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A new chaotic attractor from general Lorenz system family and its electronic experimental implementation

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Abstract

This article introduces a novel three-dimensional continuous autonomous chaotic system with six terms and two quadratic nonlinearities. The new system contains two variational parameters and exhibits Lorenz-like attractors in numerical simulations and experimental measurements. The basic dynamical properties of the new system are analyzed by means of equilibrium points, eigenvalue structures, and Lyapunov exponents. The new system examined in Matlab-Simulink® and Orcad-PSpice®. An electronic circuit realization of the proposed system is presented using analog electronic elements such as capacitors, resistors, operational amplifiers and multipliers. The behaviour of the realized system is evaluated with computer simulations.

Key Words: *Chaotic systems, chaotic circuits, chaotic attractors, chaotic oscillators.*

1. Introduction

Chaos has been shown to be useful in a variety of disciplines, such as information processing, preventing the collapse of power systems, high-performance circuits and devices, and liquid mixing with low power consumption [1]. In 1963, Lorenz found the first chaotic attractor in a three-dimensional autonomous system while studying atmospheric convection [2]. In 1976, Rossler conducted important work that rekindled the interest in low dimensional dissipative dynamical systems [3]. In 1979, Rossler himself proposed an even simpler (algebraic) system [4]. Sprott embarked upon an extensive search [5] for autonomous three-dimensional chaotic systems with fewer than seven terms in the right hand side of the model equations. Sprott considered general three-dimensional ordinary differential equations with quadratic nonlinearities. Using a numerical search, 19 cases (labeled from 'A' to 'S') appear to be distinct in the sense that there is no obvious transformation of one to another. In these 19 ('A' to 'S') cases, 'A' to 'E' (five) have five terms and two nonlinearities, while cases 'F' to 'S' (fourteen) have six terms and one nonlinearity in the right hand side. Finally, Sprott stated that his method couldnt guarantee that those were the simplest chaotic systems of ordinary differential equations, or that all of the chaotic systems with three-dimensional ordinary differential equations with five terms and two quadratic nonlinearities, or with six terms and a quadratic nonlinearity had been discovered. In fact, he reported that the cases with five terms appeared early and often in the search, and thus it wass likely they had all been found. However, new cases with six terms were still being found, indicating that additional such cases probably exist.

Purposefully creating chaos can be a nontrivial task with interesting implications in both basic research and engineering applications. To this end, Chen constructed another chaotic system using an engineering feedback control approach [6], which is not topologically equivalent to Lorenz's [6, 7, 8].

This system is the dual to the Lorenz system and similarly has a simple structure, but displays even more sophisticated dynamical behaviors [7, 8]. Here, the duality is based on a classification condition formulated by Vanecek and Celikovsky [9]. It is notable that Vanecek and Celikovsky [9] classified a generalized Lorenz system family by a condition on its linear part $A = [a_{ij}]$: $a_{12}a_{21} > 0$, which includes the familiar Lorenz system as a special case, while Chen's system satisfies $a_{12}a_{21} < 0$. Hence, Chen's system does not belong to this generalized Lorenz system family. In fact, Chen's system belongs to another canonical family of chaotic systems [10-11]. Lü and Chen found a critical new chaotic system [11-12], which satisfies the condition $a_{12}a_{21} = 0$ and represents the transition between the Lorenz and Chen attractors. In the same year, Lü, et al. constructed a unified system that contains the above three related but nonequivalent chaotic systems [13]. Lü and Chen found that the new chaotic system can display two chaotic attractors simultaneously [14]. The concept of a generalized Lorenz system is extended to a new class of generalized Lorenz-like systems in a canonical form [14]. Moreover, a multiplier-free modified Lorenz system has also been studied [15,16], in which an additional control parameter is used to verify the compound nature of the resulting butterfly-shaped attractor. By designing appropriate control gains, it is possible to confine the chaotic dynamics from one butterfly wing of the attractor to another, forming two simple attractors which, when merged together, form the entire butterfly-shaped attractor. These observations have been verified experimentally through the design of a novel circuit in [16]. None of these systems are topologically equivalent, but together they constitute a complete family of generalized Lorenz dynamical systems.

There has been increasing interest in exploiting chaotic dynamics in engineering applications, where some attention has been focused on effectively creating chaos via simple physical systems, such as electronic circuits[17]-[21]. Lately, the pursuit of designing circuits to produce chaotic attractors has become a focal point for electronics engineers, not only because of their the theoretical interest, but also due to their potential real-world applications[22] in various chaos-based technologies and information systems [22-28].

Motivated by such previous work, this article introduces another simple three-dimensional quadratic autonomous system. The aim of this article is to present a simple, interesting, and yet complex three-dimensional chaotic system, which can depict complex 2-scroll chaotic attractors simultaneously. Section 2 explains the family of general Lorenz dynamical systems. Section 3 introduces and analysis the new chaotic system. The new system is compared with the other general Lorenz family members in detail. Simulation results of the new system using Simulink modeling are also obtained in Section 3. Section 4 presents the electronic circuit schematic and actual circuit realization of the new system. Oscilloscope outputs from the actual circuit and PSpice simulation results are also given. The new circuit is also compared with the other chaotic circuitry in terms of circuit complexity and applicability. Finally, conclusions and discussions are given.

