Neutrino oscillation phase dynamically induced by $f(R)$-gravity

Salvatore Capozziello, Mariafelicia De Laurentis, Daniele Vernieri
Dipartimento di Scienze Fisiche, Università di Napoli "Federico II", INFN Sez. di Napoli, Compl. Univ. di Monte S. Angelo, Edificio G, Via Cinthia, I-80126, Napoli, Italy

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The gravitational phase shift of neutrino oscillation can be discussed in the framework of $f(R)$-gravity. We show that the shift of quantum mechanical phase can depend on the given $f(R)$-theory that we choose. This fact is general and could constitute a fundamental test to discriminate among the various alternative relativistic theories of gravity. Estimations of ratio between the gravitational phase shift and the standard phase are carried out for the electronic Solar neutrinos.

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Neutrinos are elementary particles that travel at the speed of light (or close to it if massive), are electrically neutral and are capable of passing through ordinary matter with minimal interaction. Due to these properties, they can be investigated, in principle, at all length scales, ranging from nuclei [1], to molecular structures [2], up to galaxies [3, 4] and to the whole Universe [5]. They results from radioactive decays or nuclear reactions such as those that take place in the Sun, stars or nuclear reactors. In particular, they are generated when cosmic rays hit atoms. Current evidences of dark matter and dark energy can be related to the issue that neutrinos have masses and that mass eigenstates mix and/or superimpose [6–9]. The observation of such a mixing is related to suitable constraints. Such constraints should work on observables sensitive to the effective neutrino mass as the mass in Tritium-beta decay, the sum of neutrinos masses in cosmology and the effective Majorana neutrino mass in neutrinoless double-beta decay [10].

A key role is played by the neutrino oscillations that allow the transition among the three types or *"flavor"* eigenstates, that is the electron, muon, and tauon neutrinos. It is well known that such a problem is still open and the research of new effects, in which the oscillations could manifest is one of the main goal of modern physics. For this reason, the quantum mechanical phase of neutrinos, propagating in gravitational field, has been discussed by several authors, also in view of the astrophysical consequences. More controversial is the debate concerning the redshift of flavor oscillation clocks, in the framework of the weak gravitational field of a star [11]. It has also been suggested that the gravitational oscillation phase might have a significant effect in supernova explosions due to the extremely large fluxes of neutrinos produced with different energies, corresponding to the flavor states. This result has been confirmed in [12], and it has been also derived under the assumption that the radial momentum of neutrinos is constant along the trajectory of the neutrino itself [13]. Besides, neutrino oscillations, in particular the gravitational part of the oscillation phase, could straightforwardly come into the debate, which is recently risen, to select what is the correct theory of gravity, due to the well known experimental and theoretical shortcomings of General Relativity [15, 16].

On the other hand, higher-order theories, extending in some way General Relativity, allow to pursue different approaches [17–20]. This viewpoint does not require to find out candidates for dark energy and dark matter at fundamental level (they have not been detected up to now) but it takes into account only the "observed" ingredients (*i.e.* gravity, radiation, neutrinos and baryonic matter). However, the *l.h.s.* of the Einstein equations has to be extended and modified. Despite of this modification, it is in agreement with the spirit of General Relativity since the only request is that the Hilbert-Einstein action should be generalized asking for a gravitational interaction acting, in principle, in different ways at different scales [21, 22]. This feature could be extremely interestindg for neutrino oscillations since further gravitational interaction lengths, emerging from extended theories of gravity, could be related to neutrino oscillation phase. On the other hand, the experimental identification of such a gravitational phase could be a formidable probe both for confirming or ruling out such theories at a fundamental level.

In this letter, after a short review of the gravitational phase shift in neutrino oscillations, we briefly outline the theory of $f(R)$-gravity putting in evidence the Yukawa-like correction to the gravitational potential emerging, in general, as soon as $f(R) \neq R$, that is the theory is not General Relativity. Finally, we discuss the quantum mechanical oscillation phase shift for propagation of neutrino in a generic analytic $f(R)$-gravity model.

