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# Higher Dimensional Dark Energy Investigation with Variable Λ and $G$

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Time variable Λ and $G$ are studied here under a phenomenological model of Λ through an $(n+2)$ dimensional analysis. The relation of Zeldovich [21] $|\Lambda| = 8\pi G^2 m_p^6/h^4$ between Λ and $G$ is employed here, where $m_p$ is the proton mass and $h$ is Planck's constant. In the present investigation some key issues of modern cosmology, viz. the age problem, the amount of variation of $G$ and the nature of expansion of the Universe have been addressed.

*Keywords*: Higher dimension; variable Λ; variable $G$.

## 1. Introduction

The inception of the idea of a possible time variation of the gravitational constant $G$ by Dirac [1] and subsequent supportive works, both at theoretical [2,3,4,5,6,7,8] and observational [9,10,11,12,13,14,15,16,17,18] level opened up a new pathway in cosmological research. Numerous works in this direction have been done by taking $G$ as a variable. After the discovery of the scenario of accelerating Universe [19,20], investigations within the framework of variable $G$ is also not uncommon in the literature. One of the techniques of dark energy investigation is to include a Λ term in the field equations of Einstein. Since a constant Λ is handicapped by the well known Cosmological Constant Problem and Coincidence Problem, so it is not unnatural

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to consider $\Lambda$ as a variable quantity. When the $\Lambda$ term is placed in the right hand side of the field equations and is considered as a part of the energy-momentum tensor, then instead of $T^{\mu\nu}$, the total energy-momentum tensor $T^{\mu\nu} + (\Lambda/8\pi G)g^{\mu\nu}$ is conserved. On the other hand, as an extension of Dirac's Large Number Hypothesis (LNH), Zeldovich [21] established a connection (to be shown explicitly afterwards) between the cosmological parameter $\Lambda$ and the gravitational constant $G$ by considering $\Lambda$ as the gravitational energy of the vacuum. So, variability of $\Lambda$ admits the variability of $G$ also provided one considers the other quantities of the relation of Zeldovich [21] as constant.

As a part of dark energy investigation, Ray et al. [22] considered $\Lambda$ models in four dimensional space-time by taking $G$ as time-dependent. So, it is not unnatural to make an attempt for exploring new physical features by venturing into dimensions higher than the usual $4D$. This is the motivation behind the present work where a $(n + 2)$ dimensional study of time-dependent $\Lambda$ model has been done within the framework of variable $G$. We have addressed here some key issues of modern cosmology, viz. the age problem, the amount of variation of $G$ and the nature of expansion of the Universe. The scheme of the study is as follows: Section 2 deals with field equations and their solutions while some physical features arising out of this investigation are described in different subsections of Section 3. Finally, some concluding remarks are made in Section 4.

## 2. Field Equations and Their Solutions

The $(n+2)$ dimensional metric for homogeneous and isotropic Universe is given by

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2(dx_n)^2], \tag{1}$$

where $a(t)$ is the scale factor and

$$(dx_n)^2 = d\theta^2 + sin^2\theta_1 d\theta_2^2 + ...... + sin^2\theta_1^2 sin^2\theta_2^2 ....sin^2 d\theta_n^2. \tag{2}$$

For a flat $(k = 0)$ Universe, the above metric yields the field equations given by

$$\frac{n(n+1)}{2}\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho + \Lambda \tag{3}$$

and

$$n\frac{\ddot{a}}{a} + \frac{n(n-1)}{2}\left(\frac{\dot{a}}{a}\right)^2 = -8\pi G p + \Lambda, \tag{4}$$

where $\Lambda \equiv \Lambda(t)$ and $G \equiv G(t)$.

The barotropic equation of state is given by

$$p = \omega\rho, \tag{5}$$

where the barotropic index $\omega$, assumed to be constant here, can take the values 0, 1/3, 1 and $-1$ for pressureless dust, electromagnetic radiation, stiff (Zeldovich) fluid and vacuum fluid respectively. Some other limits on $\omega$ coming from SN Ia data [23] and a combination of SN Ia data with CMB anisotropy and galaxy clustering statistics [24] are given by $-1.67 < \omega < -0.62$ and $-1.3 < \omega < -0.79$ respectively.

Now, the relation of Zeldovich [21] between $\Lambda$ and $G$ is given by

$$|\Lambda| = \frac{8\pi G^2 m_p^6}{h^4}, \tag{6}$$

where $m_p$ is the proton mass and $h$ is Planck's constant.