2. The general lorenz system family

Historically, the Lorenz System of equations is perhaps the first of the nonlinear dynamical systems found to exhibit sensitive dependence on initial conditions and chaos. The Lorenz system is described by the following nonlinear differential equations;

$$\begin{aligned}\dot{x} &= a \cdot (y - x)\\ \dot{y} &= c \cdot x - x \cdot z - y\\ \dot{z} &= x \cdot y - b \cdot z\end{aligned} \tag{1}$$

Typical parameters for a Lorenz system are a=10, c=28, and b=8/3. According to the form of the generalized Lorenz system by Vanecek and Celikovsky [9],

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0\\ a_{21} & a_{22} & 0\\ 0 & 0 & a_{33}\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + x \cdot \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} \tag{2}$$

The Lorenz system is described by

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix} -a & a & 0\\ c & -1 & 0\\ 0 & 0 & -b\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + x \cdot \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} \tag{3a}$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix} -10 & 10 & 0\\ 28 & 1 & 0\\ 0 & 0 & -8/3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + x \cdot \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} \tag{3b}$$

The Lorenz system satisfies $a_{12} \cdot a_{21} > 0, (10 \cdot 28 > 0)$.

Chen constructed another chaotic system from an engineering feedback control approach [6], which topologically differs from Lorenz's [6, 7, 8].Chen's system is of the following form:

$$\begin{aligned}\dot{x} &= a \cdot (y - x)\\ \dot{y} &= (c - a) \cdot x - x \cdot z - c \cdot y\\ \dot{z} &= x \cdot y - b \cdot z\end{aligned} \tag{4}$$

Typical parameters for the Chen system are a=35, c=28, and b=3. This system is the dual to the Lorenz system and has a similarly simple structure, but displays more sophisticated dynamical behaviors [7, 8]. Here, the duality is based on a classification condition formulated by Vanecek and Celikovsky [9]. According to the generalized Lorenz system form, the Chen system is described by

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix} -a & a & 0\\ c-a & c & 0\\ 0 & 0 & -b\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + x \cdot \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} \tag{5a}$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix} -35 & 35 & 0\\ -7 & 28 & 0\\ 0 & 0 & -3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + x \cdot \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} \tag{5b}$$

The Chen system satisfies $a_{12} \cdot a_{21} < 0, (35 \cdot (-7) < 0)$.

It is notable that Vanecek and Celikovsky [9] classified a generalized Lorenz system family using a condition on its linear part $A=[a_{ij}]$: $a_{12}a_{21}>0$, which includes the familiar Lorenz system as a special case, while Chen's system satisfies $a_{12}a_{21}<0$. Hence, Chen's system does not belong to this generalized Lorenz system family. In fact, Chen's system belongs to another canonical family of chaotic systems [10-11].

Lü and Chen found a critical new chaotic system [11-12], which satisfies the condition $a_{12}a_{21}=0$ and represents a transition between the Lorenz and Chen attractors. This chaotic attractor is generated by the following simple three-dimensional autonomous system:

$$\begin{aligned} \dot{x} &= a\cdot(y-x) \\ \dot{y} &= -x\cdot z + c\cdot y \\ \dot{z} &= x\cdot y - b\cdot z \end{aligned} \tag{6}$$

Typical parameters for the Lü system are a=35, c=28, and b=3. This system bridges the gap between the Lorenz and Chen systems. According to the for of a generalized Lorenz system, the Lu system is described by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ 0 & c & 0 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + x\cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{7a}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -36 & 36 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + x\cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{7b}$$

The Lü system satisfies $a_{12}\cdot a_{21}=0, (36\cdot 0=0)$. The new system, however, is not diffeomorphic with the Lorenz and Chen's systems since the eigenvalue structures of their corresponding equilibrium points are not equivalent. Moreover, these circuits are not topologically equivalent because no non-singular coordinate transformation from one system to the other exists [11].

Lü et al. constructed a unified system that contains the above three related, but nonequivalent chaotic systems [13]. The new unified system is described by

$$\begin{aligned} \dot{x} &= (25\cdot\alpha+10)\cdot(y-x) \\ \dot{y} &= (28-35\cdot\alpha)\cdot x - x\cdot z + (29\cdot\alpha-1)\cdot y \\ \dot{z} &= x\cdot y - \tfrac{\alpha+8}{3}\cdot z \end{aligned} \tag{8}$$

where $\alpha\in[0,1]$. According to Vanecek and Celikovsky [9], the linear part of the system in Eq. (8), a constant matrix $A=[a_{ij}]$, provides a critical value $a_{12}\cdot a_{21}$. According to this critical value, the whole family of chaotic systems in Eq. (8) can be classified as follows: when $0\le\alpha<0.8$, the system in Eq. (8) belongs to the generalized Lorenz system defined in [9], since with these values of α one has $a_{12}\cdot a_{21}>0$; when $\alpha=0.8$, the system in (8) belongs to the class of chaotic systems introduced in [11-12], since in this case $a_{12}\cdot a_{21}=0$; when $0.8<\alpha\le 1$, it belongs to the generalized Chen system formulated in [10], for which $a_{12}a_{21}<0$.

