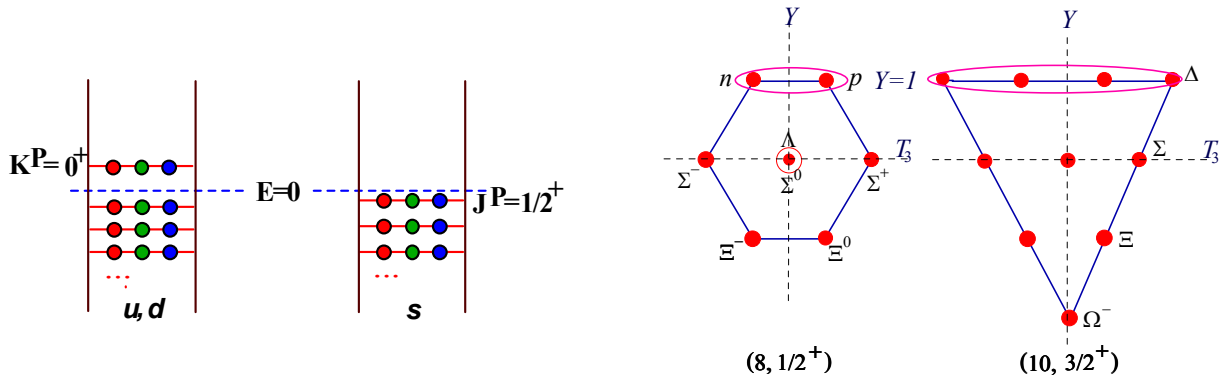

FIG. 1: Filling u, d and s shells for the ground-state baryon (left), and the two lowest baryon multiplets that follow from quantizing the rotations of this filling scheme (right).

The mass of a baryon is the aggregate energy of all filled states, and being a functional of the mesonic field, it is proportional to N_c since all quark levels are degenerate in color. Therefore quantum fluctuations of mesonic field in baryons are suppressed as $1/N_c$ so that the mean field is indeed justified.

Quantum numbers of the lightest baryons are determined from the quantization of the rotations of the mean field, leading to specific $SU(3)$ multiplets that reduce at $N_c{=}3$ to the octet with spin $\frac{1}{2}$ and the decuplet with spin $\frac{3}{2}$, see *e.g.* [14] and the Appendix. Witten's quantization condition $Y' = \frac{N_c}{3}$ [15] follows trivially from the fact that there are N_c u, d valence quarks each with the hypercharge $\frac{1}{3}$ [13]. Therefore, the ground state shown in Fig. 1 entails in fact 56 rotational states. The splitting between the centers of the multiplets

$(\mathbf{8}, \frac{1}{2}^+)$ and $(\mathbf{10}, \frac{3}{2}^+)$ is $\mathcal{O}(1/N_c)$, and the splittings inside multiplets can be determined as a perturbation in m_s [13].

The lowest baryon resonance beyond the rotational excitations of the ground state is the singlet $\Lambda(1405, \frac{1}{2}^-)$. Apparently, it can be obtained only as an excitation of the s quark, and its quantum numbers must be $J^P = \frac{1}{2}^-$ [3], see transition *1* in Fig. 2.

The existence of an $\frac{1}{2}^-$ level for s quarks automatically implies that there is a particle-hole excitation of this level by an s quark from the $\frac{1}{2}^+$ level. I identify this transition *2* with $N(1535, \frac{1}{2}^-)$ [3]. At $N_c = 3$ it is predominantly a pentaquark state $u(d)uds\bar{s}$ (although it has also a nonzero three-quark Fock component). This explains its large branching ratio in the ηN decay [16], a long-time mystery. We also see that, since the highest filled level for s quarks is lower than the highest filled level for u, d quarks, $N(1535, \frac{1}{2}^-)$ must be *heavier* than $\Lambda(1405, \frac{1}{2}^-)$: the opposite prediction of the non-relativistic quark model has been always of some concern. Subtracting $1535 - 1405 = 130$, I find that the $\frac{1}{2}^+$ s-quark level is approximately 130 MeV lower in energy than the valence 0^+ level for u, d quarks. This is an important number which will be used below. The transition entails its own rotational band discussed in the Appendix.

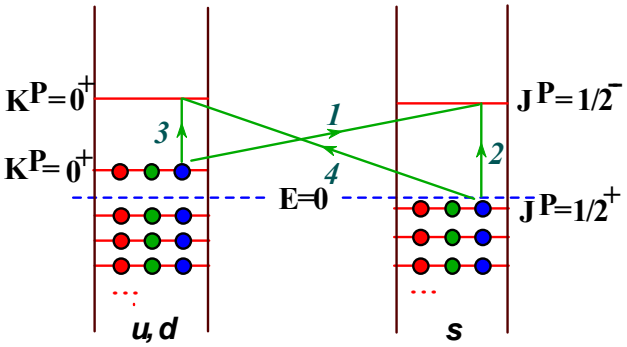

FIG. 2: The existence of the two lowest excited levels – one for the u, d quarks and another for the s quarks – implies four resonances shown by arrows. The transitions correspond to: *1*: $\Lambda(1405, 1/2^-)$, *2*: $N(1535, 1/2^-)$, *3*: $N(1440, 1/2^+)$, *4*: $\Theta^+(1530, 1/2^+)$. Each transition generally entails its own rotational band of $SU(3)$ multiplets.

The low-lying Roper resonance $N(1440, \frac{1}{2}^+)$ requires an excited one-particle u, d state with $K^P = 0^+$ (or 1^+) [3], see transition *3*. Just as the ground state nucleon, it is part of the excited $(\mathbf{8}', \frac{1}{2}^+)$ and $(\mathbf{10}', \frac{3}{2}^+)$ split as $1/N_c$. Such identification of the Roper resonance solves another problem of the non-relativistic model where $N(1440, \frac{1}{2}^+)$ must be heavier

than $N(1535, \frac{1}{2}^-)$. In our approach they are unrelated.

Given that there is an excited 0^+ level for u, d quarks, one can put there a quark taking it as well from the s-quark $\frac{1}{2}^+$ shell, see transition *4*. It is a particle-hole excitation with the valence u, d level left untouched, its quantum numbers being $S = +1,\ T = 0,\ J^P = \frac{1}{2}^+$. At $N_c = 3$ it is a pentaquark state $uudd\bar{s}$, precisely the exotic Θ^+ baryon predicted in Ref. [17] from related but somewhat different considerations. The quantization of its rotations produces the antidecuplet $(\overline{\mathbf{10}}, \frac{1}{2}^+)$. In our original prediction the $\mathcal{O}(1)$ gap between Θ^+ and the nucleon was due to the rotational energy only, whereas here the main $\mathcal{O}(1)$ part of that gap is due to the one-particle levels, while the rotational energy is $\mathcal{O}(1/N_c)$, see the Appendix. Methodologically, it is now more satisfactory.

In nuclear physics, excitations generated by the axial current $j^{\pm}_{\mu 5}$, when a neutron from the last occupied shell is sent to an unoccupied proton level or *v.v.* are known as Gamov–Teller transitions [6]. Thus our interpretation of the Θ^+ is that it is a Gamov–Teller-type resonance long known in nuclear physics.

An unambiguous feature of our picture is that **the exotic pentaquark Θ^+ is a consequence of the existence of three well-known resonances and must be light.** Indeed, the Θ^+ mass can be estimated from the apparent sum rule following from Fig. 2 [3]: $m_\Theta \approx 1440 + 1535 - 1405 \approx 1570\,\mathrm{MeV}$. Since the $N(1440)$ and $N(1535)$ resonances are broad such that their masses are not well defined, there is a numerical uncertainty in this equation. For example, if one uses the pole positions of the resonances the equation reads $m_\Theta \approx 1365 + 1510 - 1405 \approx 1470\,\mathrm{MeV}$. Therefore, it is fair to say that the sum rule predicts $m_\Theta = 1520 \pm 50\,\mathrm{MeV}$. This is in remarkable agreement with the claimed masses of the Θ^+: $m_\Theta = 1524 \pm 2 \pm 3\,\mathrm{MeV}$ [18], $1537 \pm 2\,\mathrm{MeV}$ [19], $1523 \pm 2 \pm 3\,\mathrm{MeV}$ [20], $1521.5 \pm 1.5 \pm 2.8/1.7\,\mathrm{MeV}$ [21], $1528 \pm 2.6 \pm 2.1\,\mathrm{MeV}$ [22]. For a possible explanation why Θ^+ is seen in some experiments while not observed in other see Ref. [23].

