Strong gravitational field in $R + \mu^4/R$ gravity

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We introduce a new approach for investigating the weak field limit of vacuum field equations in $f(R)$ gravity and we find the weak field limit of $f(R) = R + \mu^4/R$ gravity. Furthermore, we study the strong gravity regime in $R + \mu^4/R$ model of $f(R)$ gravity. We show the existence of strong gravitational field in vacuum for such model. We find out in the limit $\mu \to 0$, the weak field limit and the strong gravitational field can be regarded as a perturbed Schwarzschild metric.

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I. INTRODUCTIONS

Observations on supernova type Ia [1], cosmic microwave background [2] and large scale structure [3], all indicate that the expansion of the universe is not proceeding as predicted by general relativity, if the universe is homogeneous, spatially flat, and filled with relativistic matter. An interesting approach to explain the positive acceleration of the universe is $f(R)$ theories of gravity which generalize the geometrical part of Hilbert-Einstein lagrangian [4–11]. One of the initiative $f(R)$ models supposed to explain the positive acceleration of expanding universe has $f(R)$ action as $f(R) = R - \mu^4/R$ [5]. After proposing the $f(R) = R - \mu^4/R$ model, it was appeared this model suffer several problems. In the metric formalism, initially Dolgov an Kawasaki discovered the violent instability in the matter sector [12]. The analysis of this instability generalized to arbitrary $f(R)$ models [13, 14] and it was shown than an $f(R)$ model is stable if $d^2f/dR^2 > 0$ and unstable if $d^2f/dR^2 < 0$. Thus we can deduce $R - \mu^4/R$ suffer the Dolgov-Kawasaki instability but this instability removes in the $R+\mu^4/R$ model, where $\mu^4 > 0$. Furthermore, one can see in the $R-\mu^4/R$ model the cosmology is inconsistent with observation when nonrelativistic matter is present. In fact there is no matter dominant era [10, 15]. However, the recent study shows the standard epoch of matter domination can be obtained in the $R + \mu^4/R$ model [10].

It is obvious that a viable theory of gravity must have the correct newtonian limit. Indeed a viable theory of $f(R)$ gravity must pass solar system tests. After the $R - \mu^4/R$ was suggested as the solution of cosmic-acceleration puzzle, it has been argued that this theory is inconsistent with solar system tests [16]. This claim was based on the fact that metric $f(R)$ gravity is equivalent to $\omega = 0$ Brans-Dicke theory, while the observational constraint is $\omega > 40000$. But this is not quite the case and it is possible to investigate the spherical symmetric solutions of $f(R)$ gravity without invoking the equivalence of $f(R)$ gravity and scalar tensor theory [7, 9, 17–21]. It has been shown that some $f(R)$ models accept the Schwarzschild-de Sitter spacetime as a spherical symmetric solutions of field equation[17]. Hence $R - \mu^4/R$ model has a Schwarzschild-de Sitter solution with constant curvature as $R = \sqrt{3\mu^4}$ where this is not the case in $R + \mu^4/R$ model.

In this paper we study the $R + \mu^4/R$ model of $f(R)$ gravity. We find the static spherically symmetric solution of vacuum field equation in both weak field limit and strong gravity regime, moreover, the weak field analysis can be expanded on $f(R)$ models of the form $f(R) = R + \epsilon g(R)$.

II. WEAK FIELD LIMIT

In this section we investigate the weak field solution of vacuum field equation in $f(R)$ theories of gravity. We are interested in model of the form $f(R) = R + \epsilon g(R)$, with ϵ an adjustable small parameter. The motivation for discussing these models is that the nonlinear curvature terms that grow at low curvature can lead to the late time positive acceleration, but during the standard matter dominated epoch, where the curvature is assumed to be relatively high, could have a negligible effect.