From equation (6) we can write

$$G = C\sqrt{\Lambda}, \tag{7}$$

where $C = h^2/\sqrt{(8\pi)}m_p^3 =$ constant.

Let us use the *ansatz*

$$\Lambda = 3\alpha H^2, \tag{8}$$

where $\alpha$ is a parameter.

Then from (3) we have

$$\rho = \left[\frac{n(n+1) - 6\alpha}{16\pi B\sqrt{\alpha}}\right] H, \tag{9}$$

where $\dot{a}/a = H$ and $B = \sqrt{3}C$.

Using equations (9) and (5) and remembering that $\ddot{a}/a = \dot{H} + H^2$ we get from (4),

$$n\dot{H} = (1+\omega)\left[\frac{6\alpha - n(n+1)}{2}\right] H^2. \tag{10}$$

Solving equation (10) we get our solution set as

$$a(t) = C_1 t^{\frac{2n}{(1+\omega)[n(n+1)-6\alpha]}}, \tag{11}$$

$$H(t) = \frac{2n}{(1+\omega)[n(n+1)-6\alpha]} t^{-1}, \tag{12}$$

$$\rho(t) = \frac{n}{8\pi B(1+\omega)\sqrt{\alpha}} t^{-1}, \tag{13}$$

$$G(t) = \frac{2nB\sqrt{\alpha}}{(1+\omega)[n(n+1)-6\alpha]} t^{-1}, \tag{14}$$

$$\Lambda(t) = \frac{12n^2\alpha}{(1+\omega)^2[n(n+1)-6\alpha]^2} t^{-2}, \tag{15}$$

where $C_1$ is a constant.

It is interesting to note that in four dimensional case (i.e. $n = 2$), expressions for $a(t)$, $H(t)$ and $\Lambda(t)$ become identical with that of Ray et al. [22] for the same $\Lambda$ model and under the same assumptions, i.e. by taking both $\Lambda$ and $G$ as time-dependent but without choosing any particular expression for $G$. Moreover, here $G \propto 1/t$ as obtained earlier also by Dirac [1] as well as by Ray et al. [22]. Here, for physical validity, $\alpha$ cannot be equal to 1 in four dimensional case and can, in no way, be negative.

## 3. Physical Features of the Present Model

### 3.1. *Age of the Universe*

From equation (12) we find that if $\alpha = 0$ (i.e. $\Lambda = 0$), then in four dimensional case for pressureless dust,

$$t = \frac{2}{3H}. \tag{16}$$

It is clear from equation (16) that for the present accepted value of the Hubble parameter, which is $(72 \pm 8)$ kms$^{-1}$Mpc$^{-1}$ [25], the present age of the Universe becomes less than the age of the globular clusters which lies in the range 9.6 Gyr [8]to $12.5 \pm 1.5$ Gyr [26,27]. This means that dark energy models without $\Lambda$ suffer from low age problem. This justifies the necessity for including the $\Lambda$ term in the field equations as also mentioned by Sahni and Starobinsky [28].

Again, for $n = 2$ and $\alpha = 1/3$, $Ht = 1$. This implies that for pressureless dust, we can get the exact age of the present Universe which lies in the range $(14 \pm 0.5)$ Gyr [29,30,31,32]. It may be mentioned here that Ray and Mukhopadhyay [33] obtained the exact age of the Universe for stiff fluid (i.e. $\omega = 1$) for the same $\Lambda$ model with constant $G$. But stiff fluid model is not an accepted model for the present Universe. So, in that respect the present work has been successful in solving the so called age problem of the Universe without any unrealistic assumption. Moreover, it is easy to see that, with proper tuning of $\alpha$, the exact value of the present Universe can be obtained from equation (12) in dust case irrespective of the dimension. For instance, in five dimension (i.e. $n = 3$), the accepted value of the age can be obtained for $\alpha = 1$. Also for obtaining the exact age, the value of $\alpha = 1$ has to be increased with the number of dimension and for each dimension the model suffers from low-age problem if we discard $\Lambda$, i.e. if we put $\alpha = 0$.

