Evolution of near-extremal-spin black holes using the moving puncture technique

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We propose a new radial coordinate to write the Kerr metric in puncture form. Unlike the quasi-radial coordinate introduced previously, the horizon radius remains finite in our radial coordinate in the extreme Kerr limit $a/M \to 1$. This significantly improves the accuracy of the evolution of black holes with spins close to the extreme Kerr limit. We are able to evolve accurately both stationary and boosted black holes with spins as high as $a/M = 0.99$ using initial data constructed in these new puncture coordinates. Initial data of compact binaries with rapidly spinning black holes can be constructed using our proposed new puncture metric for the background conformal metric. Our simulations for single black holes suggest that such initial data can be evolved successfully by the moving puncture technique.

I. INTRODUCTION

Binary black holes (BHBHs) are among the most promising sources of gravitational waves detectable by gravitation-wave detectors such as LIGO [1, 2], VIRGO [3, 4], GEO [5], and TAMA [6, 7], as well as by the proposed space-based interferometers LISA [8], BBO [9] and DECIGO [10]. Supermassive black holes are likely formed during hierarchical mergers of halos and galaxies in the early universe. Binary black hole coalescence triggered by these mergers, followed by gas accretion onto the remnant hole, may give rise to a population of black holes with very rapid spin [11, 12]. Standard thin-disk accretion alone spins up black holes (BHs) to a maximum value of $a/M = 0.998$ [13], where a is the specific angular momentum and M is the mass of the BH. However, accretion in thick, magnetized disks tends to drive the BH spin to $a/M \approx 0.94$ [14]. There is observational evidence suggesting that rapidly spinning BHs might exist in quasars [15] and binary X-ray sources [16–18].

There is great interest in studying rapidly spinning BHs in a compact binary system. The coalesence of rapidly spinning BHs could result in a gravitational-wave induced recoil velocity of a few thousand kilometers per second in some systems [19–23]. Such recoil may have significant influence on the hierarchical evolution of supermassive BHs in galaxies [24–26] and have observable signatures in quasars and active galactic nuclei [27, 28]. Black hole-neutron star binaries with a rapidly spinning BH may produce a substantial disk about the BH after merger [29–31], which may be crucial to the formation of a short-hard gamma-ray burst.

Currently, the most common method of evolving compact binary systems in numerical relativity is the moving puncture technique [32, 33]. This technique requires initial data everywhere on the computational grid, including the BH interior. Most simulations adopt conformally-flat, puncture initial data. However, this type of initial data can only produce BHs with spins as high as ≈ 0.93 [22, 34–36], the extremal-Bowen-York limit. Moreover, conformally-flat initial data contain spurious gravitational waves even for isolated spinning BHs and thus cannot represent exact stationary Kerr BH spacetimes. BHBHs with BH spins close to this limit have been evolved using the moving puncture technique [22, 36, 37]. Initial data with BH spins higher than 0.93 have been constructed using a (non-conformally flat) Kerr-Schild background metric [35], which for isolated BHs does not contain spurious radiation. These initial data have been evolved successfully using the generalized harmonic formalism with excision, even for BHs with spins higher than the extremal-Bowen-York limit [35]. One might wonder if these initial data can also be evolved by the moving puncture technique. Since these data are excised at the horizon, it is first necessary to fill in data everywhere inside the horizon in order to evolve the spacetime by the standard moving puncture technique.

We have investigated the possibility of integrating Kerr-Schild initial data for a single, stationary, rotating BH using the moving puncture technique. We removed the physical ring singularity inside the horizon by filling the BH interior with constraint-violating "junk" initial data. It has been demonstrated that the BSSN (Baumgarte-Shapiro-Shibata-Nakamura) scheme [38, 39], coupled with moving puncture gauge conditions, guarantees that the "junk" data will not propagate out of the horizon [40–42]. We have tried various methods of filling in the "junk" data, and are able to evolve the Kerr-Schild metric for a single BH with spins as high as $a/M = 0.96$. However, when the BH spin exceeds 0.96, our code either crashes or the evolution becomes inaccurate (e.g. the BH's mass and spin deviate from their initial values significantly) after $\sim 10M$.