The vacuum field equations for these models are

$$G_{\mu\nu} = -\epsilon \left[G_{\mu\nu} + g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu + \frac{g_{\mu\nu}}{2} \times \left(R - \frac{g(R)}{\varphi(R)} \right) \right] \varphi(R), \tag{1}$$

where $\varphi(R) = dg(R)/dR$. Contracting the field equation we obtain

$$R = \epsilon \left[R - \frac{2g(R)}{\varphi(R)} + 3\Box \right] \varphi(R). \tag{2}$$

If $\epsilon = 0$ the above equations reduce to Einstein equation. Hence we suppose $G_{\mu\nu}$ and R in the r.h.s of Eqs.(1,2) can be neglected for small values of ϵ. Furthermore if the condition $\lim_{R\to 0} [g(R)/\varphi(R)] = 0$ is satisfied we can

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neglect this term too. neglecting these terms leads to the following equations

$$G_{\mu\nu} = -\epsilon \left[g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu\right] \varphi(R), \tag{3}$$

and

$$R = \epsilon 3 \Box \varphi(R). \tag{4}$$

The analysis of spherically symmetric solution can be carried out using schwarzschild coordinate

$$ds^2 = -A(r)dt^2 + B(r)^{-1}dr^2 + r^2 d\Omega^2. \tag{5}$$

In the weak field limit approximation the metric deviates slightly from the Minkowski metric, so we can write

$$\begin{aligned} &A(r) = 1 + a(r), \\ &B(r) = 1 + b(r), \\ &\mid a \mid, \mid b \mid \ll 1. \end{aligned} \tag{6}$$

When solving the field equations(3,4) we will keep only terms linear in the perturbations $a(r)$, $b(r)$. Hence equations (3,4) leads to

$$\begin{aligned} &\frac{a'}{r} + \frac{b}{r^2} = -\epsilon \frac{2}{r}\frac{d\varphi(R)}{dr} \\ &\frac{b'}{r} + \frac{b}{r^2} = -\epsilon \nabla^2 \varphi(R), \end{aligned} \tag{7}$$

and

$$R = 3\epsilon \nabla^2 \varphi(R). \tag{8}$$

where ($\prime$) indicates a derivation with respect to r.

A. $f(R) = R^{1+\epsilon}$

This model is considered in [7]. It is shown that this model has an exact spherically symmetric vacuum solution and regarding the general line-element in Eq.(5), it may be written as

$$\begin{aligned} A(r) &= r^{2\epsilon(1+2\epsilon)/(1-\epsilon)} + c\; r^{-(1-4\epsilon)/(1-\epsilon)}, \\ B(r) &= \frac{(1-\epsilon)^2}{(1-2\epsilon+4\epsilon^2)(1-2\epsilon-2\epsilon^2)} \\ &\quad \times \left(1 + c\; r^{-(1-2\epsilon+4\epsilon^2)/(1-\epsilon)}\right), \end{aligned}$$

where c is a constant. In the limit $\epsilon \to 0$, these solutions become

$$\begin{aligned} ds^2 &= -\left(1 + 2\epsilon \ln r + \frac{c}{r}\right) dt^2 + \left(1 + 2\epsilon + \frac{c}{r}\right)^{-1} dr^2 \\ &\quad + r^2 d\Omega^2. \end{aligned} \tag{9}$$

because we seek the weak field limit, in above equation we assume $c/r \ll 1$.

Since we are interested in the limit $\epsilon \to 0$, we may expand $f(R) = R^{1+\epsilon}$ around $\epsilon = 0$. Then we have

$$\begin{aligned} f(R) &= R + \epsilon R \ln R, \\ h(R) &= R \ln R, \\ \varphi(R) &= 1 + \ln R. \end{aligned}$$

It is clear that $g(R)$ satisfies the condition $\lim_{R\to 0}[g(R)/\varphi(R)] = 0$. Inserting $\varphi(R)$ in the trace equation (8), the Ricci scalar is obtained as

$$R = -\frac{6\epsilon}{r^2}. \tag{10}$$

Then we arrive at the solutions of Eqs.(7)

$$a = \frac{c}{r} + 2\epsilon \ln r, \quad b = \frac{c}{r} + 2\epsilon, \tag{11}$$

where c is a constant. We can see our solutions are in agreement with the exact solutions (9). Also one can check neglecting R, $G_{\mu\nu}$ and $g(R)/\varphi(R)$ in Eqs.(1, 2) is reasonable.