To account for higher baryon resonances one has to assume that there are higher one-particle levels, both in the u, d- and s-quark sectors, to be published elsewhere [4].

IV. BARYON RESONANCES FROM ROTATIONAL BANDS

A filling scheme of one-particle quark levels by itself does not tell us what are the quantum numbers of the state. The filling scheme treats u, d quarks and s quarks differently and

therefore violates the $SU(3)_{\rm flav}$ and also $SO(3)_{\rm space}$ symmetries. Only the $SU(2)_{\rm iso+space}$ symmetry of simultaneous isospin and compensating space rotations is preserved. In the chiral limit (which I assume for the time being) an arbitrary $SU(3)_{\rm flav}$ rotation of the mean field and hence of what we call u, d, s quarks does not change the energy of the state. The same is true for the $SO(3)_{\rm space}$ rotation. However, if $SU(3)_{\rm flav}$ and $SO(3)_{\rm space}$ rotations are slowly dependent on time, they generate a shift in the energy of the system; it is called the rotational energy. Being quantized according to the general quantization rules for rotations, it produces states with definite $SU(3)_{\rm flav}$ quantum numbers and spin.

Thus the original $SU(3)_{\rm flav} \times SO(3)_{\rm space}$ symmetry broken spontaneously by a 'hedgehog' *Ansatz* of the mean field, is restored when flavor and space rotations are accounted for. Each transition in Fig. 2 generally entails "rotational bands" of $SU(3)$ multiplets with definite spin and parity. The short recipe of getting them is: Find the hypercharge Y' of the given excitation from the number of u, d, s quarks involved; only those multiplets are allowed that contain this Y'. Take an allowed multiplet and read off the isospin(s) T' of particles at this value of Y'. The allowed spin of the multiplet obeys the angular momentum addition law:

$$\mathbf{J} = \mathbf{T}' + \mathbf{J_1} + \mathbf{J_2} \tag{2}$$

where $J_{1,2}$ are the initial and final momenta of the s shells involved in the transition. (If nonzero K shell is involved in the transition the quantization rule is more complex.) The mass of the center of an allowed rotational multiplet does not depend on $\mathbf{J}$ but only on $\mathbf{T}'$ according to the relation [8]

$$\mathcal{M} = \mathcal{M}_0 + \frac{C_2(p,q) - T'(T'+1) - \frac{3}{4}Y'^2}{2I_2} + \frac{T'(T'+1)}{2I_1} \tag{3}$$

where $C_2(p,q) = \frac{1}{3}(p^2+q^2+pq) + p + q$ is the quadratic Casimir eigenvalue of the $SU(3)$ multiplet characterized by (p,q), $I_{1,2} = \mathcal{O}(N_c)$ are moments of inertia. After the rotational band for a given transition is constructed, one has to check if the rotational energy of a particular multiplet is $\mathcal{O}(1/N_c)$ and not $\mathcal{O}(1)$, and if it is compatible with Fermi statistics at $N_c{=}3$: some *a priori* possible multiplets drop out. One gets a satisfactory description of all light baryon resonances up to about 2 GeV, to be published separately [4].

V. CHARMED AND BOTTOM BARYONS, THE LOWEST MULTIPLETS

If one of the light quarks in a light baryon is replaced by a heavy b or c quark, there are still N_c-1 light quarks left. At large N_c, they form *the same* mean field as in light baryons, with the same sequence of Dirac levels, up to $1/N_c$ corrections. The heavy quark contributes to the mean $SU(3)_{\rm flav}$ symmetric field but it is a $1/N_c$ correction, too. It means that at large N_c one can *predict* the spectrum of the $Qq\ldots q$ (and $Qq\ldots qq\bar{q}$) baryons from the spectrum of light baryons. At $N_c=3$ one does not expect qualitative difference with the $N_c\to\infty$ limit, although $1/N_c$ corrections should be kept in mind. I consider the heavy quark as a non-relativistic particle having spin $J_h=\frac{1}{2}$. $SU(4)_{\rm flav}$ symmetry is badly violated and is of no guidance.

The filling of Dirac levels for the ground-state c (or b) baryon is shown in Fig. 3, left: there is a hole in the 0^+ shell for u,d quarks as there are only N_c-1 quarks there, in an antisymmetric state in color. Adding the heavy quark makes the full state 'colorless'.

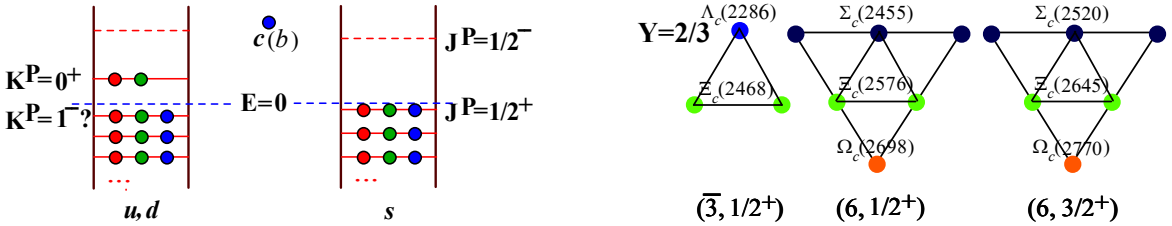

FIG. 3: Filling u,d and s shells for the ground-state charmed baryons (left), and $SU(3)$ multiplets generated by this filling scheme (right): $(\bar{\mathbf{3}},1/2^+)$, $(\mathbf{6},1/2^+)$ and $(\mathbf{6},3/2^+)$.

As in the case of light baryons, the filling scheme by itself does not tell us what are the quantum numbers of the state: they arise from quantizing the $SU(3)_{\rm flav}$ and $SO(3)_{\rm space}$ rotations of the given filling scheme. Let us do it for the ground-state baryons.

First of all, we determine the hypercharge of the filling scheme: in this case it is $Y'=\frac{1}{3}(N_c-1)$ since there are N_c-1 u,d quarks each having hypercharge one third. At $N_c=3$ one has $Y'=\frac{2}{3}$. There are two $SU(3)$ multiplets containing particles with hypercharge $\frac{2}{3}$: the anti-triplet $\bar{\mathbf{3}}$ ($p=0,q=1$) and the sextet $\mathbf{6}$ ($p=2,q=0$), therefore these are the allowed multiplets, see Fig. 3, right. What are their spins?

In the $\bar{\mathbf{3}}$ representation, there is one particle with $Y'=\frac{2}{3}$ hence its isospin $T'=0$. The possible spin of the multiplet is found from Eq. (2) which needs to be modified to include

the spin of the heavy quark J_h:

$$\mathbf{J} = \mathbf{T}' + \mathbf{J_1} + \mathbf{J_2} + \mathbf{J_h}. \tag{4}$$

In this case $J_1 = J_2 = 0$ since s quarks are not involved, $T' = 0$, and $J_h = \frac{1}{2}$. Therefore, the only possible spin of the anti-triplet is $\frac{1}{2}$, and parity plus. Its rotational energy is, according to Eq. (3),

$$E_{\rm rot}^{(\bar{\mathbf{3}})} = \frac{1}{2I_2}. \tag{5}$$

In the **6** representation, there are three particles with $Y' = \frac{2}{3}$ hence their isospin $T' = 1$. From Eq. (4) one finds then that there are *two* sextets, one with spin $\frac{1}{2}$ and another with spin $\frac{3}{2}$. They are degenerate in the leading order as the rotational energy (3) depends only on T' but not on the spin:

$$E_{\rm rot}^{(\mathbf{6})} = \frac{1}{2I_2} + \frac{1}{I_1}. \tag{6}$$

Thus the filling scheme in Fig. 3, left, implies three $SU(3)$ multiplets: $(\bar{\mathbf{3}}, \frac{1}{2}^+)$, $(\mathbf{6}, \frac{1}{2}^+)$ and $(\mathbf{6}, \frac{3}{2}^+)$, see Fig. 3, right. The last two are degenerate (but the degeneracy is lifted in the next $1/N_c^2$ order and also from the $1/m_h$ corrections) whereas the center of the anti-triplet is separated from the center of the sextets by the rotational energy $\Delta E_{\rm rot} = \frac{1}{I_1}$. The splitting *inside* multiplets owing to the explicit violation of $SU(3)$ by the strange quark mass is $\mathcal{O}(m_s N_c)$. If m_s is treated as a small perturbation, $m_s = \mathcal{O}(1/N_c^2)$, as I claim it should [3], the splitting inside the sextet must be equidistant to a good accuracy. Let us confront these predictions with current data.