Lü and Chen found a new chaotic Lorenz-like system, which can display two chaotic attractors simultaneously [14]. Consider the following simple three-dimensional quadratic autonomous system, which can display two chaotic attractors simultaneously:

$$\begin{aligned} \dot{x} &= -\tfrac{a\cdot b}{a+b}\cdot x - y\cdot z + c \\ \dot{y} &= a\cdot y + x\cdot z \\ \dot{z} &= b\cdot z + x\cdot y \end{aligned} \tag{9}$$

where a, b, c are real constants. The concept of generalized Lorenz systems is also extended to a new class of generalized Lorenz-like systems in canonical form [14]. Consider the following general Lorenz system family [14], [22]:

$$\begin{aligned}\tfrac{dx}{d\tau} &= a_1 \cdot x + a_2 \cdot y + a_{13} \cdot x \cdot z + a_{23} \cdot y \cdot z \\ \tfrac{dy}{d\tau} &= b_1 \cdot x + b_2 \cdot y + b_{13} \cdot x \cdot z + b_{23} \cdot y \cdot z + d_2 \\ \tfrac{dz}{d\tau} &= c_3 \cdot z + c_{12} \cdot x \cdot y + c_{11} \cdot x^2 \cdot z + c_{22} \cdot y^2 + c_{33} \cdot z^2 + d_3\end{aligned} \tag{10}$$

where a_i, b_i, a_{i3}, b_{i3} for i=1,2, c_{jj} for j=1,2,3, and c_3, d_2, d_3, c_{12} are real constants. The system in Eq. (13) is a general form for most typical three dimensional quadratic autonomous chaotic systems, including the Lorenz system, Chen system [6], Lü system [11], Lorenz-like systems [14], and Sprott systems [5]. The parameter settings for these three-dimensional quadratic autonomous chaotic systems are listed in Table I of Section 3.

3. A new chaotic system and its analyses

The following nonlinear autonomous ordinary differential equations comprise the proposed chaotic system.

$$\begin{aligned}\dot{x} &= y - x \\ \dot{y} &= a \cdot y - x \cdot z \\ \dot{z} &= x \cdot y - b\end{aligned} \tag{11}$$

The new system has six terms, two quadratic nonlinearities (xz, xy) and two positive real constant parameters (a, b). The state variables of the system are x, y, and z. The new system equations has two equilibrum points. The set of all points which satisfy this requirement are found by setting $\dot{x}, \dot{y}, \dot{z} = 0$, in Eq. (11), and solving for x, y, z:

$$\begin{aligned}0 &= y^* - x^* \\ 0 &= a \cdot y^* - x^* \cdot z^* \\ 0 &= x^* \cdot y^* - b\end{aligned} \tag{12}$$

Two fixed points exist, $(x^*, y^*, z^*) = (\pm\sqrt{b}, \pm\sqrt{b}, a)$. As the variables x, y, z $\in \Re$, for the fixed point to exist, b $>$ 0. The Jacobian of the system is

$$J = \begin{bmatrix} -1 & 1 & 0 \\ -z & a & -x \\ y & x & 0 \end{bmatrix} \tag{13}$$

For the case when the fixed point is $(x^*, y^*, z^*) = (\sqrt{b}, \sqrt{b}, a)$, the Jacobian becomes

$$J = \begin{bmatrix} -1 & 1 & 0 \\ -a & a & -\sqrt{b} \\ \sqrt{b} & \sqrt{b} & 0 \end{bmatrix} \tag{14}$$

The eigenvalues are found by solving the characteristic equation, $|J - \lambda I| = 0$, which is

$$\lambda^3 - a \cdot \lambda^2 + \lambda^2 + b \cdot \lambda + 2 \cdot b = 0 \tag{15}$$

yielding eigenvalues of $\lambda_1 = -1, \lambda_2 = 0.25 - 0.968245 \cdot i, \lambda_3 = 0.25 + 0.968245 \cdot i$

for a = 0.5, and b=0.5.

For the case when the fixed point is $(x^*, y^*, z^*) = (-\sqrt{b}, -\sqrt{b}, a)$, the Jacobian becomes

$$J = \begin{bmatrix} -1 & 1 & 0 \\ -a & a & \sqrt{b} \\ -\sqrt{b} & -\sqrt{b} & 0 \end{bmatrix} \tag{16}$$

The eigenvalues are found by solving the characteristic equation, $|J - \lambda I| = 0$, which is the same as before,

$$\lambda^3 - a \cdot \lambda^2 + \lambda^2 + b \cdot \lambda + 2 \cdot b = 0 \tag{17}$$

yielding eigenvalues of $\lambda_1 = -1, \lambda_2 = 0.25 - 0.968245 \cdot i, \lambda_3 = 0.25 + 0.968245 \cdot i$ for a = 0.5, and b=0.5. Note that the same eigenvalues are found, and the real parts of this eigenvalues are positive. Consequently the equilibrium points are unstable and this implies chaos. Thus, the system orbits around the two unstable equilibrium points.

