

Spacetime and Matter – a duality of partial orders ¹

Hans-Thomas Elze

Dipartimento di Fisica “Enrico Fermi”, Università di Pisa

Abstract. A new kind of duality between the deep structures of spacetime and matter is proposed here, considering two partial orders which incorporate causality, extensity, and discreteness. This may have surprising consequences for the emergence of quantum mechanics, which are discussed.

1 The context

PROLOGUE : “*What is Ultimately Possible in Physics?*” – This question must occur to every physicist, now and then, in those rare moments when the work can be put aside, when there are no grant applications to review, nor are there other earthly problems to worry about immediately. Instead, perhaps, there is an unimpeded view of the Milky Way or of the ocean surf ... In this essay, we address the question posed by successively narrowing it down.

“*Will there be a Theory Of Everything?*” – This one cannot be answered without having the TOE. Having the TOE, *we* would not recognize the TOE.

More modestly, “*Will there be quantum gravity?*”. – In trying to tackle this one, we will analyze heuristically some apparent deficiency in current elaborations of the theme, resulting in promising new perspectives on the penultimately possible in physics.

“*There is no quantum gravity*” – might this author be tempted to say. While a majority of physicists presumably would hold on to “*Not yet, but ...*”.

Adepts of various contenders for a theory of quantum gravity, such as string theory, loop quantum gravity, or quantum geometrodynamics would fill in achievements and problematic issues traded between these schools.

To wit, no consensus has been reached despite intense study of this stumbling block on the road towards a unified picture of the Universe. Brilliant thinkers have tried their best, last not least, motivated by astounding successes of the Standard Model of the constituents of matter and of the forces through which they interact.

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Purpose of this essay is *not* to propose another surprise model which could subsume gravity and spacetime to the successful paradigm of quantum field theory. Such theories commonly depart from *classical* Newtonian concepts applied to fields which represent matter (quarks, leptons, and gauge bosons), somehow existing in spacetime. These theories are then “*quantized*”. Following a precise protocol, they are reformulated and generalized according to abstract *axioms*, or derived rules, which have been distilled from research in quantum mechanics, as described elegantly and succinctly in Dirac’s famous book [1].

With rapid advances of experimental techniques, foundational questions of quantum mechanics have entered center-stage in recent years. It is also *not* the place here to discuss different related interpretations nor conceptual problems within the mathematical framework [2].

Instead, we must recall a deep structural disparity between Quantum Theory (*QT*) and the classical theory of gravity and spacetime, Einstein’s General Relativity (*GR*).

The former describes quantum evolution between an initial and a final state, pertaining to an initial and final moment of time, respectively. The law of motion, e.g., in form of the Schrödinger equation, requires a slicing of spacetime according to an external time. Dynamical changes happen from slice to slice, while a clock “ticks”, similarly as in Newtonian mechanics. This time is still universal but not absolute, since *QT* can be adjusted to Special Relativity.

But where is such clock to be found? – Reference to anyone’s time keeping device or to periodic natural phenomena may seem to suffice. But what about the whole Universe? It is a single entity and, therefore, no *external* clock exists!

Furthermore, modern theories of gravity and spacetime are founded on the principle that meaningful physical statements must be independent of a choice spatial or temporal coordinates. This symmetry principle has changed notions of *Where* and *When* profoundly. It introduces an aspect of arbitrariness into the concept of time which, nevertheless, remains essential for *QT*, as it is.

This disparity between *GR* and *QT*, produces major obstacles on the way to a unified spacetime-matter theory [3]. It motivates our heuristic argument, which we outline here, with details to follow in this essay.

At the level of quantum field theory, *QT* complies with Special Relativity. One proceeds without paying attention to the *deep structure of spacetime*. With the exception of worries caused by ubiquitous infinities in such theories.

While infinities can be dealt with by “renormalization”, they do arise from phenomena at very short distances – tacitly assuming that spacetime is a continuum manifold, admitting higher and higher energy probes that resolve shorter and shorter distances, in principle. This does not imply

open ended search for “fundamental” properties of matter, its atomistic aspects, in particular. The very assumption that a theory of spacetime compatible with quantum mechanics does exist also suggests an ultimate cut-off: by *QT*, spacetime is expected to become discrete at Planck scale, $l_P \approx 10^{-35}\text{m}$.