We next considered (nonconformally flat) puncture initial data that allow the BH spin to approach the Kerr limit. Brandt and Seidel have constructed such initial data [43, 44], which provide an exact description of Kerr spacetime with no spurious gravitational waves. Their

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metric generalizes the Schwarzschild metric in isotropic coordinates to rotating BHs. We are able to evolve this metric successfully using the moving puncture technique. However, when the BH spin approaches the extreme Kerr limit, the radius of the BH horizon shrinks to zero in their quasi-isotropic radial coordinate. We find that this shrinkage causes numerical inaccuracy during the early evolution, which results in a slow decrease in the BH spin at late times (see Sec. III). In this paper, we introduce a new radial coordinate such that the horizon coordinate radius remains nonzero in the extreme Kerr limit. We are able to evolve accurately the puncture data in this new coordinate, both for stationary and boosted BHs with spins as high as $a/M = 0.99$. Initial data for compact binaries with rapidly spinning BHs may be constructed by using a conformal background metric consisting of the superposition of two Kerr puncture metrics [45–48] in our proposed coordinates. The simulations reported below suggest that such initial data can be evolved successfully by the moving puncture technique.

This paper is organized as follows: In Sec. II, we introduce our new puncture initial data, and briefly describe our numerical method to evolve the spacetime. We present results of our simulations in Sec. III. We conclude in Sec. IV with a brief discussion of future applications of our technique.

II. FORMULATION

A. Initial data

We start from the Kerr metric in Boyer-Lindquist coordinates $(r_{\rm BL}, \theta, \phi)$. We introduce the radial coordinate η as in [43, 44]:

$$r_{\rm BL} = r_+ \cosh^2(\eta/2) - r_- \sinh^2(\eta/2) \ , \qquad (1)$$

where M is the BH's mass, a is the specific angular momentum, and $r_\pm = M \pm \sqrt{M^2 - a^2}$ are the Boyer-Lindquist radii of the outer $(+)$ and inner $(-)$ horizons of the BH. Both regions $\eta \in [0, \infty)$ and $\eta \in (-\infty, 0]$ map to $r_{\rm BL} \in [r_+, \infty)$. The BH event horizon $r_{\rm BL} = r_+$ is mapped to $\eta = 0$. Equation (1) is invariant under the inversion $\eta \to -\eta$. The spatial metric in this coordinate system is given by

$$\gamma_{ij} dx^i dx^j = \Psi_0^4 \left[e^{-2q_0} (d\eta^2 + d\theta^2) + \sin^2\theta d\phi^2 \right] \ , \qquad (2)$$

where $\Psi_0^4 = A/\Sigma$, $e^{-2q_0} = \Sigma^2/A$, $\Sigma = r_{\rm BL}^2 + a^2 \cos^2\theta$, $\Delta = r_{\rm BL}^2 - 2M r_{\rm BL} + a^2$, and $A = (r_{\rm BL}^2 + a^2)^2 - \Delta a^2 \sin^2\theta$. The spatial metric is invariant under the inversion $\eta \to -\eta$, and is asymptotically flat at $\eta \to \pm\infty$. The BH exterior is mapped twice in this metric and the two pieces are joined smoothly at the throat $\eta = 0$. The metric describes an Einstein-Rosen bridge. The nonzero components of the extrinsic curvature are

$$\begin{aligned} K_{ij} &= \Psi_0^{-2} \hat{K}_{ij} & (3) \\ \hat{K}_{\eta\phi} &= \hat{K}_{\phi\eta} = \frac{Ma\sin^2\theta}{\Sigma^2} \times \\ & \quad [2r_{\rm BL}^2 (r_{\rm BL}^2 + a^2) + \Sigma(r_{\rm BL}^2 - a^2)] & (4) \\ \hat{K}_{\theta\phi} &= \hat{K}_{\phi\theta} = -2Ma^3 r_{\rm BL} \sqrt{\Delta} \cos\theta \sin^3\theta / \Sigma^2 \ . & (5) \end{aligned}$$

The lapse and shift that give rise to a stationary spacetime are

$$\begin{aligned} \alpha &= \sqrt{\frac{\Delta\Sigma}{A}} & (6) \\ \beta^\phi &= -\frac{2Mar_{\rm BL}}{A} \quad , \quad \beta^\eta = \beta^\theta = 0 \ . & (7) \end{aligned}$$

