

Holographic Quantum Foam

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Due to quantum fluctuations, probed at small scales, spacetime is very complicated — something akin in complexity to a turbulent froth which the late John Wheeler dubbed quantum foam, aka spacetime foam. Our recent work suggests that (1) we may be close to being able to detect quantum foam with extragalactic sources once the Very Large Telescope Interferometers (VLTI) are fully operational; (2) dark energy is arguably a cosmological manifestation of quantum foam, the constituents of which obey infinite statistics; (3) in the gravitational context, turbulence is closely related to holographic quantum foam, partly validating Wheeler's picture of a turbulent spacetime.

Keywords: quantum foam, holography, turbulence, dark energy, infinite statistics

1. Introduction

At microscopic scales our world is known to obey quantum mechanics which is characterized by an indeterminism giving rise to fluctuations in measurements. If spacetime, like all matter and energy, undergoes quantum fluctuations, there will be an intrinsic limitation to the accuracy with which one can measure a distance l , for that distance fluctuates by δl . On fairly general grounds, we expect $\delta l \gtrsim l^{1-\alpha} l_P^\alpha$,¹ where l_P is the Planck length, the characteristic length scale in quantum gravity. The parameter $\alpha \sim 1$ specifies the different quantum foam models. (The canonical model² corresponds to $\alpha = 1$ with $\delta l \sim l_P$.)

Applying quantum mechanics and black hole physics (from general relativity), we³ find that a distance l fluctuates by an amount $\sim l^{1/3} l_P^{2/3}$. The corresponding quantum foam model with $\alpha = 2/3$ has become known as the holographic model since it can be shown¹ to be consistent with the holographic principle.⁴

2. Turbulence and Holography

There are deep similarities between the problem of quantum gravity and turbulence.⁵ The connection between these seemingly disparate fields is provided by the role of diffeomorphism symmetry in classical gravity and the volume preserving diffeomorphisms of classical fluid dynamics. Furthermore, in the case of irrotational fluids in three spatial dimensions, the equation for the fluctuations of the velocity potential can be written in a geometric form⁶ of a harmonic Laplace–Beltrami equation: $\frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} g^{ab} \partial_b \varphi) = 0$. Here, apart from a conformal factor, the effective space time metric has the canonical ADM form $ds^2 = \frac{\rho_0}{c} [c^2 dt^2 - \delta_{ij} (dx^i - v^i dt)(dx^j - v^j dt)]$, where c is the sound velocity and v^i are the components of the fluid's velocity vector. We observe that in this expression for the metric, the velocity of the fluid v^i plays the role of the shift vector N^i which is the Lagrange multiplier for the spatial diffeomorphism constraint (the

momentum constraint) in the canonical Dirac/ADM treatment of Einstein gravity: $ds^2 = N^2 dt^2 - h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$. Hence in the fluid dynamics context, $N^i \rightarrow v^i$, and a fluctuation of v^i would imply a fluctuation of the shift vector. This is possible provided the metric of spacetime fluctuates, which is a very loose, intuitive, semi-classical definition of the quantum foam.

But which quantum foam model? Recall $\delta\ell \sim \ell^{1-\alpha} \ell_P^\alpha$. If one defines the velocity as $v \sim \frac{\delta\ell}{t_c}$, where the natural characteristic time scale is $t_c \sim \frac{\ell_P}{c}$, then it follows that $v \sim c \left(\frac{\ell}{\ell_P}\right)^{1-\alpha}$. It is now obvious that a Kolmogorov-like scaling⁷ in turbulence has been obtained, i.e., the velocity scales as $v \sim \ell^{1/3}$ and the two-point function has the needed two-thirds power law provided that $\alpha = 2/3$. Since the velocities play the role of the shifts, they describe how the metric fluctuates at the Planck scale. The implication is that at short distances, spacetime is a chaotic and stochastic fluid in a turbulent regime⁸ with the Kolmogorov length l . The energy cascades are a property of the spacetime foam. But we emphasize that this interpretation of the Kolmogorov scaling in the quantum gravitational setting is valid only for the case of holographic quantum foam (corresponding to $\alpha = 2/3$).^a

3. Detectability of Quantum Foam with Extragalactic Sources

We¹⁰ suggested that spacetime foam might be uncovered by looking for unresolved cores in the images of distant quasars.¹¹ The point is that, due to quantum foam-induced fluctuations in the phase velocity of an incoming light wave from a distant point source, the wave front itself develops a small scale “foamy” structure. This results in the wave vector acquiring a cumulative random fluctuation in direction with an angular spread of the order of $\Delta\phi/2\pi$, where $\Delta\phi = 2\pi\delta l/\lambda = 2\pi N l^{1-\alpha} \ell_P^\alpha/\lambda$ is the fluctuation in the phase of the electromagnetic wave with wavelength λ after traveling a distance l from the distant source. In effect, spacetime foam creates a “seeing disk” whose angular diameter is¹⁰ $\Delta\phi/(2\pi) \sim (l/\lambda)^{1-\alpha} (\ell_P/\lambda)^\alpha$.^b