There are good candidates for the above ground-state multiplets: $\Lambda_c(2286)$ and $\Xi_c(2468)$ for $(\bar{\mathbf{3}}, 1/2^+)$; $\Sigma_c(2455)$, $\Xi_c(2576)$ and $\Omega_c(2698)$ for $(\mathbf{6}, 1/2^+)$; finally $\Sigma_c(2520)$, $\Xi_c(2645)$ and $\Omega_c(2770)$ presumably form $(\mathbf{6}, 3/2^+)$, see Fig. 3, right. Strictly speaking the J^P quantum numbers of most of these baryons are not measured directly but there is not much doubt they differ from the above assignments. Assuming they are correct, the observed parity-plus charmed baryons form precisely those multiplets that follow from the collective quantization.

The splittings inside the two sextets are equidistant to high accuracy, confirming that m_s can be treated as a small perturbation. Were m_s "not small", there would be substantial $\mathcal{O}(m_s^2)$ corrections to the masses, which would violate the equidistant character of the sextets spectrum.

The centers of the three multiplets are at

$$\begin{aligned} m\left(\bar{\mathbf{3}}, 1/2^+\right) &= \frac{2287 + 2*2468}{3} = 2408 \text{ MeV}, \\ m\left(\mathbf{6}, 1/2^+\right) &= \frac{3*2455 + 2*2576 + 2698}{6} = 2536 \text{ MeV}, \\ m\left(\mathbf{6}, 3/2^+\right) &= \frac{3*2520 + 2*2645 + 2770}{6} = 2603 \text{ MeV}. \end{aligned} \tag{7}$$

Although the two sextets are not exactly degenerate, their splitting 67 MeV (an unaccounted $1/N_c^2$ effect) is much less than the splitting between the anti-triplet and the mean mass of the sextets, which is

$$\frac{2536 + 2603}{2} - 2408 = 162 \text{ MeV} = E_{\rm rot}^{(\mathbf{6})} - E_{\rm rot}^{(\bar{\mathbf{3}})} = \frac{1}{I_1} = \mathcal{O}(1/N_c). \tag{8}$$

Furthermore, this number should be compared with the moment of inertia following from the splitting between *light* baryons, $(\mathbf{10}, \frac{3}{2}^+)$ and $(\mathbf{8}, \frac{1}{2}^+)$, yielding $1/I_1 = 153\,\text{MeV}$, see Section II. The proximity of the two completely different determinations of the moment of inertia supports the basic idea that it is reasonable to view both light and heavy baryons from the same large-N_c perspective [24].

VI. CHARMED AND BOTTOM BARYONS, EXCITED STATES

There are higher $SU(3)$ multiplets with $Y' = \frac{2}{3}$, however a closer inspection shows that the corresponding rotational excitations have large $\mathcal{O}(1)$ energies and not $\mathcal{O}(1/N_c)$ as requested for the rotational states. Therefore, those higher rotational states are, strictly speaking, beyond control. Higher parity-plus heavy baryon resonances should arise as one-particle and particle-hole excitations, like for light baryons.

As to parity-minus states, there are several possibilities to construct them. The first is to excite one of the u, d quarks from the 0^+ valence level to the first excited level for s quarks, which is $\frac{1}{2}^-$, see Fig. 2, transition *1*. It would be then an analogue of $\Lambda(1405, \frac{1}{2}^-)$. There is, however, an argument against it. The transition has $Y' = -\frac{1}{3}$ which is possible with the $\bar{\mathbf{3}}$ and $\mathbf{6}$ representations but now it corresponds to $T' = \frac{1}{2}$ in both cases. This difference with ground-state multiplets has a dramatic consequence: both multiplets have an $\mathcal{O}(1)$ rotational energy and hence should be discarded. [To check the analytical N_c dependence one has to construct the prototype multiplets that reduce at $N_c = 3$ to those under consideration, see the Appendix.]

The second possible way of making parity-minus states is to excite the s quark from the highest filled $\frac{1}{2}^+$ level to the $\frac{1}{2}^-$ level. It would be an analogue of $N(1535, \frac{1}{2}^-)$, see Fig. 2, transition *2*. The quantization of rotations about this excitation along the lines presented above, leads to three degenerate anti-triplets $2 \times (\bar{\mathbf{3}}, 1/2^-)$, $(\bar{\mathbf{3}}, 3/2^-)$ and many sextets with various spins. However, this interpretation is not too realistic either. First, there are only two observed anti-triplets presumably with negative parity, $\Lambda_c(2595, 1/2^-)$, $\Xi_c(2790, 1/2^-?)$ and $\Lambda_c(2625, 3/2^-?)$, $\Xi_c(2815, 3/2^-?)$. There are no candidates for the second $(\bar{\mathbf{3}}, 1/2^-)$ although it may be found in future. Second, the average mass of the two Λ_c's is only 300 MeV higher than the lowest charmed baryon $\Lambda_c(2786.5, 1/2^+)$, whereas from light baryons we expect that this excitation energy is about $1535 - 940 \approx 600$ MeV. It contradicts the basic large-N_c concept that the one-particle levels for heavy baryons do not differ much from those for light baryons.

Therefore at the moment I think that the most plausible construction of the lowest parity-minus states is to assume that there is a 1^- level for u, d quarks *below* the valence 0^+ level, see Fig. 3, left. It has to be filled in in the ground state as it has $E < 0$ but since there is a hole in the valence 0^+ level there is an excitation when one u, d quark from the 1^- level fills the hole in the 0^+ shell. Such excitation is absent for light baryons (since the valence 0^+ shell is fully filled there), therefore no previous knowledge prevents us from assuming that the excitation energy is only 300 MeV.

The quantization of rotations about such excitation produces two degenerate anti-triplets $(\bar{\mathbf{3}}, 1/2^-)$ and $(\bar{\mathbf{3}}, 3/2^-)$, exactly as needed for phenomenology. However, this transition generates also a number of higher mass almost degenerate sextets with spin from 1/2 to 5/2, none of which has been observed so far. Unfortunately, experimental knowledge of the parity-minus heavy baryons is too scarce to choose between different interpretations.

VII. CHARMED AND BOTTOM BARYONS, EXOTIC STATES

Our new observation is that there is a Gamov–Teller-type transition when the axial current annihilates a strange quark in the $\frac{1}{2}^+$ shell, and creates an u or d quark in the 0^+ shell (see Fig. 4, left), like in the case of the Θ^+. In heavy baryons it is even more trivial as there is a hole in the 0^+ valence shell from the start. Filling in this hole means making charmed (or bottom) pentaquarks which I name "Beta baryons" [26], $\mathcal{B}_c^{++} = cuud\bar{s}$, $\mathcal{B}_c^{+} = cudd\bar{s}$,

and $\mathcal{B}_b^+ = buud\bar{s}$, $\mathcal{B}_b^0 = budd\bar{s}$. While the existence of Θ^+ requires an excited ('Roper') one-particle level, the existence of the $\mathcal{B}_{c,b}$ baryons needs only the ground-state level which is undoubtedly there. In this sense, the $\mathcal{B}_{c,b}$ baryons are more basic than the Θ^+.

What are the $SU(3)$ multiplets corresponding to this excitation? The hypercharge is $Y' = 3 * \frac{1}{3} - (-\frac{2}{3}) = \frac{5}{3}$. The lowest $SU(3)$ representation containing particles with $Y' = \frac{5}{3}$ is the *anti-decapenta* ($\overline{\mathbf{15}}$)-plet ($p = 1, q = 2$) [30], see Fig. 4, right. Therefore, this is an allowed multiplet generated by the transition. There are two particles with $Y' = \frac{5}{3}$, hence their isospin is $T' = \frac{1}{2}$. The allowed spin is given by Eq. (4) where one puts $J_1 = \frac{1}{2}$, $J_2 = 0$ and obtains that the possible spins of the multiplets are $\frac{1}{2}$ (twice) and $\frac{3}{2}$, parity plus. All of them are degenerate in the leading order in $1/N_c$ but split in the next-to-leading order.

