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Comparison of Daubechies wavelets for Hurst parameter estimation

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Abstract

Time scale dependence on the working nature of wavelet analysis makes it a valuable tool for Hurst parameter estimation. Similar to other wavelet-based signal processing applications, the selection of a particular wavelet type and vanishing moment in wavelet based Hurst estimation is a challenging problem. In this paper, we investigate the best Daubechies wavelet in wavelet based Hurst estimation for an exact self similar process, fractional Gaussian noise and how Daubechies vanishing moment affects the Hurst estimation accuracy. Daubechies wavelets are preferred in analysis because increasing vanishing moment does not cause excessive increase of time support of Daubechies wavelets. Thus, limited time support of wavelets reduces the border effects. Results show that Daubechies wavelets with one vanishing moment (Daubechies 1) gives the best estimation result for short range dependent fractional Gaussian noise. Daubechies 2 is the best preference for long range dependent fractional Gaussian noise.

Key Words: *Daubechies Wavelets, Fractional Gaussian Noise, Hurst Estimation, Vanishing Moment.*

1. Introduction

Self-similarity has attracted attention in the network traffic community. It is not only a simple phenomenon related to correlation. It has changed the view of many basic artifacts in many disciplines. For example, Hurst discovered a parameter related to self-similarity while investigating the discharge time series of the river Nile in the design of a reservoir [1]. Performance-related calculations for computer networks, such as resource sharing, efficient queue management, and routing management have been studied as self-similarity artifacts occurred in most teletraffic modeling. As self-similarity is significant in several disciplines, its efficient estimation is vital. R/S statistics [2], Variance [3], Absolute Moment [4] and Variance of Residuals [5] methods are commonly used time-based estimators. Frequency-based Hurst estimation methods take the power law behavior of power spectral density into consideration. Daniell PB estimator [3], Whittle Maximum Likelihood [2] and Local Whittle ML [6] are frequency based estimators.

Wavelet analysis has been employed in Hurst estimation due for its powerful properties. The main concern which makes wavelet analysis so important for Hurst estimation is the fact that it is time scale-dependent [7–11]. Scale invariance and the bursty nature of self similar processes could be well examined via wavelet transform. An efficient wavelet-based estimator called Abry-Veitch DWB was proposed by Abry and Veitch in 1998 [12]. Abry and Veitch used Daubechies wavelets as a kernel function because of their limited time support that eases the handling of border effects [13]. The motivation behind the selection of Daubechies wavelets for self-similarity analysis is that they lead to more accurate results by a better matching self similar structure of long-range dependent processes than of other wavelet types [14]. Daubechies wavelets are also widely used in areas such as signal discontinuities, electrical disturbances and localization problems [15–17].

In this study, we investigate how vanishing moments of Daubechies wavelets affects the accuracy of Hurst estimation in various cases. A vanishing moment determines coefficient number, smoothness and time support of a wavelet. It controls the correlation structure of wavelet coefficients, which deeply affects power law behavior of the power spectrum and Fourier transform of a wavelet around zero frequency. We choose Daubechies wavelets as kernel function due to their limited time support with increasing number of vanishing moments, which eases the handling of border effects, as explained in [12]. Some conclusions were drawn on selection of wavelet type and vanishing moment for Hurst estimation from studies proposed in the literature [12, 18, 19]. Number of vanishing moments controlled correlation at a given scale and between scales and large vanishing moments lead to uncorrelated coefficients [12, 18]. In [18], the effect of vanishing moment for synthesis of fractional Brownian motion was explained. But, no conclusion has been drawn about the selection of wavelet for Hurst estimation. Only Haar wavelet (Daubechies 1) was used in Hurst estimation. A more detailed study about selection of Daubechies wavelets for Hurst estimation was performed in [19]. With $R > H + 1/2$, (R and H refer vanishing moment number and Hurst exponent, respectively), the study started the comparison with Daubechies 3 and went on to Daubechies of higher vanishing moments. Nevertheless, Daubechies 1 and Daubechies 2 also confirm the given formula with some Hurst degrees. A variable bit rate video trace was used for comparison of Daubechies wavelets in [19]. We used an exact self similar and stationary process, fractional Gaussian noise (FGN) in analysis. Considering the effect of correlation structure of wavelets coefficients on variance, we use various Daubechies with different vanishing moments during wavelet-based FGN synthesis to find which Daubechies wavelet gives the best Hurst estimation in the determined synthesis condition. This study extracts the relationship between them and gives key conclusions about to what extent self similarity which Daubechies wavelet gives the best Hurst estimation.