Let us start our discussion considering how the gravitational field contributes to the neutrino oscillations. The approach has been firstly developed in [11] and we will outline the main results reported there. If R_A is the size of a physical region where neutrinos are generated, a neutrino energy eigenstate E_ν can be denoted by $|\nu_l, R_A\rangle$ (where l $= e, \mu, \tau$ represents the weak flavor eigenstates). The three neutrino mass eigenstates can be represented by $|\nu_i\rangle$ with $i = 1, 2, 3$ corresponding to the masses m_1, m_2, m_3. The mixing between mass and flavor eigenstates is achieved by the unitary transformation

$$|\nu_l', R_A\rangle = \sum_{i=1,2,3} U_{li}|\nu_i\rangle, \tag{1}$$

where

$$U(\theta,\beta,\psi) = \begin{pmatrix} c_\theta c_\beta & s_\theta c_\beta & s_\beta \\ -c_\theta s_\beta s_\psi - s_\theta c_\psi & c_\theta c_\psi - s_\theta s_\beta s_\psi & c_\beta s_\psi \\ -c_\theta s_\beta c_\psi - s_\theta s_\psi & -s_\theta s_\beta c_\psi - c_\theta s_\psi & c_\beta c_\psi \end{pmatrix} \tag{2}$$

is a 3×3 unitary matrix parametrized by the three mixing angles $\eta = \theta, \beta, \psi$ with $c_\eta = \cos\eta$ and $s_\eta = \sin\eta$. At time $t = t_B > t_A$, the weak flavor eigenstates can be detected in a region R_B and, in general, the evolution is given by

$$|\nu_l, R_B\rangle = exp\left(-\frac{i}{\hbar}\int_{t_A}^{t_B} \mathcal{H}dt + \frac{i}{\hbar}\int_{r_A}^{r_B} \overrightarrow{P}\cdot d\overrightarrow{x}\right)|\nu_l, R_A\rangle, \tag{3}$$

where $\mathcal{H}$ is the Hamiltonian operator associated to the system representing the time translation operator and $\overrightarrow{P}$ is the momentum operator representing the spatial translation operator. The phase change in Eq.(3) is the argument of the exponential function. It can be recast in the form

$$\phi_\nu = \frac{1}{\hbar}\int_{r_A}^{r_B}\left[E\frac{dt}{dr} - p_r\right]dr\,. \tag{4}$$

The covariant formulation is

$$\phi_\nu = \frac{1}{\hbar}\int_{r_A}^{r_B} mds = \frac{1}{\hbar}\int_{r_A}^{r_B} p_\mu dx^\mu, \tag{5}$$

where $p_\mu = mg_{\mu\nu}\frac{dx^\nu}{ds}$ is the 4-momentum of the particle. The effect of gravitational field is given by $g_{\mu\nu}$ and, in general, the neutrino oscillation probability from a state $|\nu_l, R_A\rangle$ to another state $|\nu_l, R_B\rangle$ is given by

$$\begin{aligned}\mathcal{P}\left[|\nu_l, R_A\rangle \to |\nu_{l'}, R_B\rangle\right] \;=\;& \delta_{ll'} - 4U_{l'1}U_{l1}U_{l'2}U_{l2}\sin^2\left[\phi_0^{21}+\right.\\ &\left.+\,\phi_G^{21}\right] - 4U_{l'1}U_{l1}U_{l'3}U_{l3}\,\cdot\\ &\cdot\sin^2\left[\phi_0^{31}+\phi_G^{31}\right] - 4U_{l'1}U_{l1}U_{l'3}\,\cdot\\ &\cdot U_{l3}\sin^2\left[\phi_0^{31}+\phi_G^{31}\right],\end{aligned}$$