Brandt and Seidel introduce a quasi-isotropic radial coordinate [43, 44]:

$$\bar{r} = \frac{\sqrt{M^2 - a^2}}{2} e^\eta \ . \qquad (8)$$

It follows from Eq. (1) that

$$r_{\rm BL} = \bar{r} \left(1 + \frac{M + a}{2\bar{r}} \right) \left(1 + \frac{M - a}{2\bar{r}} \right) \ . \qquad (9)$$

In the Schwarzschild limit $a = 0$, the spatial metric reduces to the Schwarzschild metric in isotropic coordinates. This quasi-isotropic radial coordinate has an undesirable property that the BH horizon at $\eta = 0$ corresponds to $\bar{r} = \sqrt{M^2 - a^2}/2$, which shrinks to zero in the extreme Kerr limit. To reduce this numerical inconvenience, we generalize Eq. (8) by considering a radial coordinate of the form

$$r = \frac{\sqrt{M^2 - a^2}}{2} \lambda(a, \eta) e^\eta \ , \qquad (10)$$

where $\lambda(a, \eta)$ is an arbitrary function of a and η. One seeks to choose λ such that (1) $\eta = 0$ corresponds to a nonzero value of r for any value of $|a| \le M$, (2) $\lambda = 1$ when $a = 0$ and (3) $\lambda \to 1$ as $\eta \to \pm\infty$. Property (2) ensures that the usual isotropic radial coordinate is recovered in the Schwarzschild limit. Property (3) ensures that $r \to r_{\rm BL}$ at spatial infinity. One simple choice of r that satisfies all three properties is given by

$$r_{\rm BL} = r \left(1 + \frac{r_+}{4r} \right)^2 \ , \qquad (11)$$

which corresponds to setting λ according to

$$\lambda = \frac{e^{-\eta}}{\sqrt{M^2 - a^2}} \left[r_{\rm BL} - \frac{r_+}{2} + \sqrt{r_{\rm BL}(r_+ - r_-)} \sinh(\eta/2) \right] . \qquad (12)$$

The regions $\eta \in (-\infty, 0]$ and $\eta \in [0, \infty)$ are mapped to $r \in (0, r_+/4]$ and $r \in [r_+/4, \infty)$, respectively. The horizon is located at $r = r_+/4$. In the extreme Kerr limit, the horizon radius is $r = M/4 > 0$. The spatial metric and extrinsic curvature in this new coordinate system are given by

$$
\begin{aligned}
{}^{(3)}ds^2 &= \frac{\Sigma\left(r+\frac{r_+}{4}\right)^2}{r^3(r_{\rm BL}-r_-)}dr^2+\Sigma d\theta^2+\frac{A\sin^2\theta}{\Sigma}d\phi^2 \quad (13)\\
K_{r\phi} &= K_{\phi r}=\frac{Ma\sin^2\theta}{\Sigma\sqrt{A\Sigma}}[3r_{\rm BL}^4+2a^2r_{\rm BL}^2-a^4\\
&\quad -a^2(r_{\rm BL}^2-a^2)\sin^2\theta]\left(1+\frac{r_+}{4r}\right)\times\\
&\quad \frac{1}{\sqrt{r(r_{\rm BL}-r_-)}}\ , \quad (14)\\
K_{\theta\phi} &= K_{\phi\theta}=-\frac{2a^3Mr_{\rm BL}\cos\theta\sin^3\theta}{\Sigma\sqrt{A\Sigma}}\left(r-\frac{r_+}{4}\right)\times\\
&\quad \sqrt{\frac{r_{\rm BL}-r_-}{r}}\ . \quad (15)
\end{aligned}
$$

In our numerical evolution, we use Cartesian coordinates (x,y,z), which are related to the (r,θ,ϕ) coordinates by the usual transformation: $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$ and $z=r\cos\theta$. Cartesian components of the spatial metric γ_{ij} and extrinsic curvature K_{ij} are computed by the usual transformation formula of tensor components. The initial data of a rotating BH moving with a constant velocity as measured by a distant observer are constructed by boosting the spacetime metric derived from Eq. (13) and the lapse and shift in Eqs. (6) and (7).