This paper is organized as follows. Basic properties and mathematical equations of Hurst parameter and fractional Gaussian noise are given in Section 2. Wavelet based synthesis and self similarity analysis of fractional Gaussian noise are introduced in Section 3. We outline in detail two separate analysis studies in Section 4, then conclude the paper in Section 5.

2. Basics with self similarity and fractional gaussian noise

Hurst parameter is a numerical measure of self similarity and gives a measure of long-range dependency in a stochastic process. A randomly scattered process is characterized with Hurst value of 0.5. A continuous time stochastic process $\{X(t),\ t \in R\}$ is strictly self similar with Hurst parameter H, $0< H <1$, when

$$X(at) \stackrel{d}{=} a^H X(t). \tag{1}$$

Here, $X(at)$ is a new process which is scaled by a factor a and the symbol $\stackrel{d}{=}$ expresses equal in finite dimensional distributions.

A more general case of self similarity, wide sense self similarity is only interested in first two moments. Let X_k is a discrete time stochastic process, which is defined at discrete time points k=1, 2,..., n and $X_k^{(m)}$ is a m-aggregated time series calculated as

$$\{X_k^{(m)} = (X_{km-m+1} + \cdots + X_{km})/m, k \geq 1, m = 1, 2....\}. \tag{2}$$

$$Var[X_k^{(m)}] = m^{2H-2} Var[X_k], \tag{3}$$

$$\rho_z^{(m)} = \rho_z, \quad z \geq 0. \tag{4}$$

If variance and correlation structures obey equations (3) and (4), the process is called wide sense self similar or second order self similar. $\rho_z^{(m)}$ shows autocorrelation coefficient of m aggregated series with respect to lag z. As it can be easily seen from equation (4), m aggregated series and original series follow the same correlation structure in wide sense self similarity.

Fractional Brownian motion (FBM) and its incremental version, fractional Gaussian noise (FGN), are widely used self similar sequences in modeling studies. Both of them could be characterized with a parameter, namely the Hurst parameter [20]. FBM is a continuous time, zero mean, non stationary Gaussian process. Its non-stationarity could be easily observed via its covariance function

$$E[B_H(t_1)B_H(t_2)] = \frac{\sigma^2}{2}/2(t_1^{2H} + t_2^{2H} - |t_1 - t_2|^{2H}). \tag{5}$$

From equation (5), the variance of FBN is

$$E[B_H^2(t)] = \sigma^2 t^{2H}, \tag{6}$$

where $E[]$ is the expectation value operator. FGN is another exact self similar Gaussian process which is incremental version of FBM.

Autocorrelation function of FGN is

$$\gamma(k) = \frac{1}{2}(|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}) \quad k = 0, 1, 2... \quad . \tag{7}$$

where k denotes time lag. Its spectral density follows a power law

$$\ell_{Z_H} = \frac{\sigma^2}{|w|^{2H-1}} \qquad w \to 0. \tag{8}$$

where ℓ_{Z_H}, σ^2 and w are spectral density, variance and angular frequency, respectively.