Thus the Gamov–Teller-type transition shown in Fig. 4, left, induces three almost degenerate multiplets: $2 \times (\overline{\mathbf{15}}, 1/2^+)$ and $(\overline{\mathbf{15}}, 3/2^+)$.

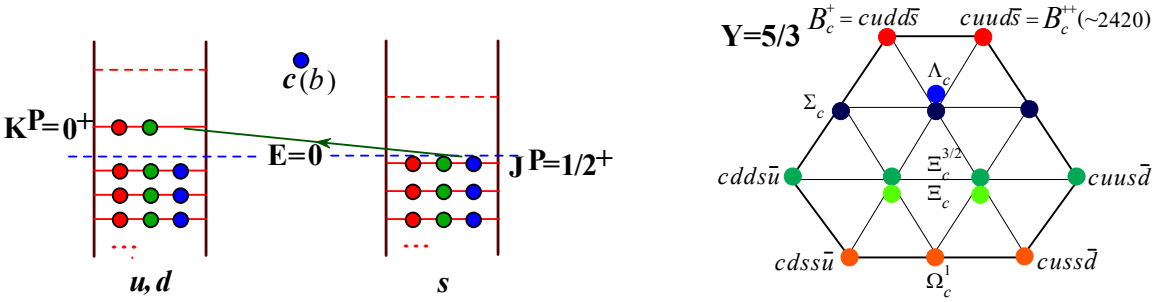

FIG. 4: The arrow shows the lowest Gamov–Teller excitation (left) leading to charmed pentaquarks forming $\overline{\mathbf{15}}$ (right).

The six baryons at the corners of the hexagon in Fig. 4, right, are explicitly exotic: their quantum numbers cannot be achieved from 3-quark states. The rest 9 baryons are crypto-exotic: they are mainly pentaquarks but have the quantum numbers of the ground-state baryons belonging to $\bar{\mathbf{3}}$ and $\mathbf{6}$ representations, and can mix with them. The mixing is an $SU(3)$ violating effect, the mixing angle being $\theta = \mathcal{O}(m_s N_c^2/\Lambda)$ where $\Lambda \sim 1\,\mathrm{GeV}$ is a typical scale in strong interactions. Actually the isotopic quadruplet $\Xi_c^{3/2}$ and the triplet Ω_c^1 mix up with the corresponding members of the $\bar{\mathbf{3}}$ and $\mathbf{6}$ only through isospin breaking, therefore this mixing can be neglected. The mixing of Λ_c, Σ_c and Ξ_c leads to a shift in the physical baryon masses, that is quadratic in m_s; it is of the order of $m_s^2 N_c^3/\Lambda$. The fact that baryons in the sextets are almost equidistant means that in practice the mixing is numerically small. Probably more important is the mixing between the two $(\overline{\mathbf{15}}, 1/2^+)$-plets

with identical quantum numbers: one goes up, the other goes down.

VIII. MASS SPLITTING IN THE EXOTIC ANTI-DECAPENTA-PLET

The $\overline{\mathbf{15}}$-plets are split in the leading $\mathcal{O}(m_s N_c)$ order, as usually. The $SU(3)$-violating term in the QCD Lagrangian, $m_s \bar{s} s$, has a singlet and an octet pieces; it is the octet piece that leads to the splitting. Since there are two ways of getting an octet in the direct product of $\overline{\mathbf{15}} \otimes \mathbf{15}$ there are two mass parameters $m_{1,2}$ that determine all mass shifts in the $\overline{\mathbf{15}}$. [This is similar to the octet baryons where there are also two parameters, and distinct from the sextet, decuplet and antidecuplet where there is only one, and consequently the masses are equidistant.] In what follows I ignore the next-to-leading mixing of the $\overline{\mathbf{15}}$-plets with the $\bar{\mathbf{3}}$ and $\mathbf{6}$, and of the two $(\overline{\mathbf{15}}, 1/2^+)$ between themselves.

A simple exercise in the $SU(3)$ algebra (which I suppress) leads to the following masses of the members of the $\overline{\mathbf{15}}$-plet, for each horizontal line in Fig. 4, right, from top to bottom:

$$\begin{aligned}
\mathcal{B}_c^{++,+}: \quad & M_1 = M_0 - m_1 - m_2, \\
\Sigma_c^{++,+,0}: \quad & M_2 = M_0 - m_1 - \frac{m_2}{4}, \\
\Lambda_c^{+}: \quad & M_3 = M_0 + \frac{m_1}{2} - \frac{5m_2}{8}, \\
\Xi_c^{++,+,0,-}: \quad & M_4 = M_0 - m_1 + \frac{m_2}{2}, \\
\Xi_c^{+,0}: \quad & M_5 = M_0 + \frac{5m_1}{4} - \frac{m_2}{16}, \\
\Omega_c^{+,0,-}: \quad & M_6 = M_0 + 2m_1 + \frac{m_2}{2},
\end{aligned} \tag{9}$$

where

$$M_0 = \frac{1}{15}(2M_1 + 3M_2 + M_3 + 4M_4 + 2M_5 + 3M_6)$$

is the center of the $\overline{\mathbf{15}}$-plet. There are 6 different masses $M_{1\text{–}6}$ expressed through 3 parameters, therefore there are 3 relations, analogous to the Gell-Mann-Okubo relation for the octet:

$$\begin{aligned}
M_1 + M_4 &= 2M_2, \\
5M_1 + 2M_2 + 2M_5 &= 9M_3, \\
4M_1 + M_2 + M_6 &= 6M_3.
\end{aligned}$$

These combinations do not depend on the $SU(3)$-violating parameters $m_{1,2}$. However, in the large-N_c approach $m_{1,2}$ are related to the splittings inside all other multiplets: $\bar{\mathbf{3}}$, $\mathbf{6}$, $\mathbf{8}$, $\mathbf{10}$ and $\overline{\mathbf{10}}$. These relations will be considered elsewhere.

We see that the lightest is the exotic doublet $\mathcal{B}_c$, and the heaviest is the exotic triplet Ω_c^1. Since we know the separation between the $1/2^+$ level for s quarks and the 0^+ level for u, d quarks from fitting the light baryon resonances (it is 130 MeV, see Section III), and assuming that it does not change for heavy baryons (as it would be at $N_c \to \infty$), we estimate the mass of the $\mathcal{B}_c^{++,+}$ pentaquarks at about $m(\Lambda_c) + 130\,\mathrm{MeV} = 2420\,\mathrm{MeV}$. The corresponding bottom pentaquarks $\mathcal{B}_b^{+,0}$ mass is about $m(\Lambda_b) + 130\,\mathrm{MeV} = 5750\,\mathrm{MeV}$. These are very light masses.

The accuracy of this prediction is $\mathcal{O}(1/N_c) \sim 150\,\mathrm{MeV}$ but there is a 360 MeV margin below the threshold for strong decays $\mathcal{B}_c \to \Lambda_c K$ (2780 MeV), $\mathcal{B}_b \to \Lambda_b K$ (6110 MeV). It means that such light charmed and bottom baryons have no strong decays, which makes their observation feasible.

Using the rule of thumb that each lower line in the $SU(3)$ weight diagram is approximately 140 MeV heavier than the previous, the expected mass of another exotic pentaquark, the quadruplet $\Xi_c^{3/2}$, is about 2700 MeV. However, this is above the threshold for the strong decay, for example $\Xi_c^-(csdd\bar{u}) \to \Xi_c^0(csd) + \pi^-(d\bar{u})$, at 2610 MeV. Therefore, the exotic pentaquarks $\Xi_c^{3/2}$ can manifest themselves as extremely narrow peaks in $\Xi_c\pi$ mass distributions. The same applies to the exotic $\Omega_c^1(\sim 2850)$ decaying into $\Omega_c\pi$ (threshold at 2840 MeV).

Charmed pentaquarks have been considered by Wu and Ma in another approach [31]; however, these authors get far larger masses and in addition pentaquarks with $\bar{c}$ quarks appear almost degenerate with those made of c quarks. This is not the case in the present scheme where *anti*-charmed pentaquarks are $\mathcal{O}(1)$, that is substantially, heavier than the charmed ones.