Using a Matlab-Simulink model, as shown in Figure 1., the xy, xz, and yz phase portraits of the new system achieved are shown in Figure 2, Figure 3, and Figure 4.

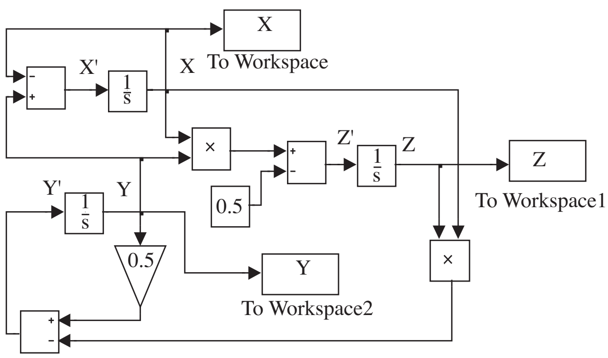

Figure 1. The Matlab-Simulink model of the new system for a=0.5, and b=0.5.

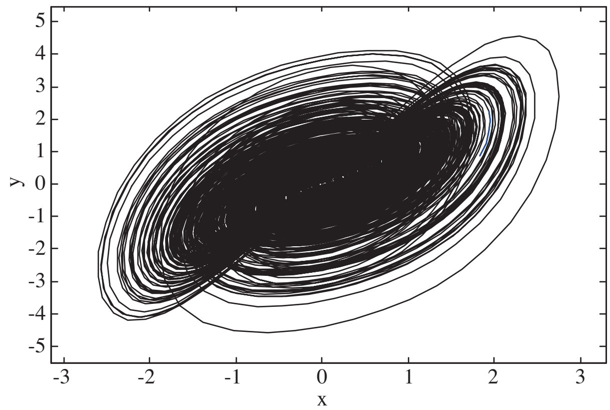

Figure 2. xy phase portrait of the new system when a=0.5, b=0.5, x_0=0.001, y_0=0.001, and z_0=0.

Accordingly to the form for generalized Lorenz systems, the novel system is described by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & a & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + x \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -b \end{bmatrix} \tag{18a}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + x \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -0.5 \end{bmatrix} \tag{18b}$$

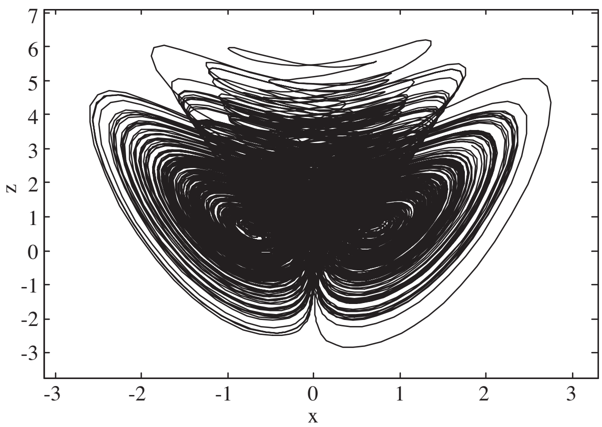

Figure 3. xz phase portrait of the new system when a=0.5, b=0.5, x_0=0.001, y_0=0.001, and z_0=0.

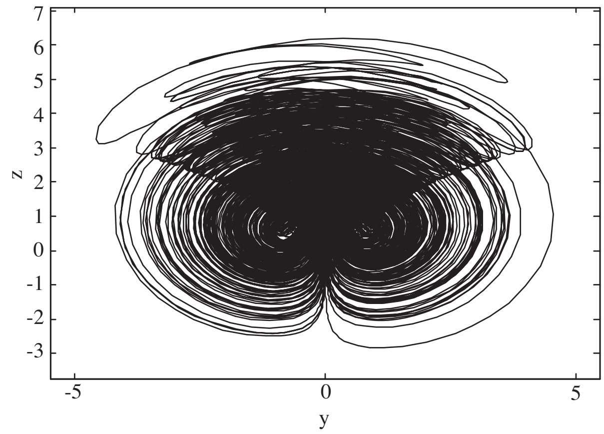

Figure 4. yz phase portrait of the new system when a=0.5, b=0.5, x_0=0.001, y_0=0.001, and z_0=0.

The novel system satisfies $a_{12} \cdot a_{21} = 0, (1 \cdot 0 = 0)$, similar to the Lü system [11,12]. The Lorenz system satisfies $a_{12} \cdot a_{21} > 0$, while Chen's system satisfies $a_{12} \cdot a_{21} < 0$. More interestingly, the new chaotic system also satisfies the condition $a_{12} \cdot a_{21} = 0$, similar to the Lü system. The novel system has two equilibrium points, whereas the Lorenz in Eq. (1), the Chen in Eq. (4) and the Lü in Eq. (6) systems have three equilibrium points, and Lorenz-like systems, such as in Eq. (11), have five equilibrium points. Despite the fact that the origin (0,0,0) is a point of equilibria for these systems, it's not an equilibrium point for the new system. The new system is not diffeomorphic with the Lorenz, Chen, Lü and Lorenz-like systems, since the eigenvalue structures of their corresponding equilibrium points are not equivalent [11,12,14]. The equilibria and eigenvalues for these three-dimensional quadratic autonomous chaotic systems are tabulated in Table 1.