For a telescope or interferometer with baseline length D , this means that dispersion (on the order of $\Delta\phi/2\pi$ in the normal to the wave front) will be recorded as a spread in the angular size of a distant point source, causing a reduction in the Strehl ratio, and/or the fringe visibility when $\Delta\phi/2\pi \sim \lambda/D$ for a diffraction limited telescope.^c Thus, in principle, for arbitrarily large distances spacetime foam

^aUpdate: Recently we⁹ have proposed a string theory of turbulence that explains the Kolmogorov scaling in $3 + 1$ dimensions and the Kraichnan and Kolmogorov scalings in $2 + 1$ dimensions. We argue that this string theory of turbulence should be understood from the viewpoint of the *AdS/CFT* dictionary. We find that not only can string theory be useful in formulating a theory of turbulence, but the physics of turbulence may provide some guidance to understanding the quantum foam phase of strong quantum gravity.

^bThis is partly based on the intuition that cumulative fluctuations in lengths are comparable in all directions. (In particular, we have assumed the *same* cumulative factors¹ for both transverse and longitudinal directions.) But we should keep in mind that this intuition, though reasonable, could be wrong; after all, spatial isotropy is here “spontaneously” broken with the detected light being from a particular direction.

^c For example, for a quasar of 1 Gpc away, at an infrared wavelength of the order of 2 microns, the

sets a lower limit on the observable angular size of a source at a given wavelength λ . Furthermore, the disappearance of “point sources” will be strongly wavelength dependent happening first at short wavelengths. Interferometer systems (like the VLTI when it reaches its design performance) with multiple baselines may have sufficient signal to noise to allow for the detection of quantum foam fluctuations.

More recently we¹² have considered the feasibility of using various existing or proposed telescopes (for wavelengths from hard X-rays down to radio waves) to test the different spacetime foam models. Figure 1 shows the prediction made for three different models of spacetime foam corresponding to $\alpha = 2/3, 0.6, 1/2$ respectively, for the size of observed haloes produced by accumulated phase dispersion for a source at two redshifts, respectively $z = 4$ and $z = 1$. The labeled arrows delineate the diffraction limited response of various telescopes. If the arrow representing a telescope’s diffraction limited response lies below the halo size curve for a given α , that model may be excluded by observations. ^d We notice that the VLTI appears to be the most promising to test the holographic model.

4. Holographic Quantum Foam Cosmology and Infinite Statistics

If there is unity of physics connecting the very small and the very large, then it is natural to apply holographic quantum foam physics to cosmology.^{1,13} We find that, according to the cosmology inspired by holographic quantum foam (dubbed HFC), the cosmic density is $\rho = (3/8\pi)(R_H l_P)^{-2} \sim (H/l_P)^2$, consistent with observation (H is the Hubble parameter of the observable universe and R_H is the Hubble radius), ^e and that the cosmic entropy is given by $I \sim (R_H/l_P)^2$. Furthermore, with the aid of archived data from the Hubble Space Telescope indicating the demise of the $\alpha = 1/2$ model, HFC has provided another argument for the existence of dark energy (independent of other cosmological/astrophysical observations of recent years). Successes of the conventional big bang cosmology, such as nucleosynthesis, can also be incorporated into HFC.

Here we concentrate on one crucial question: What is the overriding difference between conventional matter and dark energy (perhaps also dark matter) according to HFC? Since dark energy carries most of the energy of the Universe, let us focus on its constituent particles. Consider a perfect gas of N particles obeying Boltzmann

holographic model of spacetime foam predicts a phase fluctuation $\Delta\phi \sim 2\pi \times 10^{-9}$ radians. On the other hand, an infrared interferometer with $D \sim 100$ meters (like the VLTI) has $\lambda/D \sim 5 \times 10^{-9}$.

^dUpdate: Recently we¹² have elaborated on our proposal to detect quantum foam with extragalactic sources and have argued that the appropriate distance measure for calculating the expected angular broadening is the total line-of-sight comoving distance. We then deal with recent data and the constraints they put on spacetime foam models. Thus far, images of high-redshift quasars from the Hubble Ultra-Deep Field (UDF) provide the most stringent test of spacetime foam theories. Indeed, we see a slight wavelength-dependent blurring in the UDF images selected for this study. Using existing data in the HST archive we find it is impossible to rule out the $\alpha = 2/3$ model, but exclude all models with $\alpha < 0.65$.

^eSince critical cosmic energy density is the hallmark of the inflationary paradigm, HFC may supplement/implement inflation in solving some of the classic cosmological problems.