How to make a $qqqq\bar{Q}$ pentaquark in the present approach? Apparently the 0^+ shell for u, d quarks must be completed, and one has to put somewhere the fourth quark to make the state 'colorless'. The first two excited states, the $\frac{1}{2}^-$ level for s quarks and the excited ('Roper') 0^+ level for u, d quarks, are rather close in light baryons, see Fig. 2. They may be reshuffled somewhat by $1/N_c$ corrections as one goes from light to heavy baryons, therefore which one is lower in heavy baryons is not clear beforehand. Assuming it is the $\frac{1}{2}^-$ level, the lowest anti-charmed pentaquarks are $P_{\bar{c}s} = uuds\bar{c},\ uudd\bar{c}$ of Gignoux *et al.* [28] and

Lipkin [29]; they belong to the **3** representation and have negative parity. The allowed spins are $\frac{1}{2}$ (twice) and $\frac{3}{2}$, according to the quantization rules formulated in Sections IV and V, see Fig. 5.

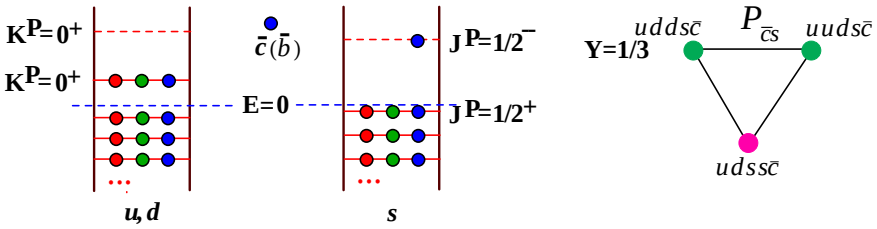

FIG. 5: *Anti*-charmed (anti-bottom) pentaquarks with negative parity; filling scheme for four quarks (left) and the multiplets it generates (right): $2 \times (\mathbf{3}, 1/2^-)$ and $(\mathbf{3}, 3/2^-)$.

Assuming the lowest excited state is the 0^+ level, the lowest anti-charmed pentaquark is $\Theta_c = uudd\bar{c}$ of Karliner and Lipkin [27] belonging to the $\bar{\mathbf{6}}$ representation; it has then spin-parity $\frac{1}{2}^+$, see Fig. 6. Such Θ_c is a direct analog of the light-quark Θ^+ as it also arises from exciting the 'Roper' level.

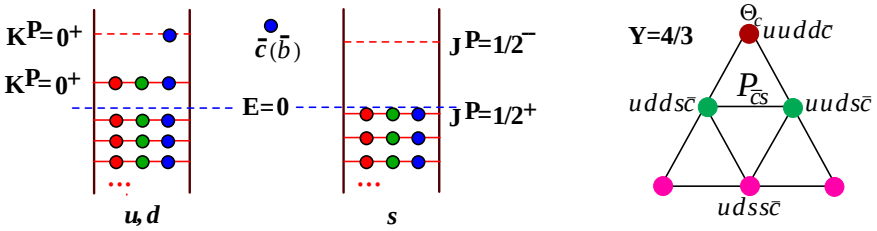

FIG. 6: *Anti*-charmed (anti-bottom) pentaquarks with positive parity; filling scheme for four quarks (left) and the multiplet it generates (right): $(\bar{\mathbf{6}}, 1/2^+)$.

To estimate the mass of anti-charmed pentaquarks, we assume that the valence 0^+ level for u, d quarks is at about 100 MeV. It must be positive otherwise it would belong to the vacuum state, but less than 130 MeV otherwise the $\frac{1}{2}^+$ level for s quarks would have positive energy. From light baryons we know that the two excited levels are about 460 MeV higher than the valence u, d shell. Therefore the lightest anti-charmed pentaquarks are expected at about $m(\Lambda_c) + 100 + 460 \approx 2850\,\text{MeV}$. This is slightly above the strong-decay threshold for the $\Theta_c \to D^- p$ (2810 MeV) and slightly below the threshold for $P_{\bar{c}s} \to D_s p$ (2910 MeV). The $\mathcal{O}(1/N_c) \sim 150\,\text{MeV}$ precision in our mass predictions does not allow at present a definite conclusion whether the anti-charmed baryons are stable against strong decays, which is critical for their observation. The important point is that anti-charmed

(anti-bottom) pentaquarks are essentially ($\sim 500\,\mathrm{MeV}$) heavier than the charmed (bottom) pentaquarks. Allowing even for a 360 MeV uncertainty in numerics, Beta baryons $\mathcal{B}_{b,c}$ remain below the threshold for strong decays!

IX. PRODUCTION RATE OF BETA BARYONS, AND DECAY SIGNATURES

In principle, $\mathcal{B}_{b,c}$ baryons can be produced whenever charm (bottom) is produced. However, the production rate is expected to be very low. It is affected by the general suppression of charm (bottom) production, and by the small coalescence factor specific for the production of objects built of many constituents. Therefore, high-energy, high-luminosity machines like LHC have better chances.

It is very difficult to make a reliable estimate of the production rate, say, at LHC, therefore I make a pessimistic estimate [32]. The number of charmed baryons produced in the central rapidity range (where it is maximal) is estimated as $dN/dy \sim 10^{-3}$. For bottom quarks it is several times less. The number of anti-deuterons produced at LHC is expected at the level of $dN/dy \sim 10^{-4}$. Deuterons are 6 quarks so the rate gives an idea of the coalescence factor for a 5-quark system, too. To get the lower bound for the production rate for the pentaquark $\mathcal{B}_c$ baryons I am inclined to multiply the two probabilities and obtain for the LHC

$$\frac{dN^{\mathcal{B}_c}}{dy} \sim 10^{-7}, \qquad y \approx 0. \tag{10}$$

This is low enough but one looses even more when a specific channel is chosen to trigger the decay of $\mathcal{B}_c$. From the experience with 'ordinary' charmed baryons we know that there are very many decay channels, the largest branching ratios being at the level of 1%. Therefore, it is important to choose a decay channel with as low background as possible, rather than seeking for a dominant decay mode. $\mathcal{B}_c^{++}$ has a remarkable decay into $p\pi^+$ proceeding through the Cabibbo-unsuppressed annihilation $c\bar{s} \to u\bar{d}$. However, this decay has probably a large background even if events are selected with protons spatially displaced from the reaction vertex. I expect that the $\mathcal{B}_c$ lifetime is of the same order as that of normal charmed baryons, *i.e.* $10^{-13}\,s$, meaning that its decay can be resolved in a vertex detector. In addition, the in-flight Cabibbo-unsuppressed decay $c \to su\bar{d}$ is probably faster than annihilation.

The $\mathcal{B}_c^{++} \to \bar{s}s\bar{d}duuu$ intermediate state is interesting because it can further proceed into $\Lambda K^+\pi^+$ or to $pK^+\bar{K}^0$ or, via a narrow resonance ϕ, to $p\phi\pi^+ \to pK^+K^-\pi^+$. These channels

may balance the branching ratio and background conditions. In fact, a similar channel $pK^+K^-\pi^-$ has been used by E791 in the search for the neutral anti-charm pentaquark $P_{\bar{c}s}$ [33] but with the trigger that four charged particles have the total zero charge. Here it must be +2. The $\mathcal{B}_c^+$ can decay into three-prong final states $p\phi \to pK^+K^-$ or ΛK^+.

Returning to the production rate (10) it should be multiplied by a typical branching ratio 10^{-2} to a particular observation channel, yielding a tiny observation rate of 10^{-9}. Given that the total number of events at LHC is 10^{15}/year, it still promises 10^6 registrations of $\mathcal{B}_c$ per year. Respectively, there could be as much as 10^5 $\mathcal{B}_b$ events per year. At Fermilab the rate is 3 orders of magnitude less but still probably accessible. It is interesting that a good fraction of $\mathcal{B}_b$ decays must be into $\mathcal{B}_c$ plus pions since the dominant weak decay is $b \to cd\bar{u}$.