Table 1. Equilibria and eigenvalues for several typical chaotic systems.

System	Parameters	Equilibria	Eigenvalues
Lorenz system(1)	a=10, b=8/3, c=28	$\begin{cases} (0,0,0) \\ (\pm 6\sqrt{2}, \pm 6\sqrt{2}, 27) \end{cases}$	$-22.8277, -2.6667, 11.8277$ $-13.8546, 0.0940 \pm 0.1945 \cdot i$
Chen system(4)	a=35, b=3, c=28	$\begin{cases} (0,0,0) \\ (\pm 3\sqrt{7}, \pm 3\sqrt{7}, 21) \end{cases}$	$-30.8359, -3, 23.8359$ $-18.4288, 4.2140 \pm 14.8846 \cdot i$
Lü system(6)	a=36, b=3, c=20	$\begin{cases} (0,0,0) \\ (\pm 2\sqrt{15}, \pm 2\sqrt{15}, 20) \end{cases}$	$-36, -3, 20$ $-22.6516, 1.8258 \pm 13.6887 \cdot i$
Lorenz-like system(9)	a=-10,b=-4,c=0	$\begin{cases} (0,0,0) \\ (2\sqrt{10}, \pm \frac{4}{7}\sqrt{35}, \frac{10}{7}\sqrt{14}) \\ -(2\sqrt{10}, \pm \frac{4}{7}\sqrt{35}, \frac{10}{7}\sqrt{14}) \end{cases}$	$-10, -4, 2.8571$ $-13.6106, 1.2339 \pm 5.6626 \cdot i$
The new system(11)	a=0.5, b=0.5	$\left\{ (\pm\sqrt{\frac{1}{2}}, \pm\sqrt{\frac{1}{2}}, \frac{1}{2}) \right.$	$-1, \frac{1}{4} \pm 0.9682 \cdot i$

It is straightforward, but somewhat tedious, to verify that there are no non-singular coordinate transforms that can convert one system to the other. Therefore, none are topologically equivalent [11,12,14]. However, it can be verified that there does not exist such a diffeomorphism between the new system and the others, since the eigenvalues of the corresponding Jacobians are not equivalent. The new system and other systems mentioned are not diffeomorphic, and furthermore, they are not topological equivalent [11,12,14].

Figure 5 shows the Lyapunov spectrum of the new system for a varying parameter b, and constant parameter a=0.5. As can be seen from the Lyapunov exponents spectrum, when b is in the range (0.035, 1.25), the new system is chaotic with a positive Lyapunov exponent. As an example, for b = 0.5, the obtained phase portraits are shown in Figure 2, Figure 3, and Figure 4.

Figure 6 shows the Lyapunov spectrum of the new system for a varying parameter a, and constant parameter b=0.5. As can be seen from the Lyapunov exponents spectrum, when a is in the range (0, 0.665), the new system is chaotic with a positive Lyapunov exponent . As an example, for a = 0.5, the phase portraits obtained are shown in Figure 2, Figure 3, and Figure 4).

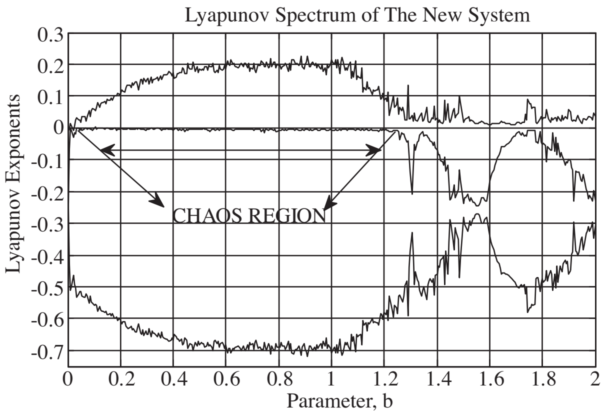

Figure 5. Lyapunov spectrum of the new system for varying parameter b, and constant parameter a=0.5.

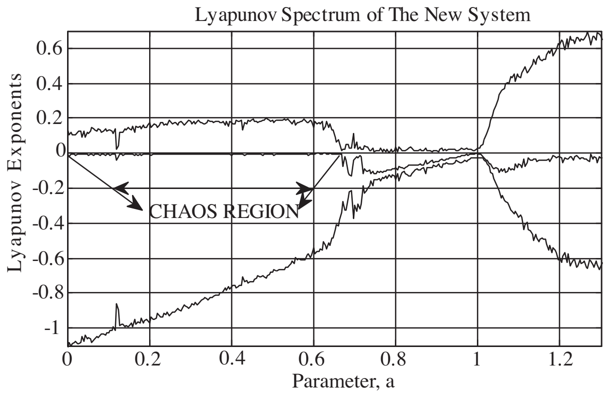

Figure 6. Lyapunov spectrum of the new chaotic system for varying parameter a, and constant parameter b=0.5.

Considering the general Lorenz system family given in Eq. (10), [14], and [23], the new chaotic system parameters are shown in Table 2.