B. $f(R) = R \pm \mu^4/R$

Based on equivalence between $f(R)$ gravity and Brans-Dicke theory with $\omega = 0$, it was argued that this theory is inconsistent with solar system tests [16]. Indeed by this approach the Post-Newtonian parameter is found as $\gamma_{PPN} = 1/2$ while the measurements indicate $\gamma_{PPN} = 1 + (2.1 \pm 2.3) \times 10^{-5}$ [22]. Also we must note that using equivalence between $f(R)$ gravity and scalar tensor gravity one can find models which are consistent with the solar system tests. This consistency can be made by giving the scalar a high mass or exploiting the so-called chameleon effect[23–25]. However, when one is using equivalence between $f(R)$ gravity and scalar tensor gravity, the continuity of scalar field or its equivalent, the Ricci scalar, at the matter boundary is crucial condition which is not the case in Einstein gravity. But in this work we don't adopt the continuity of Ricci scalar for solving the field equations. Instead, we suppose that when μ tends to zero we arrive at the Einstein gravity. Thus we find a solution for $1/R$ model which is radically different from other solutions in [26, 27].

For this model we have

$$\begin{aligned} g(R) &= \pm 1/R, \\ \varphi(R) &= \mp 1/R^2, \end{aligned} \tag{12}$$

where $g(R)$ fulfills the condition $\lim_{R\to 0}[g(R)/\varphi(R)] = 0$. Solving Eqs.(7,8) we obtain

$$\begin{aligned} &R = \mp 7\alpha\mu^{\frac{4}{3}} r^{-\frac{2}{3}}, \\ &\frac{\mu^4}{R^2} = \frac{1}{49\alpha^2}\mu^{\frac{4}{3}} r^{\frac{4}{3}}, \\ &a = -\frac{2M}{r} \pm \frac{3}{4}\alpha\mu^{\frac{4}{3}} r^{\frac{4}{3}}, \\ &b = -\frac{2M}{r} \pm \alpha\mu^{\frac{4}{3}} r^{\frac{4}{3}}. \end{aligned} \tag{13}$$

where $\alpha^3 = 4/147$ and M is a constant. Therefore the metric for space time is

$$ds^2 = -\left(1 - \frac{2M}{r} \pm \frac{3}{4}\alpha\mu^{\frac{4}{3}}r^{\frac{4}{3}}\right)dt^2 + \left(1 - \frac{2M}{r} \pm \alpha\mu^{\frac{4}{3}}r^{\frac{4}{3}}\right)^{-1}dr^2 + r^2 d\Omega^2. \quad (14)$$

From Eq.(13) it is obvious that in the limit $\mu \to 0$, μ^4/R^2 tends to zero, so there is not singularity in the field equations. Also one can check neglecting R, $G_{\mu\nu}$, and $g(R)/\varphi(R)$ in Eqs.(1, 2) is reasonable.

C. $f(R) = R + \epsilon \ln R$

For this model $\varphi(R) = 1/R$. Solving trace equation (8) and field equations (7) we obtain

$$R = \frac{\sqrt{6\epsilon}}{r}, \quad (15)$$

and

$$a = b = -\frac{2M}{r} - \sqrt{\frac{\epsilon}{6}}r. \quad (16)$$

where M is a constant. Therefore the space time metric for empty space in this model is

$$ds^2 = -\left(1 - \frac{2M}{r} - \sqrt{\frac{\epsilon}{6}}r\right)dt^2 + \left(1 - \frac{2M}{r} - \sqrt{\frac{\epsilon}{6}}r\right)^{-1}dr^2 + r^2 d\Omega^2. \quad (17)$$

We can see, the generalized Newtonian potential is

$$\Phi_G = -\frac{M}{r} - \frac{1}{2}\sqrt{\frac{\epsilon}{6}}r. \quad (18)$$

This generalized gravitational potential has two terms. The first term is the standard Newtonian potential and the second term make a constant acceleration, $+\sqrt{\epsilon/24}$, which is independent of the mass of star. In [28] this metric is used to address the Pioneer's anomalous.