Spacetime, therefore, should reflect a similar (or the same?) *atomism* as attributed to matter in modern physical theories. Such ideas can be traced back all the way to ancient Greece and the pre-Socratic philosophers, as described, for example, in Schrödinger’s marvellous book [4]. Problematic aspects of space as continuum have already been noticed then, with much insight, as well as in later periods when natural philosophy thrived [5, 6].

This leads us to conclude our brief exposition of ideas addressing “spacetime and matter” with the *hypothesis*:

- Spacetime and matter must reflect each others atomistic structure.

Hence, unequal footing for matter and spacetime may cause obstructions to the search for “quantum gravity” from the outset. Of course, this results from our ignorance of the relevant degrees of freedom close to l_P , where quantum numbers and degrees of freedom of the Standard Model not necessarily play a role.

Furthermore, spacetime without matter or matter in a “background” spacetime are justifiable as approximate yet accurate mathematical models under certain circumstances. However, these abstractions found their way into the discussed disparity between *GR* and *QT*.

Therefore, we will address the fundamental mutual dependence of spacetime and matter, and its consequences. We assume that the categories *causality*, embedded in the deepest spacetime structure, and *extensity*, a defining quality of matter here, are *dual* to each other, in a sense to be defined. Physical reality ultimately rests inseparably on both. ²

2 Atomism all the way – locally finite partially ordered sets

A *locally finite partially ordered set* is mathematically defined as a set C together with a binary relation \prec satisfying:

- *transitivity*: $x \prec y \prec z \Rightarrow x \prec z$, $\forall x, y, z \in C$;

²This author has a hard time to imagine a piece of matter to exist without an accompanying spacetime volume and finds it just as difficult to believe that a spacetime can exist deprived of matter, which provides rulers and light signals to reveal its geometry.

- *irreflexivity*: $x \not\prec x$, $\forall x \in C$;
- *local finiteness*: $\forall x, z \in C$ the set $\{y \in C | x \prec y \prec z\}$ has a finite number of elements.

A relation between two elements that is *not* implied by transitivity is called a *link*.

Such structures made their entrance into physics only recently, while having much older roots in related atomistic ideas [4, 5, 6]. Myrheim [7] and 't Hooft [8] initiated modern considerations of discrete sets as foundation for a theory of spacetime, incorporating also causality as a fundamental principle. This subsequently led to the *causal set hypothesis* [9] and research program; among the most interesting results has been a *prediction* of the cosmological constant [6].³

No attempts to merge such discrete structures with *QT* have been reported. In fact, it may be wrong to pursue this, since quantum phenomena possibly *emerge* only at larger scales than l_P , the scale of spacetime discreteness. We expand on this issue in Section 3.

2.1 Spacetime as a causal set

The binary relation \prec of a locally finite partially ordered set can be interpreted as stating *causal order* between two elements $x, y \in C$:

$$x \prec y \text{ reads as } x \text{ precedes } y . \tag{1}$$

Elements of the set are naturally interpreted as elementary “events” and C is termed a *causal set* or *causet*. – Elements which are not related by \prec are *spacelike* to each other, which is denoted by: $x \not\prec y \Leftrightarrow x \not\prec y$ and $y \not\prec x$.

Transitivity and irreflexivity together imply that there can be *no causal loops*. Furthermore, local finiteness introduces the notion of discreteness, when this definition is applied to (re)construct spacetime.

The motivation for a causet as the structure possibly underlying spacetime stems from the fact that causal ordering together with a four-dimensional volume element is sufficient to determine all metric information, topology, and differentiable structure of a continuum spacetime, under plausible technical assumptions. This has been stated as [6]:

$$\text{Order} + \text{Number} = \text{Geometry} , \tag{2}$$

where numbers of set elements encode volume information in the discrete case. – Thus, Minkowski space, \mathbb{M} , can be reconstructed as continuum limit of a causet.

³For the state of the art, see papers by Rideout and Walden, Sorkin, Sverdlov and Bombelli, Henson, Johnston, Philpott with Dowker and Sorkin, Brightwell with Henson, and Surya collected in [10].