Because it is the main b-quark decay, $\mathcal{B}_c$ can be looked for at B-factories, Belle and Babar. As a conservative estimate of the $\mathcal{B}_c$ production probability I would take the product of the probability to create a charmed baryon of comparable mass (say, $\Sigma_c(2455)$ or $\Xi_c(2468)$), and of the probability to create a deuteron. Searching for $\mathcal{B}_c$ in relativistic heavy ion collisions may be also promising since the coalescence factor may be more favorable there.

X. CONCLUSIONS

If the number of colors N_c is treated as a free algebraic parameter, baryon resonances are classified in a simple way. At large N_c all baryon resonances are basically determined by the "intrinsic" quark spectrum which takes certain limiting shape at $N_c \to \infty$. This spectrum is the same in light baryons $q\ldots qq$ with N_c light quarks q, and in heavy baryons $q\ldots qQ$ with N_c-1 light quarks and one heavy quark Q, since the difference is a $1/N_c$ effect.

One can excite quark levels in various ways called either one-particle or particle-hole excitations; in both cases the excitation energy is $\mathcal{O}(1)$. On top of each one-quark or quark-antiquark excitation there is generically a band of $SU(3)$ multiplets of baryon resonances, that are rotational states of a baryon as a whole. Therefore, the splitting between multiplets is $\mathcal{O}(1/N_c)$. The rotational band is terminated when the rotational energy reaches $\mathcal{O}(1)$. Some multiplets which differ only by spin are degenerate in the leading order but become split in the next $\mathcal{O}(1/N_c^2)$ order.

In reality N_c is only 3, and the above idealistic hierarchy of scales is somewhat blurred. Nevertheless, a close inspection of the spectrum of baryon resonances reveals certain hierar-

chy schematically summarized as follows:

- Baryon mass: $\mathcal{O}(N_c)$, numerically 1200 MeV, the average mass of the ground-state octet
- One-quark and particle-hole excitations in the intrinsic spectrum: $\mathcal{O}(1)$, typically 400 MeV, for example the excitation of the Roper resonance
- Splitting between the centers of $SU(3)$ multiplets arising as rotational excitations of a given intrinsic state: $\mathcal{O}(1/N_c)$, typically 133 MeV
- Splitting between the centers of rotational multiplets differing by spin, that are degenerate in the leading order: $\mathcal{O}(1/N_c^2)$, typically 44 MeV
- Splitting inside a given multiplet owing to the nonzero strange quark mass: $\mathcal{O}(m_s N_c)$, typically 140 MeV.

In practical terms, the lowest light baryon multiplets $(\mathbf{8}, 1/2^+)$ and $(\mathbf{10}, 3/2^+)$ form the "rotational band" about the ground state, with the splitting between their centers being $\frac{3}{2I_1} = 230\,\mathrm{MeV} = \mathcal{O}(1/N_c)$. The ground state of a heavy baryon (where one light quark is replaced by a heavy one so that there is a hole in the light quarks valence shell) generates a rotational band of three multiplets, $(\overline{\mathbf{3}}, 1/2^+)$, $(\mathbf{6}, 1/2^+)$ and $(\mathbf{6}, 3/2^+)$. These are precisely the observed multiplets, and the prediction is that the two sextets are degenerate in the leading order whereas the splitting between the $\overline{\mathbf{3}}$ and $\mathbf{6}$ is $\frac{1}{I_1} = 153\,\mathrm{MeV}$. In reality the two sextets are not degenerate but their splitting 67 MeV (an $1/N_c^2$ effect) is substantially less than the splitting between the mean mass of the sextets and the anti-triplet, which is 162 MeV, off by only 6% from the large-N_c prediction.

This coincidence encourages to look what is the *lowest non-rotational excitation* of a heavy baryon in the large-N_c limit. Apparently, it is the particle-hole excitation where one takes an s quark from the highest filled shell and puts an u or d quark at the lowest u, d valence shell, filling in the hole there, see Fig. 4. The corresponding baryon resonances have the (penta) quark content $\mathcal{B}_c^{++} = cuud\bar{s}$, $\mathcal{B}_c^{+} = cudd\bar{s}$ with mass $m(\Lambda_c) + 130\,\mathrm{MeV} = 2420\,\mathrm{MeV}$ and $\mathcal{B}_b^{+} = buud\bar{s}$, $\mathcal{B}_b^{0} = budd\bar{s}$ with mass $m(\Lambda_b) + 130\,\mathrm{MeV} = 5750\,\mathrm{MeV}$. I call them "Beta baryons" (implying, of course, that "Alpha baryons" are the standard, mainly three-quark baryons). Actually, $\mathcal{B}_{b,c}$ baryons are part of the larger $\overline{\mathbf{15}}$ multiplet of pentaquarks, and

there must be three of them: two with spin-parity $1/2^+$ and one with $3/2^+$. The splitting of these $\overline{\mathbf{15}}$-plets is expected to be less than 100 MeV.

The arithmetic for the masses would be exact in the limit of infinite N_c, however in reality $\mathcal{O}(1/N_c) \sim 150\,\mathrm{MeV}$ corrections are allowed. However, there is still quite some room below the threshold for strong decays, which is at 2780 MeV. Therefore, I believe that at least one but maybe two or even three exotic pentaquarks $\mathcal{B}_{b,c}$ are stable with respect to strong decays. This makes their discovery feasible, despite that the production rate is probably very low, see Section IX.

I think that the presented case for the heavy $\mathcal{B}_{b,c}$ pentaquarks is even stronger that it has been for the Θ^+ pentaquark [17], whose mass I confirm here from a new, unified point of view.

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Appendix. ARBITRARY N_c PROTOTYPES OF $SU(3)$ MULTIPLETS

The usual $SU(3)$ multiplets, **8** and **10** for light baryons, $\bar{\mathbf{3}}$ and **6** for charmed and bottom baryons, arise for baryons made minimally of three quarks. If N_c is considered a free parameter baryons made of N_c quarks fall into other $SU(3)$ multiplets. The aim of this Appendix is to construct the generalizations of the ordinary multiplets to arbitrary N_c, such that the 'prototype' multiplets reduce to the usual ones at $N_c=3$.

A generic $SU(3)$ multiplet or irreducible representation is uniquely determined by two non-negative integers (p,q) having the meaning of upper (lower) components of the irreducible $SU(3)$ tensor $T^{\{f_1\ldots f_p\}}_{\{g_1\ldots g_q\}}$ symmetrized both in upper and lower indices and with a contraction with any $\delta^{g_n}_{f_m}$ being zero. Schematically, q is the number of boxes in the lower line of the Young tableau depicting an $SU(3)$ representation and p is the number of extra boxes in its upper line, see Fig. 7.

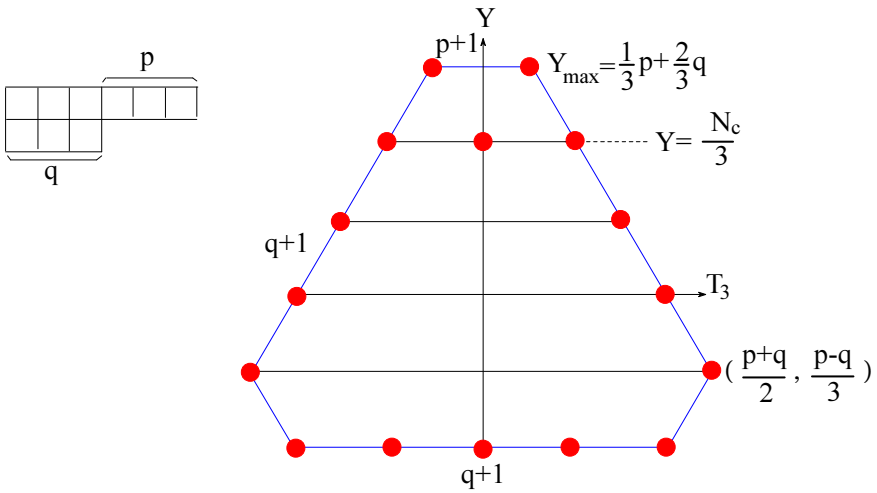

FIG. 7: A generic $SU(3)$ multiplet is, on the one hand, defined by the Young tableau and on the other hand can be characterized by quantum numbers (T_3, Y) of its members filling a hexagon in the (T_3, Y) axes (the weight diagram).