Table 2. System parameters of several typical chaotic systems.

a_1	a_2	a_{13}	a_{23}	b_1	b_2	b_{13}	b_{23}	d_2	c_3	c_{12}	c_{11}	c_{22}	c_{33}	d_3	System
-10	10	0	0	28	-1	-1	0	0	$-\frac{8}{3}$	1	0	0	0	0	Lorenz
-35	35	0	0	-7	28	-1	0	0	-3	1	0	0	0	0	Chen
-36	36	0	0	0	20	-1	0	0	-3	1	0	0	0	0	Lü
2.86	0	0	-1	0	-10	1	0	1	-4	1	0	0	0	0	Lorenz -like
-1	1	0	0	0	0.5	-1	0	0	0	1	0	0	0	0.5	The New

4. Circuit realization of the new attractor

A simple electronic circuit is designed that can be used to study chaotic phenomena. The circuit employs simple electronic elements, such as resistors, and operational amplifiers, and is easy to construct. Figure 7. shows the circuit schematic for implementing the new chaotic system in Eq. (11). There are 3 capacitors, 8 resistors, 4 operational amplifiers and 2 multipliers in the circuit.

Zhong and Tang introduced the circuitry realization of Chen's attractor [29] in 2001. As can be seen in Figure 8., this e circuit contains 3 capacitors, 20 resistors, 8 opamps and 2 multipliers. Note that the new system is simpler than the Chen system in terms of circuit complexity.

Cuomo and Oppenheim introduced the circuit realization of the Lorenz attractor[30]. As can be seen in Figure 9., this circuit contains 3 capacitors, 20 resistors, 8 opamps and 2 multipliers. Thus, the new system is also simpler than the Lorenz system in terms of circuit complexity.

The new system has two equilibrium points. But the Lorenz in Eq. (1), the Chen in Eq. (4) and the Lüin Eq. (8) systems have three equilibrium points, while Lorenz-like system as in Eq. (14) have five equilibrium points. Despite the fact that the origin(0,0,0) is a point of equilibrium for these systems, it is not an equilibrium point for the new system. Thus, it does not require initial condition voltages for executing the circuit. Consequently, realization of the new circuit is very easy.

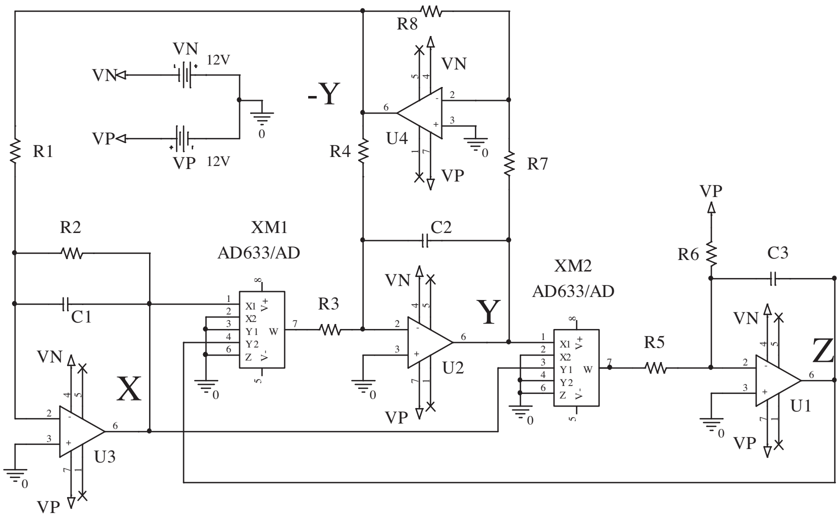

Figure 7. The electronic circuit schematic of the new chaotic system.

Chaotic differential equations for the new circuit are given below.

$$\begin{aligned} \dot{x} &= \frac{1}{R_1C_1}y - \frac{1}{R_2C_1}x \\ \dot{y} &= \frac{1}{R_4C_2}y - \frac{1}{R_3C_2}x \cdot z \\ \dot{z} &= \frac{1}{R_5C_3}x \cdot y - \frac{1}{R_6C_3 \cdot V_p} \end{aligned} \tag{19}$$

An experimental electronic circuit for the new chaotic system is implemented with parameters of a=0.5, b=0.5, and initial conditions x_0=0, y_0=0, z_0=0. LM741 opamps, and the Analog Devices AD633JN multipliers are used with R_1=R_2=400K, R_3=R_5=40K, R_4 = 800K, R_6 = 9600K, R_7 = R_8 = 100K, C_1 = C_2 = C_3 = 1nF, V_N = -12V, and V_P=12V. Also, an Orcad-PSpice simulation is conducted for initial conditions x_0=0.001,

y_0=0.001, z_0=0 and the same parameter values as in the experiment. All electronic components are easily available. Acceptable inputs to the AD633 multiplier IC are −10 to +10 V. The output voltage is the product of the inputs divided by 10 V. The experimental electronic circuit realization of the new system is shown in Figure 10. Oscilloscope outputs of circuitry of the new system are shown in Figure 11, Figure 12, and Figure 13 for xy, xz, and yz attractors, respectively.