Generally, a representative causet can always be obtained by a Lorentz invariant Poisson process (“*sprinkling*”): elements of the set are generated randomly by sprinkling points with uniform average density of $1/l_P^4$ into a Lorentzian four-dimensional continuum manifold, from which they inherit causal ordering.⁴

Dynamics of causets will not be further discussed. It is, of course, an important topic to take the stringent requirements of general coordinate invariance and causality into account, since evolving discrete structures are to replace dynamical continuum manifolds governed by *GR* at large [6, 10]. It may suffice to mention that through the growth of a causet by new elements time can be seen to “happen”.

However, another question looms, as surmised in Section 1, and shall be addressed: Has matter been overlooked in recent attempts to formulate a theory of spacetime valid down to shortest distances?

2.2 Matter and extensity

A causet correlate of spacetime is well defined *without* mentioning matter. For physics, however, matter has to be dealt with. It is necessary, in order to derive phenomenological consequences of the atomistic structure of spacetime, which one would hope to test experimentally.

So far, this has been done by adapting continuum notions of propagation of particles or fields to terms of “order and number” which are available for a causet; cf. Philpott, Dowker and Sorkin or Sverdlov and Bombelli in [10].

Still, this comes with assumptions which detract from simplicity and beauty of the causal set hypothesis:

- Causet elements have to *register* additional physical data, such as a field amplitude or presence of a particle, and causal relations between them must *act as transport channels* for these quantities or related information.
- This might signify that such causet is a *coarse-grained* version of a finer structure, with manifestations of matter *emerging* together with the causet.

The latter would be in the spirit of Kaluza-Klein models, with additional degrees of freedom arising from the geometry of extra-dimensions beyond the perceived four-dimensional spacetime. It cannot be ruled out at present.

However, as a radical alternative, we introduce a *locally finite partial order* to describe the presence of matter as follows.

⁴It has been conjectured that this is the only way to produce a Lorentz invariant discretization.

Lacking a general dynamical theory, consider for a moment that matter is positioned or moving relative to a *given* causet. It may be naturally allocated “in between” causet elements that are *spacelike* to each other. To this end, we choose a locally finite partial order that incorporates the category of “*extensity*”, in distinction to *causality*.

The corresponding binary relation between spacelike elements, $u, v \in C$, with $u \not\prec v$, is denoted by \prec^e and interpreted by:

$$u \prec^e v \text{ reads as } u \text{ is extended by } v . \quad (3)$$

Extensity here is a fundamental feature of matter which, in its primordial form, is represented by the extensional relation between events.

2.3 Spacetime-matter duality

Oriented loops of extensional links also do *not* exist. This does not rule out all-spacelike extended matter, including tilings of Euclidean space and loops in general.

By definition, a causal link between two events *cannot* coincide with an extensional link. In this sense, both orders are complementary to each other.

This leads us to consider *transformations* of the causal and extensional relations among events according to one of the rules:

$$A := \left((\prec, \succ, \prec^e, \succ^e) \mapsto (\prec^e, \succ^e, \prec, \succ) \right) , \quad (4)$$

$$B := \left((\prec, \succ, \prec^e, \succ^e) \mapsto (\succ^e, \prec^e, \succ, \prec) \right) , \quad (5)$$

$$C := \left((\prec, \succ, \prec^e, \succ^e) \mapsto (\succ, \prec, \succ^e, \prec^e) \right) . \quad (6)$$

Representing each map by a 4×4 matrix, it is easy to see that they form an abelian group, with multiplication rules: $AB = C$, $BC = A$, $CA = B$, $A^2 = B^2 = C^2 = e$, e denoting the identity. It is known as *Klein four-group* \mathbf{V} .⁵

Consider a causet sprinkled into four-dimensional Minkowski space, \mathbb{M}^4 , with extensional relations representing matter. Then, global application of rule C can be interpreted as discrete analogue of combined *time reversal* and *parity* transformations, under which such a set may or may not be invariant.

A causet can only be *globally invariant* under (either) transformation A (or $B = CA = AC$), if the directed network of causal and extensional links can be mapped one-to-one, necessitating equal numbers of causal and extensional links. Elements which are neither causally nor

⁵The symmetric traceless matrices representing A , B , and C can serve as generators of a continuous group. Transformations effected by its elements can produce *superpositions* of spacetime-matter configurations. We do not investigate this interesting possibility here.

extensionally related to any other element are not affected. Such a highly symmetric state is characterized by *spacetime-matter duality*: causal and extensional structures imply each other.