3. Wavelet-based self similarity analysis and synthesis of fractional gaussian noise

Wavelet representation provides a multiresolution expression of a signal with localization in both time and frequency. A function $X(t)$ is decomposed into a family of functions, called wavelets, in wavelet representation. Each wavelet is created by scaling and translating of a special function, called the mother wavelet, which oscillates, has finite energy and zero mean [16]:

$$\int_R \psi(t)dt = 0. \tag{9}$$

A mother wavelet is required to satisfy admissibility condition and has at least one vanishing moment,

$$\int_R t^k \psi(t)dt = 0, \quad k = 0, 1, 2, \ldots, N-1 \quad , \tag{10}$$

where N denotes the number of vanishing moments [21].

Scale dependent working nature of wavelet analysis makes it a valuable tool for self similar sequence generation and self similarity estimation. For an efficient wavelet based FGN synthesis, we consider input coefficients as they must have identical statistical properties to that of obtained coefficients from wavelet analysis of FGN. Corresponding detail coefficients $d_j[n]$ and approximation coefficients $a_j[n]$ from wavelet analysis of a FGN process ($Z_H(t)$) is calculated as

$$d_j[n] = 2^{-j/2} \int_{-\infty}^{+\infty} Z_H(t)\psi(2^{-j}t - n)dt, \quad j \in Z, n \in Z \tag{11}$$

$$a_j[n] = 2^{-j/2} \int_{-\infty}^{+\infty} Z_H(t)\phi(2^{-j}t - n)dt, \quad j \in Z, n \in Z, \tag{12}$$

where $\phi(t)$ is the scaling function which is associated with $\psi(t)$, j denotes scale and n denotes time. For any given resolution 2^J (J also denotes scale), wavelet mean square representation of FGN is

$$Z_H(t) = 2^{-J/2} \sum_{-\infty}^{+\infty} a_J[n]\phi(2^{-J}t - n) + \sum_{j=-\infty}^{J} 2^{-j/2} \sum_{-\infty}^{+\infty} d_j[n]\psi(2^{-j}t - n). \tag{13}$$

Correlation between detail coefficients in the same scale is calculated as

$$E\{d_j[n]d_k[m]\} = 2^{-j/2} \int_{-\infty}^{+\infty} A_\psi(2^{j-k}), \tau-(2^{j-k}n - m))|\tau|^{2H} d\tau (2^k)^{2H-1}, \tag{14}$$

where

$$A_\psi(\alpha, \tau) = \sqrt{a} \int_{-\infty}^{+\infty} \psi(t)\psi(\alpha t - \tau)dt. \tag{15}$$

$A_\psi(\alpha,\tau)$ is called a wide-band ambiguity function of $\psi(t)$ [22, 23].

From equation (13), detail coefficients of FGN are stationary and their variance is

$$var(d_j[n]) = \frac{\sigma^2}{2} V_\psi(H)(2^j)^{(2H-1)}, \tag{16}$$

where $V_\psi(H)$ depends on both selected wavelet and Hurst parameter:

$$V_\psi(H) = -\int_{-\infty}^{+\infty} \gamma_\psi(\tau)|\tau|^{2H} d\tau. \tag{17}$$

Hurst parameter of FGN is easily calculated from the relationship between wavelet coefficient variance and corresponding scale [24]:

$$\log_2(var(d_j[n])) = (2H-1)j + \text{consant}. \tag{18}$$

Due to the correlation structure of detail coefficients, they cannot be generated randomly for wavelet-based FGN synthesis. They must follow the given correlation structure in equation (14). However, considering synthesis of FGN with some wavelets that approximate Karhunen-Loeve expansions, a generated sequence would adapt to self similar process [9]. Thus, wavelet coefficients could be generated randomly with selection of these particular wavelets for wavelet based FGN synthesis.