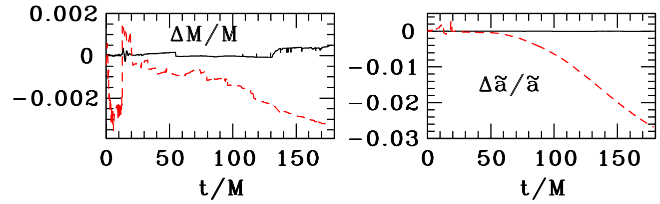

FIG. 1: Fractional error of the BH mass $\Delta M/M$ (left graph) and spin parameter $\Delta\tilde{a}/\tilde{a}$ (right graph) vs time for a stationary BH with spin parameter $\tilde{a}\equiv a/M=0.99$. Dash (red) lines show the results for the quasi-isotropic radial coordinate, and solid (black) lines show the results for our new radial coordinate. The resolution in the innermost refinement level is $M/50$ for both cases.

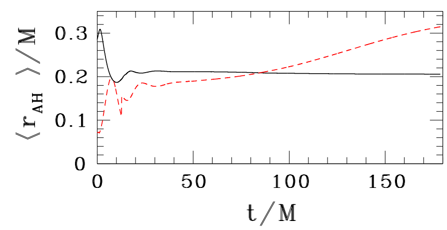

FIG. 2: Evolution of the average coordinate radius of the BH's horizon evolved with our radial coordinate (black solid line) and the quasi-isotropic coordinate (red dashed line).

B. Numerical evolution scheme

The formulation and numerical scheme for our simulations are basically the same as those already reported in [30, 49], to which the reader may refer for details. We adopt the BSSN formalism coupled to the standard moving puncture gauge conditions to evolve the spatial metric and extrinsic curvature. The evolution equations are given by Eqs. (9)–(13) in [49]. The gauge conditions are given by Eqs. (2)–(4) in [30], with the gauge parameter η set to $1/M$. During the evolution, we adopt Eqs. (29), (30) in [49] and Eq. (11) in [50] to help enforce/control additional constraints in the BSSN variables.

We evolve the BSSN equations with sixth-order accurate, centered finite-differencing stencils, except on shift advection terms, where we use sixth-order accurate upwind stencils. We apply Sommerfeld outgoing wave boundary conditions to all BSSN fields. Our code is embedded in the Cactus parallelization framework [51], and our fourth-order Runge-Kutta timestepping is managed by the `MoL` (Method of Lines) thorn, with a Courant-Friedrichs-Lewy (CFL) factor set to 0.25 in all simulations. We find that we get better results if we add a seventh-order Kreiss-Oliger dissipation of the form

$$(\epsilon/256)(\Delta x^7\partial_x^7+\Delta y^7\partial_y^7+\Delta z^7\partial_z^7)f \quad (16)$$

to the lapse and shift variables f, with the parameter ϵ set to 0.9. We use the Carpet [52] infrastructure to implement moving-box adaptive mesh refinement. The apparent horizon of the BH is computed with the `AHFinderDirect` Cactus thorn [53]. The BH's mass and angular momentum are computed using the isolated horizon formalism [54], with the axial Killing vector computed using the numerical technique described in [55].

For the initial lapse and shift, we have implemented the lapse and shift obtained from the analytic spacetime metric (6), (7) and (13) (boosted in the case of a moving BH), as well as the standard choice setting $\alpha=\psi^{-2}$ and $\beta^i=0$ [where $\psi=(\det\gamma_{ij})^{1/3}$]. We find that these two different sets of initial lapse and shift data yield similar evolution results for stationary BHs. The first set of lapse and shift yields a slightly better result for boosted BHs. We show the results for the second set of initial lapse and shift for stationary BHs and the first set for boost BHs in the next section.

III. RESULTS

We perform simulations on rapidly rotating BHs with spin parameter $\tilde{a}\equiv a/M=0.99$ for cases where the BH is stationary and boosted to give a momentum $P=0.5M$, relative to observers at spatial infinity. We use seven refinement levels for all these simulations. The resolution in the innermost refinement level is $M/50$ for a typical run. We also try resolutions $M/40$, $M/60$ and $M/80$ for

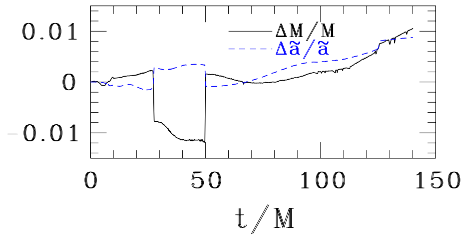

FIG. 3: Fractional error of the BH mass $\Delta M/M$ (black solid line) and spin parameter $\Delta\tilde{a}/\tilde{a}$ (blue dash line) vs time for a BH with spin parameter $\tilde{a} \equiv a/M = 0.99$ and moving with a momentum $P = 0.5M$ relative to observers at spatial infinity. The resolution in the innermost refinement level is $M/50$.

the stationary BH case and confirm that the code converges at approximately fourth order, as expected. The outer boundary is a rectangular box with a length of $512M \times 512M \times 256M$ in the (x, y, z) directions. We impose reflection symmetry about the equatorial ($z = 0$) plane.