III. STRONG GRAVITY REGIME IN $R + \mu^4/R$ MODEL

In this section we investigate the existence of strong gravitational field for $f(R) = R + \mu^4/R$ model of $f(R)$ gravity. We can rewrite the field equation (1) as

$$G^\nu_\mu\left(1 - \frac{\mu^4}{R^2}\right) = -\frac{1}{3}\delta^\nu_\mu R - \nabla_\mu\nabla^\nu\left(\frac{\mu^4}{R^2}\right), \quad (19)$$

where we have used the trace equation

$$R = -3[R + \Box]\left(\mu^4/R^2\right). \quad (20)$$

In the above equation we have neglected the energy-momentum tensor of matter because we investigate the strong gravitational field around a spherically symmetric distribution of matter. Adopting the general spherically symmetric metric (5), we can rewrite the trace equation (20) and (rr),(tt) components of field equation (19) as

$$-\left[B\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right) + \frac{1}{2}\left(B' + \frac{BA'}{A}\right)\frac{d}{dr} + R\right] \times \left(\mu^4/R^2\right) = \frac{R}{3}, \quad (21a)$$

$$\left(\frac{BA'}{rA} + \frac{B-1}{r^2}\right)\left(1 - \mu^4/R^2\right) + \left(B\frac{d^2}{dr^2} + \frac{B'}{2}\frac{d}{dr}\right)\left(\mu^4/R^2\right) = -\frac{R}{3}, \quad (21b)$$

$$\left(\frac{B'}{r} + \frac{B-1}{r^2}\right)\left(1 - \mu^4/R^2\right) + \frac{BA'}{2A}\frac{d}{dr}\left(\mu^4/R^2\right) = -\frac{R}{3}, \quad (21c)$$

where($\prime$) denotes derivation with respect to the (r). In the previous section we showed , $(R + \mu^4/R)$ model has the week field solution as

$$ds^2 = -\left[1 - \frac{2M}{r} + \frac{3}{4}\alpha(\mu r)^{\frac{4}{3}}\right]dt^2 + \left[1 - \frac{2M}{r} + \alpha(\mu r)^{\frac{4}{3}}\right]^{-1}dr^2 + r^2 d\Omega^2, \quad (22)$$

where $\alpha = (4/147)^{1/3}$. It is obvious this metric reduces to Schwarzschild metric in the limit $\mu \to 0$. Now we seek the solution of field equation in the limit $(r \to 2M)$. Without loss of generality we can assume $2M = 1$. In order to solve equations (21) we use some definitions as

$$\begin{aligned} \phi &= \gamma/R, \\ \gamma &= -\mu^{4/3}, \\ A &= 1 - \frac{1}{r} + \gamma a(r), \\ B &= 1 - \frac{1}{r} + \gamma b(r). \end{aligned} \quad (23)$$

Because we seek the solution in the limit $r \to 1$, we may define a new variable as $x = r - 1$. Using these definitions we can rewrite Eqs.(21) as

$$\gamma\left(b\frac{d}{dx} + \frac{2b}{x+1} + \frac{b' + a'}{2} + \frac{(x+1)(b-a)}{2(x + \gamma a(x+1))}\left(\frac{1}{(x+1)^2} + \gamma a'\right)\right)\frac{d\phi^2}{dx} = \left(\frac{1}{3} - \gamma\phi^2\right)\frac{1}{\phi} - \left(\frac{x}{x+1}\frac{d^2}{dx^2} + \frac{2x+1}{(x+1)^2}\frac{d}{dx}\right)\phi^2 \quad (24a)$$

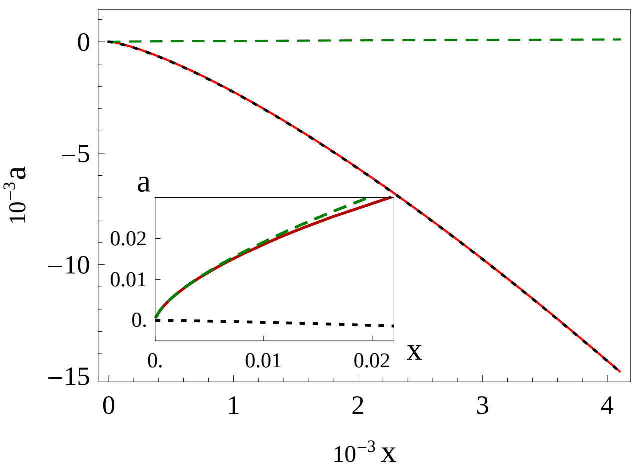

FIG. 1: a against x. The red-dashed line shows numerical results of Eqs.(25). The green-dashed line represents approximate solution for $x \ll 1$ (Eq.(26a)) and the black-dotted line is the approximate solution for $x \gg 1$ (Eq.(28a)). A close up on the origin of main figure is presented .