Selection of a wavelet function for FGN synthesis depends on the behavior of the Fourier transform of a given wavelet $\{\psi(w)\}$ at zero frequency [22]. Therefore decay properties of coefficient correlations need frequency in the characterization:

$$\begin{aligned} E\{d_j[n]d_k[m]\} = \; & \tfrac{\sigma^2}{2}(2\sin(\textstyle\prod H)\Gamma(2H-1) \\ & \times \int_{-\infty}^{+\infty} e^{iw(n-m)} \tfrac{|\psi(w)|^2 dw}{2H-1})2^{-j/2} \\ & \times \int_{-\infty}^{+\infty} A_\psi(2^{j-k}), \tau-(2^{j-k}n-m))|\tau|^{2H} d\tau (2^k)^{2H-1}, \end{aligned} \tag{19}$$

If the inequality

$$H < N - \frac{1}{2} \tag{20}$$

is provided, it follows $|\psi(w)|^2 \approx |w|^{-(2H-1)}$ at zero frequency and provides 1/f type spectrum behavior. N denotes the order at which the moment vanishes. The character diverges from self similar behavior when $H \geq N - \frac{1}{2}$.

As shown by Wornell [9], convenient simplification for FGN synthesis is to ignore the correlation between wavelet coefficients. Considering a collection of uncorrelated coefficients, Wornell shows that their spectrum represents precisely the power-law of 1/f processes.

The need for a vanishing moment plays an important role in wavelet based Hurst estimation and FGN synthesis. Selection of wavelets with particular vanishing moments is necessary so that Hurst estimation provide a flat behavior at the origin and give less variance. When $N > H-1$, regression plot is asymptotically unbiased

[12]. Due to admissibility condition, all Daubechies wavelets including Haar wavelet provides this equation. However, increasing N causes border effects, diminishes number of wavelet coefficients and increases variance [12].

4. Synthesized FGN sequences

Each generated FGN sequence give rise to many an observation. As stated by Wornell [9], ignoring the correlations between wavelet coefficients, 45 different Daubechies wavelets are used for FGN synthesis. Sellan-Meyer synthesis method [25] is used for wavelet based FBM, hence FGN generation. Two strategic approaches are proposed to find the best Daubechies function for Hurst estimation. Abry-Veitch DWB estimation method is used for self similarity estimation procedure. Totally 101500 FGN sequences are generated; 81000 of them is used for first case. Hurst index of generated FGN sequences are determined as 0.1, 0.2…0.9 and generated sequences are indexed as $Z_{1,H}^{Db}(t), Z_{2,H}^{Db}(t)...Z_{200,H}^{Db}(t)$, where H is Hurst degree and Db denotes the Daubechies type used in the synthesis.

4.1. First approach

The aim of the first approach is to ascertain which Daubechies wavelet is better, for which Hurst degree, for a particular FGN synthesis. For the sake of analysis, 9,000 iterations are used for each Hurst index. Conclusions are derived by comparing mean absolute errors (MAE) and variance of errors. Graphics and tables show clearly how vanishing moment affects the reliability of Hurst estimation.

Steps in the iteration to realize this approach are shown in Figure 1.

Two matrices are obtained for a Hurst index of 0.1 at the end of 9000 iterations: one for MAEs and one for variance of errors. Calculated MAE values for H =0.1, for dB1 through dB18, are given in Table 1. Note that the estimation associated with dB1 gives the best and smallest MAE = 0.0732451.

Table 1. MAE values for H =0.1, 9000 iterations.

dB1	**dB2**	**dB3**	**dB4**	**dB5**	**dB6**
0.0732451	0.1167764	0.09886235	0.08565505	0.0881924	0.08333965
dB7	**dB8**	**dB9**	**dB10**	**dB11**	**dB12**
0.0839509	0.0837934	0.1114627	0.10048105	0.1454993	0.1490307
dB13	**dB14**	**dB15**	**dB16**	**dB17**	**dB18**
0.14322665	0.1207729	0.0917636	0.19271385	0.18460985	0.1644204 s

Figure 2 and Figure 3 clearly show dB1 gives the best estimation for H =0.1 for FGN synthesis. MAE is 0.0732451 and aggregate variance is 0.162647.

MAEs were estimated via the same number of iterations for H = 0.2, 0.3, . . . , 0.9. We do not provide all obtained results and plots in this paper, nonetheless, it is easily seen from Figure 2, Figure 3 and Figure 5, there are obvious differences in Hurst estimation for short range dependent and long range dependent FGN's. For H =0.4 and H =0.5, again dB1 gives optimum estimation results. The graphical result for MAE values are shown together for H =0.4 and 0.5 in Figure 4.