Figure 1 shows the fractional error in the mass and spin parameter of the BH as a function of time for a stationary BH, evolved with both the quasi-isotropic radial coordinate and our proposed new radial coordinate. The grid structure, resolution and gauge conditions are identical for these two runs. We see that the BH's spin slowly decreases with time when evolved with the quasi-isotropic radial coordinate. Such a secular shift of the BH's spin is also observed in the evolution of a near-extremal-Bowen-York-spin BH, and can be reduced by using higher resolution [37]. By contrast, we see no such drift when evolved with our proposed radial coordinate using the same resolution. The BH's spin is conserved to within 10^{-4} during the entire evolution of $180M$. We attribute this result to the fact that the BH interior is better resolved with our radial coordinate during the early simulation. At $t = 0$, the coordinate radius of the horizon is $0.07M$ in the quasi-isotropic coordinate and $0.285M$ in our radial coordinate. Hence the initial BH's diameter is covered by 7 grid points in the quasi-isotropic coordinate and 28 grid points in our radial coordinate. Figure 2 shows the average coordinate radius of the BH as a function of time, evolved with our radial coordinate (black solid line) and the quasi-isotropic coordinate (red dashed line). We find that the average coordinate radius approaches a constant value after $t \gtrsim 50M$ when evolved with our radial coordinate, and the metric approaches a "trumpet" geometry [56, 57] in which the conformal factor $\psi \propto r^{-1/2}$ near the puncture. By contrast, we find the radius increases slowly at late time when evolved with the quasi-isotropic coordinates, which correlates with the slow decrease in the BH spin due to accumulated numerical inaccuracy during the early evolution.

We have tried to evolve a BH with spin parameter $a/M = 0.999$. We find that although the initial horizon radius is $0.261M$, the puncture evolution quickly drives the horizon radius to below $0.1M$ after $\sim 5M$. The BH spin slowly decreases with time due to insufficient resolution. It has been reported that the horizon radius increases when the parameter η in the shift equation increases [58]. We have observed this behavior for lower spin BHs. However, we find that the evolution of the horizon size is insensitive to the values of η for the high-spin BHs. For example, for a BH with $a/M = 0.99$, the horizon sizes evolved with different values of η change by less than 5%. Our numerical experiments seem to suggest that the puncture evolution will eventually drive the coordinate size of the horizon to a constant value tending towards zero as the BH spin approaches the extreme Kerr limit. However, for a given BH spin a close to the extreme Kerr limit, the final size of the horizon is still larger than the initial horizon radius in the quasi-isotropic coordinate. Hence our proposed radial coordinate is better suited for evolving high-spin BHs than the quasi-isotropic coordinate.

Finally, Fig. 3 shows the boosted case in which the BH moves with a momentum $P = 0.5M$. We see that the errors are less than 1.2% throughout the entire evolution lasting about $150M$, during which the BH has traveled a coordinate distance of about $60M$. This demonstrates that stable evolution of rapidly rotating BHs can be achieved using the moving puncture technique for the puncture initial data in our radial coordinate.

IV. CONCLUSION

We construct a new radial coordinate to write the Kerr metric in puncture form. This new radial coordinate has the advantage that the BH horizon radius remains finite in the extreme Kerr limit, which is useful for numerical simulations. By contrast, the horizon radius approaches zero in the quasi-isotropic coordinate originally adopted for this metric. We have demonstrated that higher accuracy is achieved by using our coordinate rather than the quasi-isotropic coordinate when evolving a high-spin BH. With our new coordinate, we are able to evolve, using the moving puncture technique, rapidly rotating BHs, both stationary and boosted, with spin parameters as high as 0.99.