The dimension of a representation or the number of particles in the multiplet is

$$\mathrm{Dim}(p,q) = (p+1)(q+1)\left(1+\frac{p+q}{2}\right). \tag{A.1}$$

In the weight (T_3, Y) diagram where T_3 is the third projection of the isospin and Y is the hypercharge, a generic $SU(3)$ representation takes the form of a hexagon, whose the upper horizontal side contains $p+1$ 'dots' or particles, the adjacent sides contain $q+1$ particles, with alternating $p+1$ and $q+1$ particles in the rest sides, the corners included – see Fig. 7. If either p or q is zero, the hexagon reduces to a triangle.

Particles at the upper (horizontal) side of the hexagon have the hypercharge

$$Y_{\max} = \frac{1}{3}\,p + \frac{2}{3}\,q \tag{A.2}$$

being the maximal possible hypercharge of a multiplet with given (p,q). The number of particles in a horizontal line with given Y is

$$n(Y) = \frac{4}{3}\,p + \frac{2}{3}\,q + 1 - Y. \tag{A.3}$$

(possible non-unity multiplicities of particles with fixed Y, T_3 are neglected here.)

To find what $SU(3)$ multiplets are generated as rotation states from a given intrinsic quark ground state or excitation, one has first to determine the hypercharge Y' by counting the number of u, d, s quarks involved in the intrinsic quark state,

$$Y' = \frac{1}{3}\,(\#\text{ of } u,d \text{ quarks}) - \frac{2}{3}\,(\#\text{ of } s \text{ quarks}). \tag{A.4}$$

For example, for the ground state light baryon $Y' = \frac{1}{3}\, N_c$ as there are N_c u, d quarks at the ground-state valence level, Fig. 1. The number of particles with this hypercharge is related to the isospin T' by the equation

$$2T' + 1 = n(Y') = \frac{4}{3}\, p + \frac{2}{3}\, q + 1 - Y'. \tag{A.5}$$

The logic of constructing the prototype multiplets is as follows. One first finds the allowed multiplets that contain a given Y' for $N_c = 3$, and reads off the T' for those multiplets from the weight diagrams. By construction, $Y' \leq Y_{\rm max}$, the highest possible hypercharge of an allowed multiplet. Let us write $Y_{\rm max} = Y' + X$ where X is the number of steps in the weight diagram, by which the top of the diagram is separated from Y'.

In principle, there are many ways how to generalize the real-world $SU(3)$ multiplets to arbitrary N_c. The natural one [7, 8] is to fix at all N_c the shape of the weight diagram at its upper part, meaning fixing T' and X for all N_c as they appear at $N_c = 3$. Physically, it corresponds to the generalization where one adds more u, d quarks to the baryon as one increases N_c, and not s quarks. The (p, q) numbers of the prototype multiplet in question is then found from Eq. (A.5) and from

$$Y' + X = Y_{\rm max} = \frac{1}{3}\, p + \frac{2}{3}\, q. \tag{A.6}$$

The rotational energy of the prototype multiplet is given by Eq. (3). Let us consider several examples of building the prototype multiplets at arbitrary N_c. We assume N_c is odd such that baryons are fermions.

Light baryons, ground state

In this case $Y' = 1$ at $N_c = 3$, and the allowed multiplets are **8**, **10** since these multiplets contain a line with $Y' = 1$; for the octet there are two such particles, hence $T' = \frac{1}{2}$, whereas for the decuplet there are four such particles, hence $T' = \frac{3}{2}$. In both cases it is the upper line, therefore $X = 0$.

Generalizing these multiplets to arbitrary N_c we fix T' and X what they are at $N_c = 3$ but change Y' according to Eq. (A.4). For the ground state $Y' = \frac{N_c}{3}$. Solving Eqs.(A.5,A.6)

with respect to (p,q) we find the prototype 'octet':

$$\begin{Bmatrix} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{Bmatrix} = \begin{Bmatrix} 1 \\ \frac{N_c-1}{2} \\ \frac{(N_c+1)(N_c+5)}{4} \\ \frac{N_c}{4I_2}+\frac{3}{8I_1} \end{Bmatrix} \stackrel{N_c\to 3}{\longrightarrow} \begin{Bmatrix} 1 \\ 1 \\ \mathbf{8} \\ \frac{3}{4I_2}+\frac{3}{8I_1}. \end{Bmatrix} \tag{A.7}$$

The prototype 'decuplet' is

$$\begin{Bmatrix} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{Bmatrix} = \begin{Bmatrix} 3 \\ \frac{N_c-3}{2} \\ \frac{(N_c-1)(N_c+7)}{4} \\ \frac{N_c}{4I_2}+\frac{15}{8I_1} \end{Bmatrix} \stackrel{N_c\to 3}{\longrightarrow} \begin{Bmatrix} 3 \\ 0 \\ \mathbf{10} \\ \frac{3}{4I_2}+\frac{15}{8I_1}. \end{Bmatrix} \tag{A.8}$$

The rotational energies of the prototype "**8**" and "**10**" differ by $\frac{3}{2I_1} = \mathcal{O}(1/N_c)$. The term $\frac{N_c}{4I_2} = \mathcal{O}(1)$ is a common shift in this (and subsequent) examples. Higher multiplets containing $Y' = \frac{N_c}{3}$ have $\mathcal{O}(1)$ rotational splitting and should be discarded for this reason.

The spins of these two prototype multiplets are found from the vector addition rule (2). In this case $J_1 = J_2 = 0$, hence the spin $J = \frac{1}{2}$ for the 'octet', and $J = \frac{3}{2}$ for the 'decuplet'.

Light baryons, $0^+ \to \frac{1}{2}^-$ transition

This is the intrinsic one-quark excitation *1* in Fig. 2. At $N_c{=}3$ it corresponds to $Y' = 0$, and the allowed multiplets are, in principle, the singlet, the octet and the decuplet, with $T' = 0$, $(0,1)$, 1, respectively, and $X = 1$ in all cases. The generalization changes $Y' = \frac{N_c-3}{3}$. The prototype 'singlet' is

$$\begin{Bmatrix} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{N_c-3}{2} \\ \frac{(N_c-1)(N_c+1)}{8} \\ \frac{N_c-3}{4I_2} \end{Bmatrix} \stackrel{N_c\to 3}{\longrightarrow} \begin{Bmatrix} 0 \\ 0 \\ \mathbf{1} \\ 0. \end{Bmatrix} \tag{A.9}$$

The spin of this 'singlet' is $J = \frac{1}{2}$.

To build the 'octet' we take $T'=1$ and $X=1$ and find

$$\left\{\begin{array}{l} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{array}\right. = \left\{\begin{array}{c} 1 \\ \frac{N_c-1}{2} \\ \frac{(N_c+1)(N_c+5)}{4} \\ \frac{N_c-2}{2I_2}+\frac{1}{I_1} \end{array}\right. \stackrel{N_c\to 3}{\longrightarrow} \left\{\begin{array}{c} 1 \\ 1 \\ \mathbf{8} \\ \frac{1}{2I_2}+\frac{1}{I_1}. \end{array}\right. \tag{A.10}$$

We note that the rotational energy differs from that of the 'singlet' by $\mathcal{O}(1)$. Therefore, this multiplet, strictly speaking, is *not* a rotational excitation of the intrinsic state. In this case the rotational band consists of only one state, the 'singlet'. At $N_c=3$ it is the $\Lambda(1405, 1/2^-)$.

Light baryons, $\frac{1}{2}^+ \to \frac{1}{2}^-$ transition

This is the intrinsic particle-hole excitation *2* in Fig. 2. Y' is the same as for the ground state (see above), hence the rotational band consists of 'octets' and 'decuplets'. Their spins are found from Eq. (2): the rotational band about this transition consists of the following multiplets: $(\mathbf{8}, 1/2^-)$ (twice), $(\mathbf{8}, 3/2^-)$, $(\mathbf{10}, 1/2^-)$, $(\mathbf{10}, 3/2^-)$ (twice) and $(\mathbf{10}, 5/2^-)$. In the leading $1/N_c$ order the three octets are degenerate and so are the four decuplets.