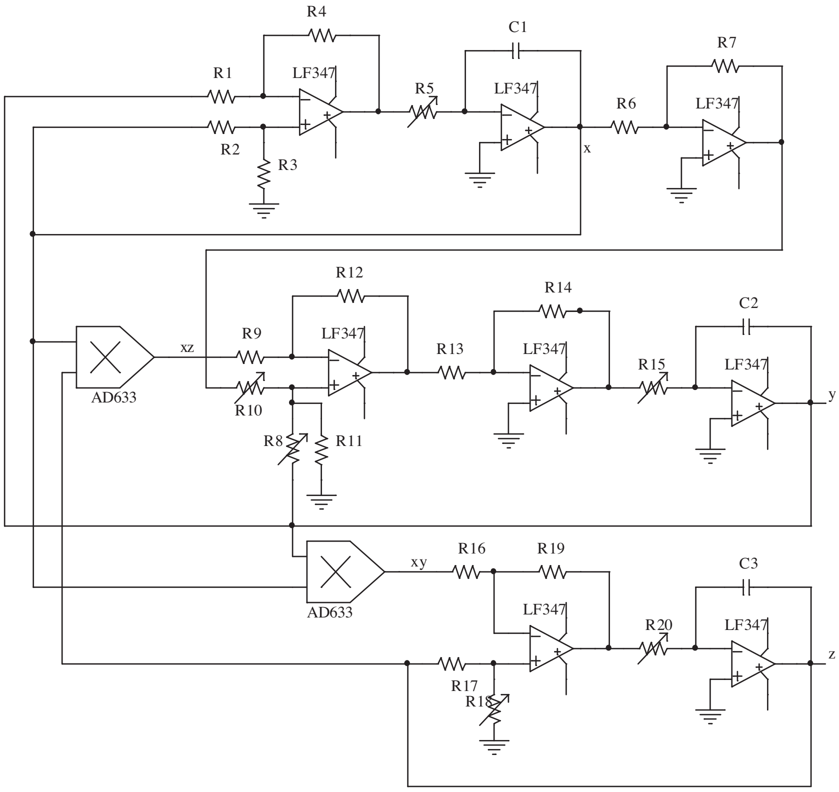

Figure 8. The electronic circuit schematic of the Chen system [30].

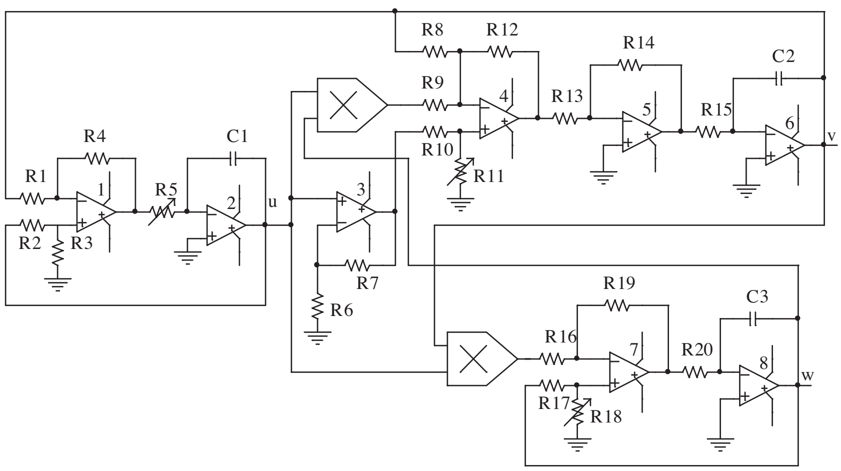

Figure 9. The electronic circuit schematic of the Lorenz system [31].

PSpice simulations of the new chaotic system are also attained in Figure 14, Figure 15, and Figure 16 for xy, xz, and yz attractors, respectively. In this simulation, the parameters a and b are set at a value of 0.5, and all initial conditions are zero.

Figure 10. Electronic circuit realization, and oscilloscope output of the new system.

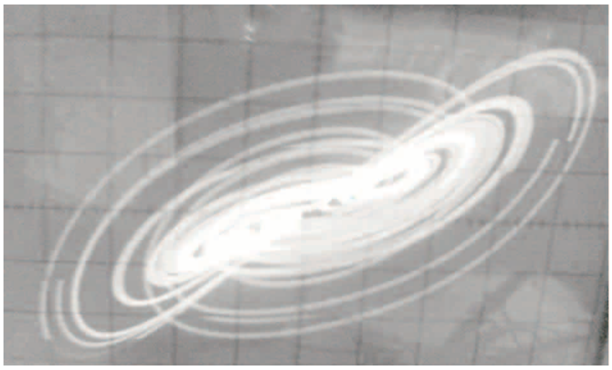

Figure 11. *xy* strange attractor as oscilloscope output of experimental circuit in Figure 10.