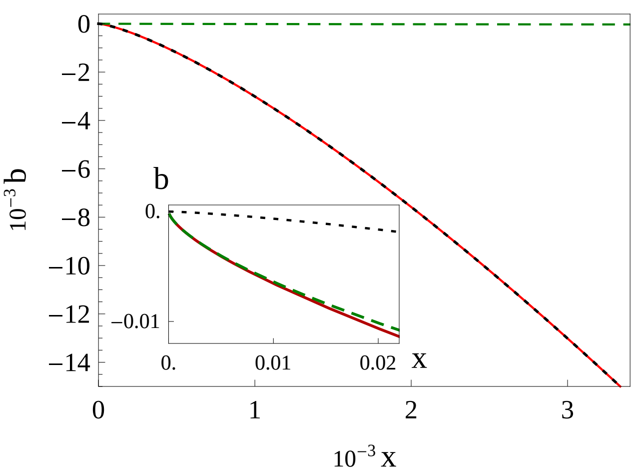

FIG. 2: b against x. The red-dashed line shows numerical results of Eqs.(25). The green-dashed line represents approximate solution for $x \ll 1$ (Eq.(26b)) and the black-dotted line is the approximate solution for $x \gg 1$ (Eq.(28b)). A close up on the origin of main figure is presented .

$$\left(\frac{x}{x+1}\frac{d^2}{dx^2}+\frac{1}{2(x+1)^2}\frac{d}{dx}+\gamma\left(b\frac{d^2}{dx^2}+\frac{b'}{2}\frac{d}{dx}\right)\right)\phi^2$$
$$=\frac{1}{3\phi}+\left(\frac{b}{(x+1)^2}+\frac{a'}{x+1}+\frac{b-a}{x+\gamma a(x+1)}\right.$$
$$\left.\times\left(\frac{1}{(x+1)^2}+\gamma a'\right)\right)(1+\gamma\phi^2) \tag{24b}$$

$$\frac{1}{2}\left(1+\gamma\frac{(x+1)(b-a)}{x+\gamma a(x+1)}\right)\left(\frac{1}{(x+1)^2}+\gamma a'\right)\frac{d\phi^2}{dx}$$
$$=\frac{1}{3\phi}+\left(\frac{b}{(x+1)^2}+\frac{b'}{x+1}\right)\left(1+\gamma\phi^2\right), \tag{24c}$$

where ($\prime$) denotes derivation with respect to the (x). For

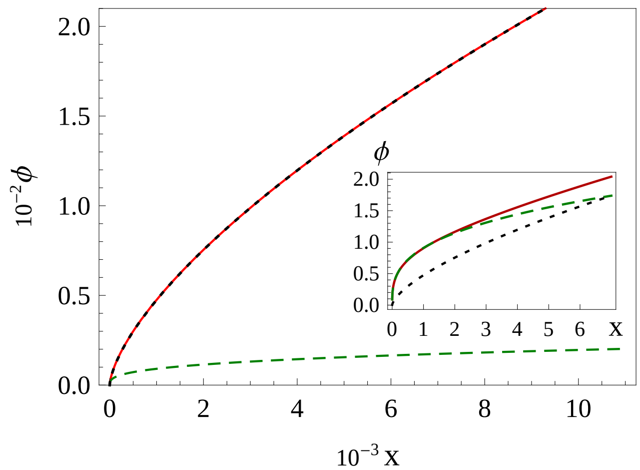

FIG. 3: φ against x. The red-dashed line shows numerical results of Eq.(25a). The green-dashed line represents approximate solution for $x \ll 1$ (Eq.(26c)) and the black-dotted line is the approximate solution for $x \gg 1$ (Eq.(28c)). A close up on the origin of main figure is presented .