At first sight, this duality seems artificial. Looking outside, local features of \mathbb{M}^4 with matter could not deviate more from duality between two partial orders. However, this is due to asymmetric circumstances with spacetime appearing static while matter evolves with respect to it.

The motion of this *dilute matter* can be described simply. Similarly to propagation of a scalar field from a point source [10], the “endpoint” vertices of an extensional link propagate forward, each into its “future lightcone”: all outgoing causal links are followed; however, only spacelike pairs of new endpoints contribute, which allow and consequently obtain an extensional link between them.⁶

For *dense matter*, we consider a \mathbb{M}^2 background. A corresponding causet set inherits causal and spacelike relations through sprinkling from \mathbb{M}^2 . The latter define extensional links, with directionality given by the one-dimensional spatial order of the background. This discrete spacetime-matter structure is *invariant* under transformation A , hence *selfdual* (apart from statistical fluctuations). It is an extended matter distribution resembling the world sheet of a straight line string. By thinning the distribution of extensional links, the amount of matter could be locally varied.

How can the duality be realized in *three spatial dimensions*?

All sets are locally finite with denumerable elements and the expected number of events per unit fourvolume of spacetime is finite. Therefore, discretized patches of higherdimensional spacetime-matter can be built by randomly stacking (1+1)-dimensional sheets together, eventually reaching the continuum limit. This implies *dimensional reduction* of spacetime from 3+1 to 1+1 dimensions, as shorter and shorter distances are probed. Indeed, there are various independent arguments supporting this picture [11]. – A macroscopic patch of \mathbb{M}^4 (with matter) *can* result, if spatial orientations of the microscopic (extensional) sets are randomized, while keeping lightcones aligned. Clearly, it is not trivial to arrive at the Poincaré symmetry. Predictable deviations would be most interesting.⁷

Finally, consider any element of a causet discretizing \mathbb{M}^4 and take its “*lightcone*”, consisting of causally related elements. The lightcone becomes an extensionally ordered “*matter cone*” by transformation A . The extensionally unrelated *outside* of the matter cone can be identified

⁶Such propagation on a causet sprinkled into \mathbb{M}^4 can be analyzed in detail. If limited to overlapping future lightcones of the endpoints, this leads to a *formation time* effect.

⁷Extensional ordering introduces a spacelike correlation, if stringlike structures thread spacetime, as is implicit here. It might have interesting consequences for quantum mechanics.

with the causally unrelated outside of the lightcone, since A does not act on unrelated elements. Statistical properties of these sets are not influenced by the transformation. Therefore, dimension measures will indicate a four-dimensional space in both cases [6].

Here, the reader must wonder, whether we are only changing names between causal and extensional order. – Without *interactions*, this must be so. However, as soon as interactions affect causal and matter links, or vertices, in distinctive ways, these entities obtain distinctive roles, differentiating aspects of matter from those of spacetime.

A theory with interactions phrased in terms of the partial orders awaits elaboration. However, duality of spacetime and matter here already suggests that a Lorentzian space *together* with its dual should be relevant for physics after sufficient coarse-graining of sufficiently large partially ordered sets. Known phenomena may then take place as perturbations of perfect spacetime-matter duality, rendering physical a *doubled number of dimensions*.

3 Emergent quantum mechanical aspects

We have argued for duality between spacetime and matter in the foregoing, suggesting also that physics at large scales must refer to *two* copies, say, of \mathbb{M}^4 .

Indeed, we have encountered this doubling before. – Classical Hamiltonian mechanics employs phase space, consisting of coordinates *paired* with momenta. The latter can be replaced by a second set of coordinates through *Fourier transformation*. Both sets describe \mathbb{M}^4 spaces in Special Relativity.

The author has recently studied classical ensemble theory in terms of such *doubled set of coordinates*; see Ref. [12] with numerous related references. We summarize here how this, quite surprisingly, produces important aspects of quantum theory (*QT*).