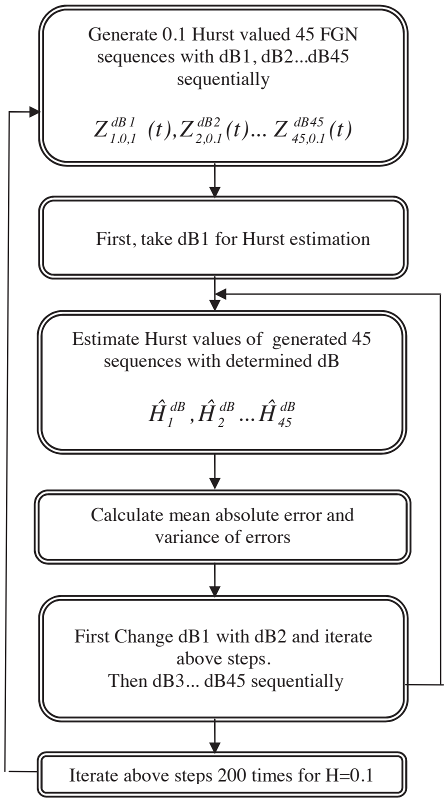

Figure 1. Iterations steps for first approach.

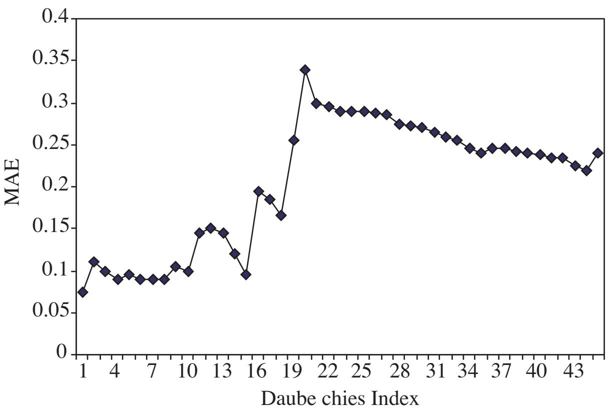

Figure 2. MAE values for $H = 0.1$, 9000 iterations.

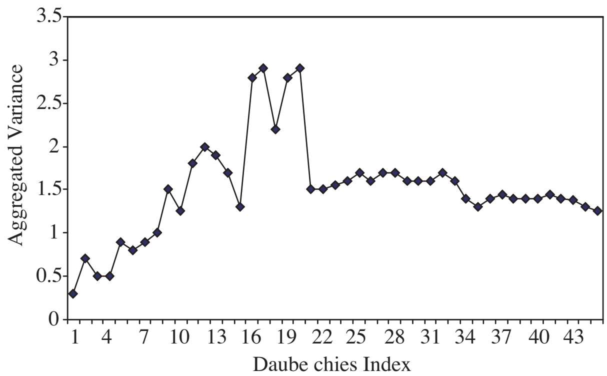

Figure 3. Aggregated variance for $H = 0.11$, 200 iterations.

Difference in estimation errors for dB1 and dB2 lessen with increasing Hurst quantity for short range dependent FGN's; but at all times dB1 gives better results the measure of mean absolute errors and error variances. Obtained estimation results are quite different for long range dependent FGN's. Daubechies with two vanishing moments is the best choice for wavelet-based self similarity estimation. For $H = 0.7$ and 0.9, MAE values are plotted together in Figure 5. As a measure of bias calculated relative mean, inaccuracy results are listed in Table 2. Only estimation error of the first 5 Daubechies are given and they are enough to make a conclusion about which Daubechies is better in Hurst estimation for long range dependent FGN's. Relative mean inaccuracy is calculated as

$$\Delta H = \frac{|\hat{H} - H|}{H} * 100\%, \tag{22}$$

where $\hat{H}$ is estimated Hurst value and H is the exact Hurst value.