the limit $\mu \to 0$, in the above equations we suppose that we can neglect terms containing γ . After solving equations we check this assumption. By neglecting these terms, equations 24 can be rewritten as

$$\frac{1}{3\phi}=\left(\frac{x}{x+1}\frac{d^2}{dx^2}+\frac{2x+1}{(x+1)^2}\frac{d}{dx}\right)\phi^2 \tag{25a}$$

$$\frac{b}{(x+1)^2}+\frac{a'}{x+1}+\frac{b-a}{x(x+1)^2}=-\frac{1}{3\phi}$$
$$+\left(\frac{x}{x+1}\frac{d^2}{dx^2}+\frac{1}{2(x+1)^2}\frac{d}{dx}\right)\phi^2 \tag{25b}$$

$$\frac{1}{2}\frac{1}{(x+1)^2}\frac{d\phi^2}{dx}=\frac{1}{3\phi}+\frac{b}{(x+1)^2}+\frac{b'}{x+1}. \tag{25c}$$

In the limit $x \ll 1$, solutions of Eqs. (25) are

$$a_0=\frac{3}{8}\left(\frac{4}{3}\right)^{1/3}x^{2/3}, \tag{26a}$$

$$b_0=-\frac{1}{8}\left(\frac{4}{3}\right)^{1/3}x^{2/3}, \tag{26b}$$

$$\phi_0=\left(\frac{3}{4}\right)^{1/3}x^{1/3}. \tag{26c}$$

Thus we obtain the metric for $x \ll 1$ as

$$\begin{aligned} ds^2 &= -\left(1-\frac{1}{r}-\frac{3}{8}\left(\frac{4}{3}\right)^{1/3}\mu^{4/3}(r-1)^{2/3}\right)dt^2 \\ &+ \left(1-\frac{1}{r}+\frac{1}{8}\left(\frac{4}{3}\right)^{1/3}\mu^{4/3}(r-1)^{2/3}\right)dr^2 \\ &+ r^2 d\Omega^2. \end{aligned} \tag{27}$$

Furthermore, for $x \gg 1$, we can obtain the solutions of equations (25) as

$$a_\infty = -\frac{3}{4}\alpha x^{4/3}, \tag{28a}$$

$$b_\infty = -\alpha x^{4/3}, \tag{28b}$$

$$\phi_\infty = \frac{1}{7\alpha} x^{2/3}, \tag{28c}$$

which are in agreement with week field limit (22). Now we can check the validity of our assumption. Considering the solutions (28), shows that neglecting terms containing γ in Eqs. (24) is valid only for $x \gg\mid \gamma^3 \mid$ or $x \gg \mu^4$. Hence the metric (26) is solution of field equations in the range of $\mu^4 \ll x \ll 1$. By performing a conformal transformation and changing coordinate we can see the strong field solution (27) is

$$\begin{aligned}
ds^2 =& \\
&-\left(1-\frac{2M}{r}-\frac{3}{8}\left(\frac{4}{3}\right)^{1/3}(2M\mu)^{4/3}(\frac{r}{2M}-1)^{2/3}\right)dt^2 \\
&+\left(1-\frac{2M}{r}+\frac{1}{8}\left(\frac{4}{3}\right)^{1/3}(2M\mu)^{4/3}(\frac{r}{2M}-1)^{2/3}\right)dr^2 \\
&+r^2 d\Omega^2,
\end{aligned}$$

which is valid in the range of $(2M\mu)^4 \ll r/2M - 1 \ll 1$ and farther where $r \gg 2M$, the metric of space time can be approximated by the metric (22). Furthermore, we have solved field equations (25)numerically. figure shows that the numerical solutions are in agreement with the approximate solutions (26,28).

IV. DISCUSSION

We have studied spherically symmetric solution of $f(R)$ gravity. At first we have introduced a new approach for investigating the weak field limit of vacuum field equations in $f(R)$ gravity and we find the weak field limit of $f(R) = R + \mu^4/R$ gravity, which differs slightly from the schwarzschild metric. Moreover we have investigated the strong field regime for this model. We have shown that if $(r-2M)/(2M)^5 \gg \mu^4$, where $2M$ and r are Schwarzschild radius and radius in the Schwarzschild coordinate, the gravitational field is a perturbed Schwarzschild metric even in strong gravity regime.

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