In reality, there is indeed a well-grouped triad of octets with the approximate centers at $1615(\mathbf{8}, 1/2^-)$, $1710(\mathbf{8}, 1/2^-)$ and $1680(\mathbf{8}, 3/2^-)$. Given that degeneracy is always lifted by any additional interaction (for example by the strange quark mass) such that one state goes up and the other goes down, this seems to be a success of the description. The situation is worse with parity-minus decuplets. Two of them are rather well identified at approximately $1758(\mathbf{10}, 1/2^-)$ and $1850(\mathbf{10}, 3/2^-)$. As to the rest decuplets, there is only a one-star $\Delta(1940, 3/2^-)$ and a three-star $\Delta(1930, 5/2^-)$ in the PDG, which can fit into the picture but the experimental situation is inconclusive.

Light baryons, $\frac{1}{2}^+ \to 0^+$ transition

This is the intrinsic particle-hole excitation *4* in Fig. 2. At $N_c = 3$ it corresponds to $Y' = 2$, and the allowed multiplets are the $\overline{\mathbf{10}}$ and the $\mathbf{27}$, with $T' = 0$, 1, respectively, and

$X=0$ in both cases. The generalization is $Y'=\frac{N_c+3}{3}$. The prototype 'anti-decuplet' is

$$\begin{cases} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{cases} = \begin{cases} 0 \\ \frac{N_c+3}{2} \\ \frac{(N_c+5)(N_c+7)}{8} \\ \frac{N_c+3}{4I_2} \end{cases} \stackrel{N_c\to 3}{\longrightarrow} \begin{cases} 0 \\ 3 \\ \overline{\mathbf{10}} \\ \frac{3}{2I_2}. \end{cases} \tag{A.11}$$

The spin of this '$\overline{\mathbf{10}}$' is $J=\frac{1}{2}$.

To build the prototype '**27**'-plet we put $T'=1$ and $X=0$ and find

$$\begin{cases} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{cases} = \begin{cases} 2 \\ \frac{N_c+1}{2} \\ \frac{3(N_c+3)(N_c+9)}{8} \\ \frac{N_c+3}{4I_2}+\frac{1}{I_1} \end{cases} \stackrel{N_c\to 3}{\longrightarrow} \begin{cases} 2 \\ 2 \\ \mathbf{27} \\ \frac{3}{2I_2}+\frac{1}{I_1}. \end{cases} \tag{A.12}$$

The spins of the '**27**'-plet are found from the vector addition law $\mathbf{J}=\mathbf{1}+\mathbf{\frac{1}{2}}$, and can be both $J=\frac{1}{2}$ and $J=\frac{3}{2}$. In the leading order in $1/N_c$ they are degenerate but both separated by $\frac{1}{I_1}\approx 150\,\mathrm{MeV}$ from the '$\overline{\mathbf{10}}$'. In practical terms it means that there must be an exotic triplet $\Theta^{++},\Theta^{+},\Theta^{0}$ with spins $\frac{1}{2}$ and $\frac{3}{2}$ about 150 MeV heavier that the singlet Θ^+ belonging to the anti-decuplet.

Heavy baryons, ground state

In this case $Y'=\frac{2}{3}$ at $N_c=3$, and the allowed multiplets are $\bar{\mathbf{3}}$, $\mathbf{6}$ since these multiplets contain particles with $Y'=\frac{2}{3}$; for the $\bar{\mathbf{3}}$ there is one such particle, hence $T'=0$, whereas for the $\mathbf{6}$ there are three such particles, hence $T'=1$. In both cases it is the upper line, therefore $X=0$.

Generalizing these multiplets to arbitrary N_c we fix T' and X what they are at $N_c=3$ but change Y' according to Eq. (A.4). For the ground state $Y'=\frac{N_c-1}{3}$. Solving Eqs.(A.5,A.6) with respect to (p,q) we find the prototype 'anti-triplet':

$$\begin{cases} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{cases} = \begin{cases} 0 \\ \frac{N_c-1}{2} \\ \frac{3(N_c+1)(N_c+3)}{8} \\ \frac{N_c-1}{4I_2} \end{cases} \stackrel{N_c\to 3}{\longrightarrow} \begin{cases} 0 \\ 1 \\ \bar{\mathbf{3}} \\ \frac{1}{2I_2}. \end{cases} \tag{A.13}$$

Its spin is determined from $\mathbf{J} = \mathbf{T}' + \mathbf{J_h}$ and can be only $\frac{1}{2}$.

The prototype 'sextet' is

$$\left\{\begin{array}{l} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{array}\right. = \left\{\begin{array}{c} 2 \\ \frac{N_c-3}{2} \\ \frac{3(N_c-1)(N_c+5)}{8} \\ \frac{N_c-1}{4I_2} + \frac{1}{I_1} \end{array}\right. \stackrel{N_c\to 3}{\longrightarrow} \left\{\begin{array}{c} 2 \\ 0 \\ \mathbf{6} \\ \frac{1}{2I_2} + \frac{1}{I_1}. \end{array}\right. \tag{A.14}$$

The spins are $\frac{1}{2}$ and $\frac{3}{2}$.

Heavy baryons, $0^+ \to \frac{1}{2}^-$ transition

This is the intrinsic one-quark excitation analogous to transition *1* in Fig. 2. At $N_c = 3$ it corresponds to $Y' = -\frac{1}{3}$, and the allowed multiplets are the anti-triplet and sextet with $T' = \frac{1}{2}$ and $X = 1$ in both cases. The generalization is $Y' = \frac{N_c-4}{3}$. The prototype 'anti-triplet' is

$$\left\{\begin{array}{l} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{array}\right. = \left\{\begin{array}{c} 0 \\ \frac{N_c-1}{2} \\ \frac{3(N_c+1)(N_c+3)}{8} \\ \frac{2N_c-5}{4I_2} + \frac{3}{8I_1} \end{array}\right. \stackrel{N_c\to 3}{\longrightarrow} \left\{\begin{array}{c} 0 \\ 1 \\ \bar{\mathbf{3}} \\ \frac{1}{4I_2} + \frac{3}{8I_1}. \end{array}\right. \tag{A.15}$$

We see that already the lowest possible multiplet has an unacceptable $\mathcal{O}(1)$ energy as compared to the ground state rotational energy. It means that from the large-N_c viewpoint the existence of this multiplet strictly speaking cannot be claimed. One has to explain the parity-minus heavy baryons by other means – see Section V.

Heavy baryons, $\frac{1}{2}^+ \to 0^+$ transition

This is the intrinsic particle-hole excitation depicted in Fig. 4, left. At $N_c = 3$ it corresponds to $Y' = \frac{5}{3}$, and the allowed multiplet is the anti-decapenta-plet shown in Fig. 4, right, with $T' = \frac{1}{2}$ and $X = 0$. Its arbitrary-N_c generalization has $Y' = \frac{N_c+2}{3}$. The prototype

'anti-decapenta-plet' is characterized by

$$\left\{\begin{array}{c} p \\ q \\ \mathrm{Dim} \\ E_{\rm rot} \end{array}\right. = \left\{\begin{array}{c} 1 \\ \frac{N_c+1}{2} \\ \frac{(N_c+3)(N_c+7)}{4} \\ \frac{N_c+2}{4I_2}+\frac{3}{8I_1} \end{array}\right. \stackrel{N_c\to 3}{\longrightarrow} \left\{\begin{array}{c} 1 \\ 2 \\ \overline{\mathbf{15}} \\ \frac{5}{4I_2}+\frac{3}{8I_1}. \end{array}\right. \tag{A.16}$$

The spins of the 'anti-decapenta-plet' are found from the relation $\mathbf{J} = \mathbf{\frac{1}{2}} + \mathbf{\frac{1}{2}} + \mathbf{\frac{1}{2}}$. Therefore, there are two multiplets with spin $\frac{1}{2}$ and one multiplet with spin $\frac{3}{2}$, all degenerate in the leading order in $1/N_c$. Their lightest members are the exotic Beta-baryons $\mathcal{B}_{b,c}$, the main prediction of this paper.

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$qqqs\bar{Q}$ have been hypothesized by Gignoux, Silvestre-Brac and Richard [28] and Lipkin [29] and denoted as $P_{\bar{c}s}$. I propose here a very different type of pentaquarks $qqqQ\bar{s}$ which I suggest to call "Beta baryons" and denote as $\mathcal{B}_{c,b}$ (calligraphic 'Bee' in LaTeX). The implication is that "Alpha baryons" are mainly the three-quark ones, of course.

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