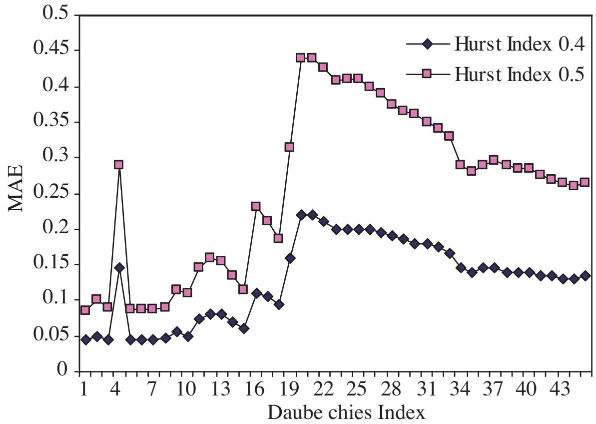

Figure 4. MAE values for $H = 0.4$ and $H = 0.5$, 9000 Iterations.

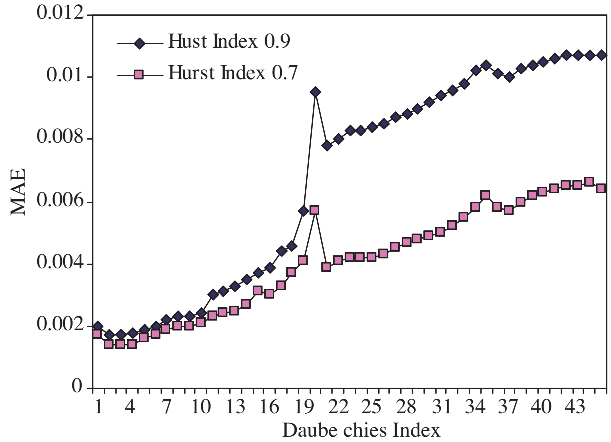

Figure 5. MAE values for $H = 0.7$ and $H = 0.9$.

Table 2. Relative mean inaccuracies for long range dependent FGNs, 9000 Iterations.

Exact Hurst Values	Relative mean inaccuracies for 5 Daubechies (%)				
	dB1	**dB2**	**dB3**	**dB4**	**dB5**
0.6	0.029783	0.024395	0.025177	0.027801	0.029285
0.7	0.030147	0.02287	0.023474	0.025689	0.027476
0.8	0.026378	0.020011	0.02054	0.022478	0.024042
0.9	0.028266	0.021881	0.022031	0.023568	0.025235

4.2. Second approach

In the second case study, the steps of iteration are as shown in Figure 6. 2250 iterations are carried out for each determined Hurst value. This approach aims to get how well a particular Daubechies wavelet for Hurst estimation in comparison to obtained minimum error results. Figures 7 through 10 obviously show how Hurst estimation MAE's, with particular Daubechies, follow minimum MAE values obtained in 45 different Daubechies functions.

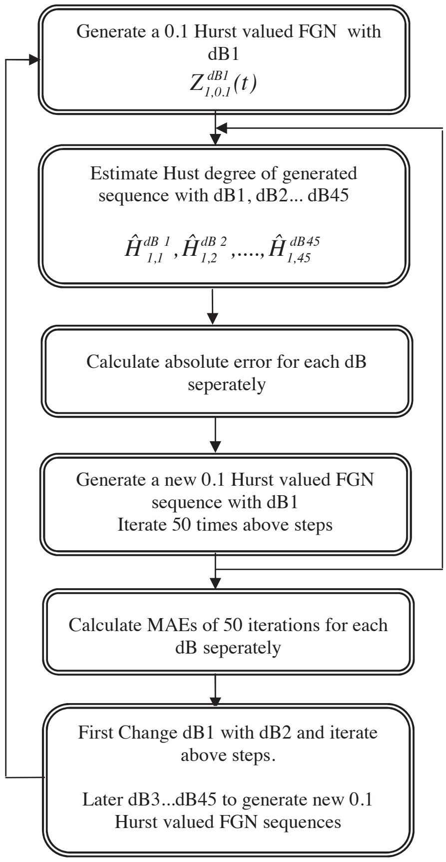

Figure 6. Iteration steps for second approach.