Binary black hole initial data with rapidly spinning BHs may be constructed using a conformal background metric consisting of the superposition of two Kerr-like conformal metrics in puncture form. This type of initial data for binary black hole head-on collision has been constructed in quasi-isotropic coordinates [45–48]. It will be useful to generalize this technique to construct quasi-circular, rapidly spinning binary black hole initial data using superposed puncture Kerr metrics in our proposed radial coordinate for the background metric. Our numerical results presented in this paper suggest that such initial data can be evolved reliably using the moving puncture technique. This type of initial data has the additional advantages that the BH spins can be higher and the amount of spurious gravitational radiation will be significantly less than the conformally flat initial data, as demonstrated in [47] for the head-on collision case.

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[1] B. Abbott and the LIGO Scientific Collaboration, Phys. Rev. D **77**, 062002 (2008).
[2] D. A. Brown, S. Babak, P. R. Brady, N. Christensen, T. Cokelaer, J. D. E. Creighton, S. Fairhurst, G. Gonzalez, E. Messaritaki, B. S. Sathyaprakash, et al., Class. Quant. Grav. **21**, S1625 (2004).
[3] F. Acernese and the VIRGO Collaboration, Class. Quant. Grav. **23**, S635 (2006).
[4] F. Beauville and the LIGO-VIRGO Working Group, Classical and Quantum Gravity **25**, 045001 (2008).
[5] H. Lück and the GEO600 collaboration, Class. Quant. Grav. **23**, S71 (2006).
[6] M. Ando and the TAMA collaboration, Class. Quant. Grav. **19**, 1409 (2002).
[7] D. Tatsumi and the TAMA collaboration, Classical and Quantum Gravity **24**, S399 (2007).
[8] G. Heinzel, C. Braxmaier, K. Danzmann, P. Gath, J. Hough, O. Jennrich, U. Johann, A. Rüdiger, M. Sallusti, and H. Schulte, Class. Quant. Grav. **23**, S119 (2006).
[9] E. Phinney et al, NASA Mission Concept Study (2003).
[10] S. Kawamura and the DECIGO collaboration, Class. Quant. Grav. **23**, S125 (2006).
[11] M. Volonteri, P. Madau, E. Quataert, and M. J. Rees, Astrophys. J. **620**, 69 (2005).
[12] E. Berti and M. Volonteri, Astrophys. J. **684**, 822 (2008).
[13] K. S. Thorne, Astrophys. J. **191**, 507 (1974).
[14] C. F. Gammie, S. L. Shapiro, and J. C. McKinney, Astrophys. J. **602**, 312 (2004).
[15] J.-M. Wang, Y.-M. Chen, L. C. Ho, and R. J. McLure, Astrophys. J. Lett. **642**, L111 (2006).
[16] M. Middleton, C. Done, M. Gierliński, and S. W. Davis, Mon. Not. R. Astro. Soc. **373**, 1004 (2006).
[17] J. M. Miller, C. S. Reynolds, A. C. Fabian, G. Miniutti, and L. C. Gallo, Astrophys. J. **697**, 900 (2009).
[18] L. Gou, J. E. McClintock, J. Liu, R. Narayan, J. F. Steiner, R. A. Remillard, J. A. Orosz, S. W. Davis, K. Ebisawa, and E. M. Schlegel, Astrophys. J. **701**, 1076 (2009).
[19] M. Campanelli, C. Lousto, Y. Zlochower, and D. Merritt, Astrophys. J. Lett. **659**, L5 (2007).
[20] M. Campanelli, C. O. Lousto, Y. Zlochower, and D. Merritt, Physical Review Letters **98**, 231102 (2007).
[21] J. A. González, M. Hannam, U. Sperhake, B. Brügmann, and S. Husa, Physical Review Letters **98**, 231101 (2007).
[22] S. Dain, C. O. Lousto, and Y. Zlochower, Phys. Rev. D **78**, 024039 (2008).