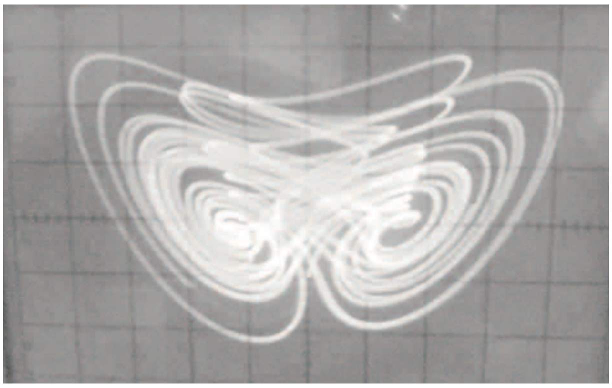

Figure 12. *xz* strange attractor as oscilloscope output of experimental circuit in Figure 10.

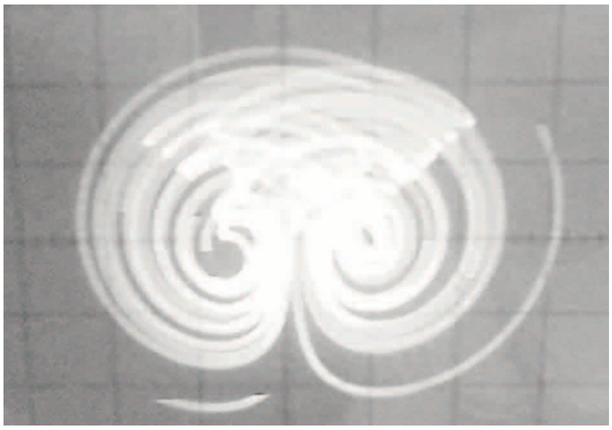

Figure 13. *yz* strange attractor as oscilloscope output of experimental circuit in Figure 10.

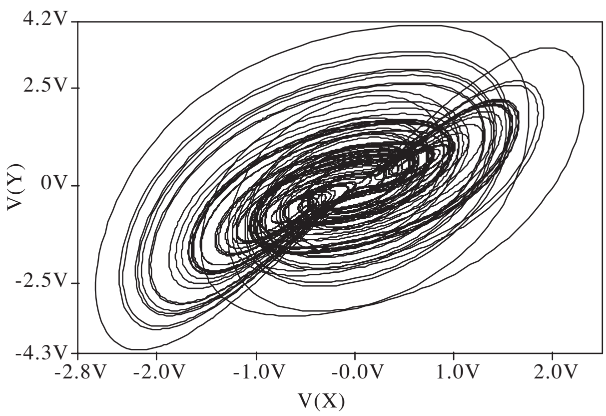

Figure 14. PSpice simulation result of the new chaotic system's electronic oscillator (Figure 7) for xy strange attractor.

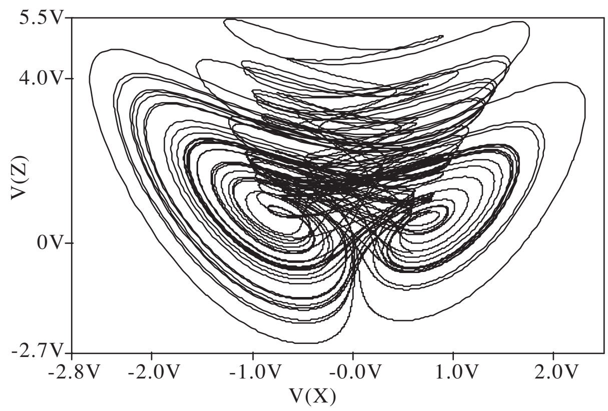

Figure 15. PSpice simulation result of the new chaotic system's electronic oscillator (Figure 7)for xz strange attractor.

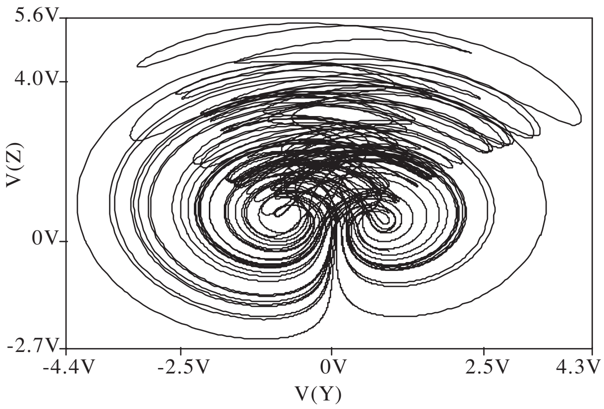

Figure 16. PSpice simulation result of the new chaotic system's electronic oscillator (Figure 7) for yz strange attractor.

5. Conclusions

This article introduces a novel simple three-dimensional quadratic autonomous chaotic system, which can generate complex 2-scroll chaotic attractors simultaneously. The objective of this article is to present and further study a simple, interesting, and yet complex three-dimensional quadratic autonomous chaotic system. Our investigation was completed using a combination of theoretical analysis, simulations and experiments. Electronic circuitry of the new chaotic system is very simple. The simulation results were produced using Matlab-Simulink® and Orcad-PSpice® programs. The study of chaotic oscillators is of interest in electrical engineering education. Introducing a laboratory project that integrates experimental and simulation results may prove an exciting experience. Building the electronics of this new chaotic system is very easy, due to its having zero initial conditions. The new system has a small margin for varying the output signal for easy implementation, as shown in the phase portraits of Figure 14, Figure 15, and Figure 16, respectively.

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