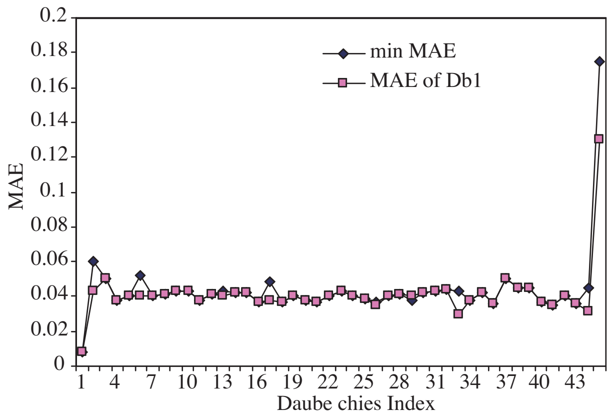

Figure 7. MAE result of dB1 in comparison with minimum MAEs for $H = 0.2$, 50 iterations.

Table 3 shows Hurst estimated MAE's of Daubechies wavelets for $H = 0.1$ FGN's generated with dB1. 50 Iterations are performed for Db1 at $H = 0.1$. Results of only the first 18 Daubechies estimates are given in Table 3. Similar calculations are done for other 0.1 Hurst valued FGN's generated with different vanishing moments.

Table 3. MAE values for $H = 0.1$ valued FGN sequences generated with dB1, 50 iterations.

dB1	**dB2**	**dB3**	**dB4**	**dB5**	**dB6**
0.003539	0.13489	0.11677	0.096803	0.10416	0.10131
dB7	**dB8**	**dB9**	**dB10**	**dB11**	**dB12**
0.10695	0.09714	0.12992	0.11347	0.14987	0.1369
dB13	**dB14**	**dB15**	**dB16**	**dB17**	**dB18**
0.12911	0.10875	0.086048	0.18354	0.17326	0.15596

The iterative procedure shown in Figure 6 was also followed for $H = 0.2,\ 0.3, \ldots,\ 0.9$. To give results for all synthesized FGN sequences and Hurst values is difficult in this paper. Instead, Table 4 gives a summary. It shows how many times a particular Daubechies wavelet gives the best estimation in 45 FGN sequences synthesized with dB1 to dB45. Each step was iterated 50 times under the same conditions. Hurst estimation with dB1 give best results for 45 FGN sequences, dB1 to dB45, sequentially, for $H = 0.1$. The Daubechies given in Table 4 are enough to derive a conclusion.

Table 4. Number of best estimations in 45 different FGN sequences generated with 45 Daubechies, 50 Iterations.

Hurst Values									
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
dB1	23	32	42	42	19	2			
dB2					3	21	19	17	24
dB3					11	14	20	16	13
dB4						7		6	5

Another important observation that could be derived from Approach 2 is how well an estimation value, with a particular Daubechies, in comparison to minimum error value obtained from the 45 Daubechies function. As could be seen from Figure 7 and Figure 8, for Hurst values $H = 0.2$ and $H = 0.4$ sequences, dB1 gives most of the min MAE results. Only two or three times did other Daubechies wavelets give better results. Obtained quantities are in agreement with the first approach. For short range dependent FGN sequences and white noise, estimation with dB1 is better in the case of whatever Daubechies or which Daubechies used for synthesis of FGN.