where ϕ_0^{ij} are the usual kinematic phase while ϕ_G^{ij} are the gravitational contributions. It can be shown that, in a flat space-time, the ϕ_G^{ij} contributions are zero. In fact, a particle passing nearby a point mass feels a Schwarzschild geometry so the trajectories is

$$dx \simeq \left[1 - \frac{2G_N M}{c^2 r}\right]cdt\,. \tag{6}$$

If the effects of gravitational field are vanishing, Eq. (6) becomes $dx \simeq cdt$. Considering two generic neutrino mass eingestates in a Schwarzshild geometry, the standard phase of neutrino oscillation is

$$\phi_0 = \frac{\Delta m^2 c^3}{4E\hbar}\left(r_B - r_A\right), \tag{7}$$

while the total gravitational phase shift is

$$\phi_{grav} = \frac{G_N \triangle m^2 Mc}{4\hbar E}\log\frac{r_B}{r_A}, \tag{8}$$

as shown in [11], where $\triangle m^2$ is the mass squared difference, $\triangle m^2 = \left|m_2^2 - m_1^2\right|$, E the neutrino energy, r_A and r_B the point where neutrinos are created and detected, respectively. Nevertheless, assuming that the neutrino energy is constant along the trajectory, the term (8) could be cancelled out at typical astrophysical scales [14].

With this considerations in mind, let us take into account how possible corrections to the Newtonian potential could affect this result. We will follow the discussion developed in [23, 24].

Let us consider the general gravitational action

$$\mathcal{A} = \int d^4x\sqrt{-g}\,[f(R) + \mathcal{X}\mathcal{L}_m] \,, \tag{9}$$

where $f(R)$ is an analytic function of the Ricci scalar R, g is the determinant of the metric $g_{\mu\nu}$, $\mathcal{X} = \dfrac{16\pi G_N}{c^4}$ is the coupling constant and $\mathcal{L}_m$ is the perfect-fluid matter Lagrangian. Such an action is the straightforward generalization of the Hilbert-Einstein action of General Relativity obtained for $f(R) = R$. In the metric approach [17], the field equations are obtained by varying (9) with respect to the metric:

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - f'(R)_{;\mu\nu} + g_{\mu\nu}\Box f'(R) = \frac{\mathcal{X}}{2}T_{\mu\nu} \,, \tag{10}$$

that are fourth-order field equations in the metric derivatives. $T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$ is the energy momentum tensor of matter, the prime indicates the derivative with respect to R and $\Box = {}_{;\sigma}{}^{;\sigma}$ is the d'Alembert operator. We adopt the signature $(+,-,-,-)$. We do not want to impose a particular forms for $f(R)$-model but only consider analytic Taylor expansion where the cosmological term and terms higher than second are discarded. The Lagrangian is then

$$f(R) \sim a_1R + a_2R^2 + ... \tag{11}$$

where the parameters $a_{1,2}$ specifies the particular models. Performing the Newtonian limit of the above field equations with the theory given by Eq.(11), it is possible to demonstrate, in general, that the gravitational potential, generated by a point-like matter distribution is (see [23, 24] for details):

$$\Phi(r) = -\frac{3G_NM}{4a_1r}\left(1 + \frac{1}{3}e^{-\frac{r}{L}}\right) = \Phi(r)_{Newton} + \Phi(r)_{Yukawa} \,, \tag{12}$$

where

$$L \equiv L(a_1, a_2) = \left(-\frac{6a_2}{a_1}\right)^{1/2} \,. \tag{13}$$

L is an *interaction gravitational length* due to the correction to the Newtonian potential. However, as soon as $a_1 = 3/4$, $a_2 = 0$ and $\Phi(r)_{Yukawa} \to 0$, the standard Newtonian limit of General Relativity is fully recovered (for a discussion on this point, see [22]).