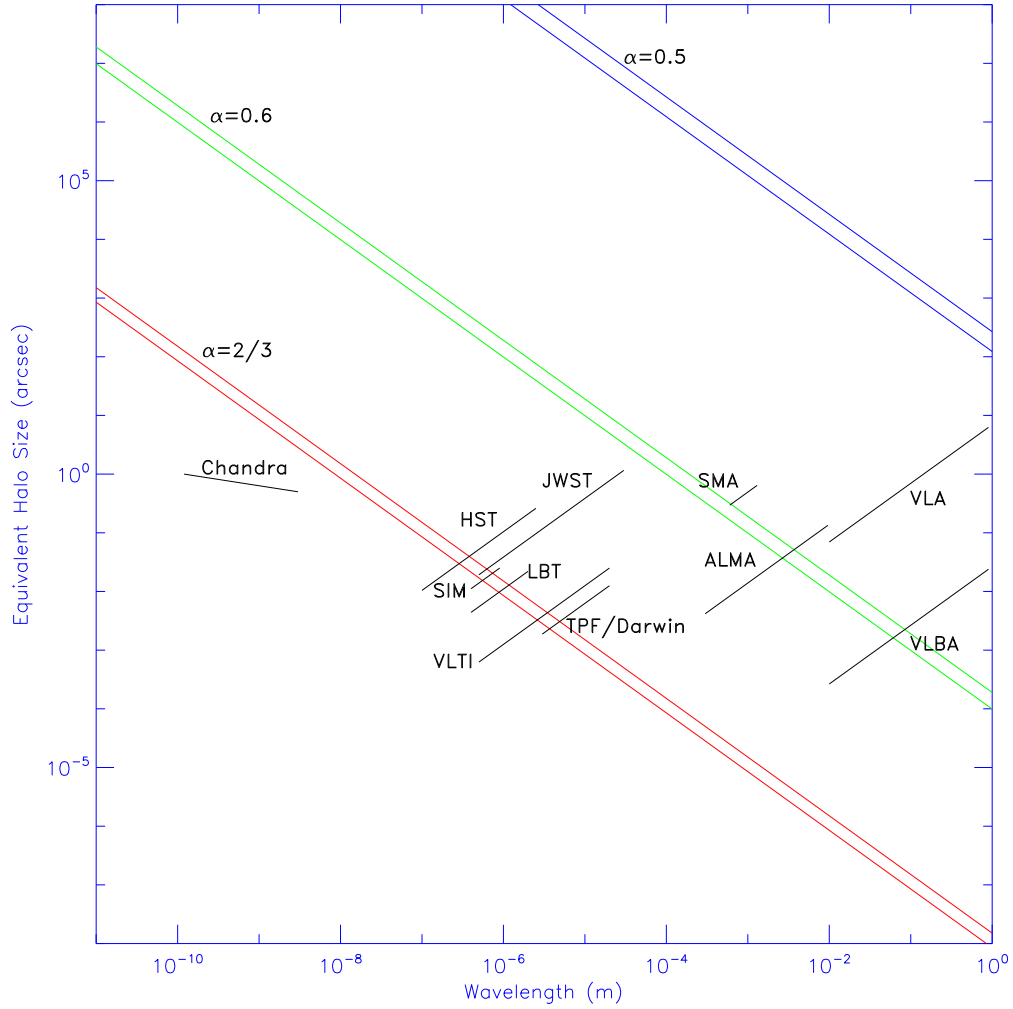


Fig. 1. Detectability of various models of foamy spacetime with existing and planned telescopes.

statistics at temperature T in a volume V . For the problem at hand, we take $V \sim R_H^3$, $N \sim (R_H/l_P)^2 \gg 1$ and $T \sim R_H^{-1}$ (the average energy carried by each particle). A standard calculation yields the partition function $Z_N = (N!)^{-1}(V/\lambda^3)^N$, where $\lambda \sim T^{-1}$. We get, for the entropy of the system, $S = N[\ln(V/N\lambda^3) + 5/2]$. The important point to note is that, since $V \sim \lambda^3$, the entropy S becomes nonsensically negative. But the solution is pretty obvious: the N inside the log in S somehow must be absent. In that case, the Gibbs $1/N!$ factor must be absent from the partition function Z_N , accordingly the “particles” are distinguishable and nonidentical, and the entropy becomes $S = N[\ln(V/\lambda^3) + 3/2]$.

Now the only known consistent statistics in greater than two space dimensions without the Gibbs factor is infinite statistics (sometimes called “quantum Boltzmann statistics”).^{14,15} Thus the “particles” constituting dark energy obey infinite statistics, instead of the familiar Fermi or Bose statistics.¹³ (Using the Matrix theory approach, Jejjala, Kavic and Minic¹⁶ have also argued that dark energy quanta obey infinite statistics.) This is the crucial difference between the constituents of dark energy and ordinary matter, according to HFC. Since each “particle” has such long wavelength ($\sim R_H$), dark energy acts like a (dynamical) cosmological constant. But to get the correct equation of state and an appropriate transition from an earlier decelerating to a recent accelerating cosmic expansion, one may need to take into account the coupling between pressureless dark matter and holographic dark energy.¹⁷ Finally we note that a theory of particles obeying infinite statistics cannot be local.¹⁵ But intuitively holographic theories also possess non-locality. Thus it is not surprising that non-locality is present in HFC, a cosmology that is intimately connected to holography.^{1,13} The appearance of infinite statistics in HFC is suggestive of a holographic view of spacetime.

Acknowledgments

This work was supported in part by the US Department of Energy.

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