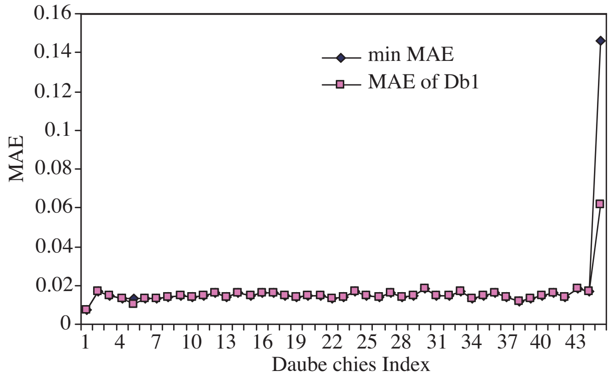

Figure 8. MAE result of dB1 in comparison with minimum MAEs for $H = 0.4$, 50 iterations.

However, for long range dependent processes nearly in all cases Daubechies with two vanishing moments gives the best Hurst estimation results. This is shown in Figure 9 and Figure 10 for $H = 0.7$ and $H = 0.9$, respectively. dB2 finds most of the minimum MAE points in both figures.

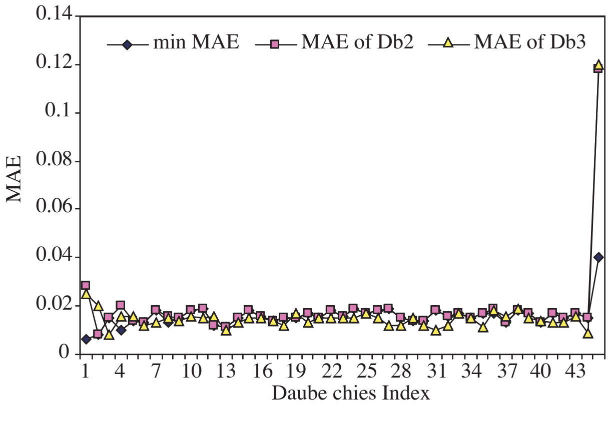

Figure 9. MAE results of dB2 and dB3 in comparison with minimum MAEs for $H = 0.7$, 50 iterations.

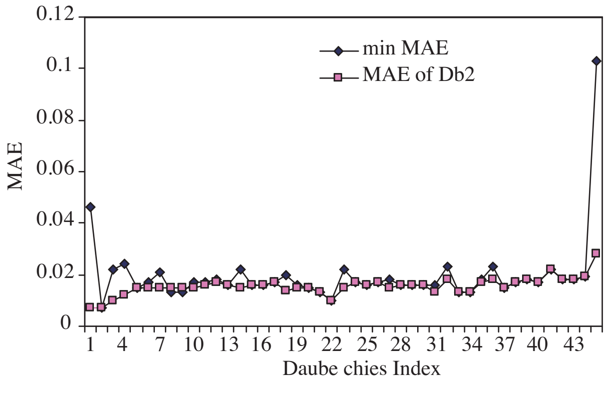

Figure 10. MAE results of dB2 in comparison with minimum MAEs for $H = 0.9$, 50 iterations.

5. Conclusion

Daubechies wavelets are the most-used wavelet in Hurst parameter estimation. A Daubechies wavelet is distinguished from others with its vanishing moment number. As its vanishing moment gets larger, it becomes smoother and gives wider temporal support. It also plays a vital role in the correlation structure of wavelet coefficients. In this study we find the best Daubechies wavelet to estimate Hurst parameter under various conditions. To what extent, and which Daubechies estimates best Hurst parameters for FGN synthesis?

Most teletraffic data types show long range dependency in its correlation structure. According to experimental simulations, Hurst estimation with Daubechies 2 gives better estimation results for long range dependent FGN's. Calculated relative mean inaccuracies with Daubechies 2 are 0.24395%, 0.02287%, 0.020011% and 0.021881% for FGN sequences with Hurst values 0.6, 0.7, 0.8 and 0.9, respectively, and they are minimum error quantities. On the other hand, for short range dependent FGN sequences and also for white noise Daubechies 1 is the best preference. Db1 gives 42 times best estimation results in 45 different 0.3 and 0.4 Hurst valued FGN's generated with Db1, Db2, . . . , Db45, sequentially.

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