[23] J. Healy, F. Herrmann, I. Hinder, D. M. Shoemaker, P. Laguna, and R. A. Matzner, Physical Review Letters **102**, 041101 (2009).
[24] M. Volonteri, Astrophys. J. Lett. **663**, L5 (2007).
[25] M. Volonteri, S. Callegari, M. Colpi, M. Dotti, and L. Mayer, Memorie della Societa Astronomica Italiana **79**, 1231 (2008).
[26] L. Blecha and A. Loeb, Mon. Not. R. Astro. Soc. **390**, 1311 (2008), 0805.1420.
[27] S. Komossa and D. Merritt, Astrophys. J. Lett. **689**, L89 (2008), 0811.1037.
[28] A. Gualandris and D. Merritt, Astrophys. J. **678**, 780 (2008).
[29] E. Rantsiou, S. Kobayashi, P. Laguna, and F. A. Rasio, Astrophys. J. **680**, 1326 (2008).
[30] Z. B. Etienne, Y. T. Liu, S. L. Shapiro, and T. W. Baumgarte, Phys. Rev. D **79**, 044024 (2009).
[31] M. Ruffert and H. Janka, ArXiv e-prints (2009), 0906.3998.
[32] M. Campanelli, C. O. Lousto, P. Marronetti, and Y. Zlochower, Physical Review Letters **96**, 111101 (2006).
[33] J. G. Baker, J. Centrella, D.-I. Choi, M. Koppitz, and J. van Meter, Physical Review Letters **96**, 111102 (2006).
[34] S. Dain, C. O. Lousto, and R. Takahashi, Phys. Rev. D **65**, 104038 (2002).
[35] G. Lovelace, R. Owen, H. P. Pfeiffer, and T. Chu, Phys. Rev. D **78**, 084017 (2008).
[36] M. Hannam, S. Husa, and N. Ó. Murchadha, ArXiv e-prints (2009), 0908.1063.
[37] P. Marronetti, W. Tichy, B. Brügmann, J. González, and U. Sperhake, Phys. Rev. D **77**, 064010 (2008).
[38] M. Shibata and T. Nakamura, Phys. Rev. D **52**, 5428 (1995).
[39] T. W. Baumgarte and S. L. Shapiro, Phys. Rev. D **59**, 024007 (1998).
[40] Z. B. Etienne, J. A. Faber, Y. T. Liu, S. L. Shapiro, and T. W. Baumgarte, Phys. Rev. D **76**, 101503 (2007).
[41] D. Brown, O. Sarbach, E. Schnetter, M. Tiglio, P. Diener, I. Hawke, and D. Pollney, Phys. Rev. D **76**, 081503 (2007).
[42] D. Brown, P. Diener, O. Sarbach, E. Schnetter, and M. Tiglio, Phys. Rev. D **79**, 044023 (2009).
[43] S. R. Brandt and E. Seidel, Phys. Rev. D **52**, 856 (1995).
[44] S. R. Brandt and E. Seidel, Phys. Rev. D **54**, 1403 (1996).
[45] S. Dain, Physical Review Letters **87**, 121102 (2001).
[46] S. Dain, Phys. Rev. D **64**, 124002 (2001).
[47] M. Hannam, S. Husa, B. Brügmann, J. A. González, and U. Sperhake, Classical and Quantum Gravity **24**, S15 (2007).
[48] W. Krivan and R. H. Price, Phys. Rev. D **58**, 104003 (1998), arXiv:gr-qc/9806017.
[49] Z. B. Etienne, J. A. Faber, Y. T. Liu, S. L. Shapiro, K. Taniguchi, and T. W. Baumgarte, Phys. Rev. D **77**, 084002 (2008).
[50] M. D. Duez, S. L. Shapiro, and H.-J. Yo, Phys. Rev. D **69**, 104016 (2004).
[51] `http://www.cactuscode.org/`.
[52] E. Schnetter, S. H. Hawley, and I. Hawke, Class. Quantum Grav. **21**, 1465 (2004).
[53] J. Thornburg, Class. Quant. Grav. **21**, 743 (2004).
[54] A. Ashtekar and B. Krishnan, Living Reviews in Relativity **7**, 10 (2004), arXiv:gr-qc/0407042.

[55] O. Dreyer, B. Krishnan, D. Shoemaker, and E. Schnetter, Phys. Rev. D **67**, 024018 (2003).
[56] M. Hannam, S. Husa, D. Pollney, B. Brügmann, and N. Ó. Murchadha, Physical Review Letters **99**, 241102 (2007).
[57] M. Hannam, S. Husa, F. Ohme, B. Brügmann, and N. Ó Murchadha, Phys. Rev. D **78**, 064020 (2008).
[58] B. Brügmann, J. A. González, M. Hannam, S. Husa, U. Sperhake, and W. Tichy, Phys. Rev. D **77**, 024027 (2008).