Consider a (1+1)-dimensional nonrelativistic object with equations of motion derived from a Hamiltonian function, $H(x, p) := p^2/2 + v(x)$, where x , p , and v denote coordinate, momentum, and *true potential*, respectively.⁸ – The Liouville equation describes evolution of a *statistical ensemble* of such objects by its evolving phase space probability distribution, f , with dynamics given by H . Combining Fourier transformation and linear coordinate transformations, the *classical* equation is:

$$i\partial_t f(x, y; t) = \left\{ \hat{H}_x - \hat{H}_y + \mathcal{E}(x, y) \right\} f(x, y; t) , \quad (7)$$

$$\hat{H}_\chi := -\frac{1}{2}\partial_\chi^2 + v(\chi) , \quad \text{for } \chi = x, y , \quad (8)$$

⁸The following works for Lagrangian field theories as well.

$$\mathcal{E}(x, y) := (x - y)v'\left(\frac{x + y}{2}\right) - v(x) + v(y) , \quad (9)$$

in terms of coordinates x, y and $v'(x) := dv(x)/dx$.

This seems to be *QT!* – The Eq. (7) looks like the *quantum mechanical* von Neumann equation for a density operator $\hat{f}(t)$, considering $f(x, y; t)$ as its matrix elements.⁹ We recover the usual operator \hat{H} for the Hamiltonian function H . Yet an essential difference consists in the interaction $\hat{\mathcal{E}}$ between *bra*- and *ket*-states. Hilbert space and its dual (*nota bene*) here are coupled by an unfamiliar *superoperator*.¹⁰ – We find that:

$$\hat{\mathcal{E}} \equiv 0 \Leftrightarrow \text{true potential } v \text{ is constant, linear, or harmonic .}$$

To appreciate this fact, we emphasize that we are here concerned with physics in continuum spacetime as a valid approximation.

Different from a commonly used *macroscopic potential* V , the *true potential* v becomes piecewise defined, when approaching smaller scales.¹¹ An arbitrarily differentiable V , especially, is an approximation to v and differences between the two give rise to local fluctuations δV : $v(\mathbf{x}) = V(\mathbf{x}) + \delta V(\mathbf{x})$. Following Section 2, there are two sources of *fluctuations*: matter and spatiotemporal discreteness.

Furthermore, there is an “asymptotic freedom” effect, due to spatiotemporal discreteness.¹²

These remarks make it plausible to assume that the true potential v is *piecewise linear*, with pieces characterized by a typical “linearity length” δ , with $\delta \gg l_{Pl}$, such that the continuum description is meaningful.

Based on these considerations, it has been shown that the “Liouville equation” (7) *does* become the von Neumann equation of *QT* [12]. – Furthermore, the local fluctuations introduce a *Lindblad term*, i.e., a natural decoherence and localization mechanism. This has been much looked for in recent studies, since it may lead to testable predictions. In particular, it would be a wellcome feat to derive the (non)existence of macroscopic “Schrödinger cat” states from such an equation [2].

The inferred emergence of quantum mechanics here needs further study of the possibility and interpretation, or of the elimination, of negative probabilities, of entanglement, and with

⁹From here, one proceeds to find the probabilistic interpretation of *QT* [12].

¹⁰How it disturbs the emergence of *QT*, has been discussed in various ways recently, cf. [12, 13].

¹¹Recall the random stacking of (1+1)-dimensional sheets mentioned in Section 2.3.

¹²The “No Interaction Theorem” can be illustrated as follows: *Imagine each particle or wave-packet as a “world tube” in Minkowski space. In the center of mass frame, the two tubes make an “X”. The center of the X, where they meet, is the interaction region. Now boost each particle to very high energy. Because of Lorentz contraction, the tubes become so flattened that the interaction region shrinks to less than a Planck volume. Hence there is (very likely) no element of the causet to represent this interaction region, whence there is no interaction.* (R. Sorkin)

respect to the Born rule. Quantitative determination of induced fluctuations, e.g., δV , needs an understanding of the transition between microscopic spacetime-matter structure and emerging field theories, such as the Standard Model. Possibly, this requires a duality breaking mechanism.

For this, it is essential to add to the two partial orders – incorporating causality of spacetime, extensity of matter, and discreteness of both – a model of their interactions, which invites further exploration.

ÉPILOGUE : “*Nobody understands quantum mechanics.*” (R.P. Feynman) The arguments presented in this essay may open a new vista on this, “The Problem”. A careful (re)examination of the fundamental notions of spacetime and matter may lead to the *unforeseen possibility* to understand quantum mechanics by the methodology of and *within* physics.

After all, “*We can*” ...

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