### 3.2. *Rate of change of G*

From equation (14) we get,

$$\frac{\dot{G}}{G} = -\frac{1}{t}. \tag{17}$$

Equation (17) tells us that the rate of change of $G$ is of the order of $t^{-1}$. If we take the present age of the Universe as 14 Gyr, then (since 1 Gyr is nearly $4 \times 10^{17}$ seconds and 1 year is about $3 \times 10^{7}$ seconds) the value of $\dot{G}/G$ is of the order of $10^{-11}$ per year which is supported by various theoretical and observational results (a comprehensive list in this regard is provided in Ref. 22). Moreover, $\dot{G}/G$ does not depend on $n$, $\alpha$, mass of proton and Planck's Constant. This means that the rate of change of $G$ is independent of the space-time dimension, the parameter associated with dark energy and two constants of microscopic world. This result justifies the claim that dark energy does not exhibit any gravitational effect like clustering and gravity is fundamentally different from other types of forces of nature.

### 3.3. *Calculation of the deceleration parameter*

The deceleration parameter $q$ is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(1 + \frac{\dot{H}}{H^2}\right). \tag{18}$$

Then, using equation (12), we get from equation (18),

$$q = -\left[\frac{n(1+\omega) + (1+\omega)(6\alpha - n^2)}{2n}\right]. \tag{19}$$

Then, for pressureless dust ($\omega = 0$) we get from equation (19),

$$q = \left(\frac{n^2 - n - 6\alpha}{2n}\right). \tag{20}$$

It has been already shown that in four dimensional case ($n = 2$) we can get the exact age of the Universe, so far as observational results are concerned, for $\alpha = 1/3$. Now, for $n = 2$ and $\alpha = 1/3$ we get $q = 0$ which means that the Universe expands with a uniform velocity. So, we are not getting an accelerating Universe what the present cosmological scenario demands. Another interesting point is, corresponding to every value of $\alpha$ for which we get the exact age, we get the value of $q$ as zero whatever may be the spatial dimension. This means that the Universe expands with uniformly in every dimension. This situation can be compared with the so called 'hesitation period' (when the quasars were supposed to be formed) of the Eddington-Lemaîtré model. While in the Eddington-Lemaîtré model the Universe behaves like de-Sitter model for large $t$, here the Universe expands uniformly for all the time. However, an accelerating Universe can be obtained from this model also under proper tuning of $\alpha$. For example, in four dimensional case ($n = 2$) with pressureless dust, $q$ becomes $-0.1$ for $\alpha = 0.4$ and in that case the age of the Universe comes out as 14.9 Gyr which is not an unreasonable estimate. Similar is the case for all other dimensions.It may be mentioned here that Khadekar and Butey [34] also obtained $q = 0$ for $N = 1$ where under the assumption $\rho_m = \gamma/R^N$, $\gamma$ and $N(> 0)$ being constants It is interesting to note here that for all those tuned values of $\alpha$ which give us the exact age of the Universe, the scale factor increases linearly. It may be mentioned here that linear expansion in a Robertson-Walker Universe is shown to be possible also by Usmani et al. [35] for the $\Lambda$ model $\dot{\Lambda} \sim H^3$.

## 4. Conclusions

In the present work, some new features of an widely used Λ-dark energy model has been explored through an $(n+2)$ dimensional analysis with variable $\Lambda$ and variable $G$. Recently Khadekar and Butey [34] have studied a similar higher dimensional cosmological model where both $\Lambda$ and $G$ are variable. While Khadekar and Butey [34] have chosen $G$ by following Eddington-Weinberg empirical relation [36] and Singh [37], here in the present case a particular relation of Zeldovich's [21] in connection to the cosmological parameter is used for obtaining an expression for the so-called gravitational constant. The approach of the present work is also completely different from that of Khadekar and Butey [34].

In the present investigation we have been addressed, by using a well known $\Lambda$ model, some key issues of modern cosmology, viz. the age problem, the amount of variation of $G$ and the nature of expansion of the Universe. The results we have shown are as follows:

(i) For any dimension, the accepted age of the present Universe can be attained under proper tuning of the parameter $\alpha$.

(ii) The amount of variation of the gravitational constant is shown to be compatible with theoretically and observationally established values without any prior assumption on the value of the parameter $\alpha$. This is an improvement over the work of Ray and Mukhopadhyay [33] where the fine-tuning of the parameters were essential for obtaining the currently accepted values so far as the amount of variation of $G$ is concerned. But, here no such precondition is required.

(iii) The present model has been successful in obtaining both negative and zero values of the deceleration parameter $q$ which correspond to accelerating and uniformly expanding Universe respectively.

Finally, it should be mentioned that although in the present work emphasis is given on the zero value of the equation of state parameter $\omega$, but the present model also admits negative values of $\omega$ with the restriction $0 < \omega < -1$.

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