Now we calculate the gravitational phase shift of neutrino oscillation in $f(R)$-gravity using the potential (12) in the Eq. (4). We obtain the general expression

$$\phi_{grav} = \frac{\Delta m^2Mc}{4\hbar E}\left(\frac{3G_N}{4a_1}\right)\int_{r_A}^{r_B}\left(\frac{1}{r} + \frac{1}{3r}e^{-\frac{L}{r}}\right), \tag{14}$$

from which we have the following result

$$\phi_{grav} = \frac{\Delta m^2Mc}{4\hbar E}\left(\frac{3G_N}{4a_1}\right)\left[\log\frac{r_B}{r_A} + \sum_{n=1}^{\infty}(-1)^n\left(\frac{\left(\frac{r_B-r_A}{L}\right)^n}{n\cdot n!}\right)\right] = \phi_{Newton} + \phi_{Yukawa} \,, \tag{15}$$

where ϕ_{grav} is the total gravitational phase shift in Eq. (8) and

$$\phi_{Yukawa} = \frac{\Delta m^2Mc}{4\hbar E}\left(\frac{3G_N}{4a_1}\right)\left[\sum_{n=1}^{\infty}(-1)^n\left(\frac{\left(\frac{r_B-r_A}{L}\right)^n}{n\cdot n!}\right)\right] \,. \tag{16}$$

The Yukawa term disappears in standard Einstein gravity, that is $f(R) = R$. Note that the series in above equation is absolutely convergent. If we consider Solar neutrinos, we can use the following values: $M \sim M_\odot \sim 1.9891 \times 10^{30}$Kg,

$r_A \sim r_\oplus \sim 6.3 \times 10^3$Km, and $r_B \sim r_A + D$, where $D \sim 1.5 \times 10^8$Km is the Sun-Earth distance. In order to estimate the phases differences (7), (8) and (16), we introduce the ratio Q_{grav} defined as

$$Q_{grav} = \frac{\phi_{Newton}}{\phi_0} \sim \frac{G_N M \log \frac{r_B}{r_A}}{c^2(r_B - r_A)} \sim 10^{-7}, \tag{17}$$

and the ratio Q_{Yukawa} defined as

$$Q_{Yukawa} = \frac{\phi_{Yukawa}}{\phi_0} \sim \frac{G_N M}{c^2(r_B - r_A)} \sum_{n=1}^{\infty} (-1)^n \left(\frac{\left(\frac{r_B - r_A}{L}\right)^n}{n \cdot n!} \right), \tag{18}$$

where we have assumed that $3/4a_1 \sim 1$. Note that both Q_{Newton} and Q_{Yukawa} do not depend on the squared-mass difference Δm^2 and on the neutrino energy E. The ratio Q_{Yukawa} can be calculated for different values of the *interaction lenght* L. For example from Eq. (18), after summing the series, we obtain the results:

$$L \sim 1.5 \cdot 10^7 Km \Longrightarrow Q_{Yukawa} \sim -2.9 \cdot 10^{-8}, \tag{19}$$

$$L \sim 1.5 \cdot 10^8 Km \Longrightarrow Q_{Yukawa} \sim -8 \cdot 10^{-9}. \tag{20}$$

In this way the values of Q_{Yukawa} can be seen as corrections to the standard gravitational phase shift of neutrino oscillations depending on the particular choice of L and so, through Eq.(13), directly on the particular $f(R)$-model considered.

We remark that the calculated correction to gravitational phase shift in Eq. (16) depends on the *interaction lenght* L defined in Eq. (13). This is directly related to the particular $f(R)$-gravity model through the coefficients a_1 and a_2 in Eq. (11). This fact could be used as an experimental test in order to probe a given gravity theory through the neutrino oscillation induced by means of the gravitational field itself. On the other hand, interpreting L as the characteristic wavelenght of the neutrino interaction with the gravitational field, the gravitational phase correction could be used as a method to constrain the mass of electronic neutrinos travelling from the Sun to the Earth surface, or, eventually, also from other neutrinos sources as Supernovae or neutron stars.

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