Explaining Holographic Dark Energy

Sheldon Gao

Unit for History and Philosophy of Science, Faculty of Science

& Centre for Time, Department of Philosophy, University of Sydney

Email: sgao7319@uni.sydney.edu.au

The physical origin of holographic dark energy (HDE) is reexamined. It is shown that the well-accepted explanation in terms of the UV/IR connection argument of Cohen *et al* is wrong. Moreover, Thomas's bulk holography argument, which is considered as another physical basis of the HDE model, is not consistent with observations either. A new conjecture is then proposed to explain the HDE model. It is suggested that the dark energy of the universe may originate from the quantum fluctuations of space-time limited in the event horizon of the universe. The energy density of such fluctuations is shown to assume the same form as that in the HDE model. Moreover, both theoretical considerations and latest observations suggest $c \approx \sqrt{\pi}/2$.

1. Introduction

Recently a holographic dark energy (HDE) model is proposed to explain the observed dark energy of the universe [1-4]. According to the model, the dark energy density is

$$\rho_{DE} = 3c^2 M_P^2 L^{-2} \qquad (1)$$

where c is an undetermined numerical constant of order of unity, M_P is the reduced Planck mass $M_P^2 = 1/8\pi G$, L is the event horizon of the universe. It has been shown that the model is favored by the latest observational data (see, e.g. Refs. [5,6]). The best-fit results in Ref. [5] and Ref. [6] are respectively $c = 0.88^{+0.24}_{-0.06}$ and $c = 0.818^{+0.113}_{-0.097}$ for 68.3% confidence level, when using the observational data including the sample of Type Ia supernovae (SNIa), the shift parameter of the cosmic microwave background (CMB), and the baryon acoustic oscillation (BAO) measurement. As thus, it seems that holographic dark energy is a plausible candidate for the dark energy of the universe.

However, most investigations concentrate on the application of Eq. (1) and its comparison with empirical data, while the physical basis of the HDE model has not been deeply studied (the first half of Ref. [7] is an exception). Moreover, a recent analysis has shown that the well-accepted explanation of Eq. (1), based on the UV/IR connection argument of Cohen *et al* [1], has serious problems when applying the model to different eras of the universe [8, 9]. Since Eq. (1) itself seems to be empirically favored when being used to explain the dark energy, a further analysis of its physical basis becomes necessary and even very urgent. This will be the main purpose of this paper.

The plan of this paper is organized as follows. In Section 2, we first examine Cohen *et al*'s UV/IR connection argument. It is shown that an effective quantum field theory (QFT), with a relationship between UV and IR cutoffs suggested by Cohen *et al*, cannot consistently describe all epochs of the universe and further explain the dark energy. As a result, the argument is actually not the physical basis of the HDE model. We then examine Thomas's bulk holography argument in Section 3. Thomas suggested that the finite quantum zero-point energy consistent with observations may come from the holographic reduction in the number of independent degrees of freedom and the holographic energy per degree of freedom. However, a concrete calculation shows that Thomas's method gives more vacuum energy than the observed dark energy. Therefore, the bulk holography argument cannot provide a plausible explanation of the HDE model either. These negative results imply that the dark energy of the universe probably does not originate from the quantum zero-point energy. In Section 4, we propose a new conjecture on the origin of dark energy, according to which dark energy may originate from the quantum fluctuations of space-time. It is shown that the energy density of such fluctuations assumes the same form as that in the HDE model. Thus, the new conjecture may provide a plausible explanation of the HDE model. Conclusions are given in the last section.

2. The UV/IR connection argument of Cohen *et al*

The well-accepted explanation of the HDE model is that HDE comes from the quantum zero-point energy predicted by an effective quantum field theory (QFT) with a proper UV/IR connection. The argument was first given by Cohen *et al* to solve the fine-tuning problem of the cosmological constant [1], and then developed to explain the dark energy by Hsu and Li [3, 4]. In the following, we will reexamine the argument and analyze whether it is the physical basis of the HDE model.

The argument of Cohen *et al* can be basically formulated as follows. For an effective quantum field theory in a box of size L with UV cutoff Λ, the entropy S scales extensively, $S \sim L^3\Lambda^3$. According to the holographic principle [10-12], the entropy S should be limited by the Bekenstein-Hawking entropy bound, namely

$$L^3\Lambda^3 \le S_{BH} \sim L^2 M_P^2 \qquad (2)$$

where S_{BH} is Bekenstein-Hawking entropy bound. Therefore, the length L, which acts as an IR cutoff, cannot be chosen independently of the UV cutoff, and scales as Λ^{-3}. However, there is evidence that the above entropy bound is still loose, and in particular, a local quantum field theory cannot be used as an effective low energy description of any system containing a black hole (e.g. particle states whose size is smaller than their corresponding Schwarzschild radius) [11, 12]. As thus, there should exist a stronger constraint on the IR cutoff L, which excludes all states that lie within their Schwarzschild radius:

$$L^3\Lambda^4 \le L M_P^2 \qquad (3)$$

where Λ^4 is the maximum energy density in the effective theory. Here the IR cutoff scales like Λ^{-2}. When Eq. (3) is near saturation, the entropy is $S_{\max} \approx S_{BH}^{3/4}$. Cohen *et al* suggested that an effective local quantum field theory will be a good approximate description of physics when Eq. (3) is satisfied, because those states that cannot be described by it has been excluded. In other words, when the UV cutoff and the IR cutoff are properly connected, an effective local quantum field theory will be still viable.

It is worth noting that Eq. (3) can also be derived by invoking the Bekenstein bound [8][13]. For a weakly gravitating system in which self-gravitation effects can be omitted, the Bekenstein bound is given by a product of the energy and the linear size of the system, EL. In the context of the effective quantum field theory as described above, it is proportional to $L^4\Lambda^4$. Then according to the holographic principle, we have $L^4\Lambda^4 \leq S_{BH} \sim L^2 M_P^2$, and again we can obtain Eq. (3). Note that this requirement automatically prevents formations of black holes, as the Bekenstein bound does not involve the Newton gravitational constant. Thus, the above two derivations are actually equivalent.

Now we analyze the applicability of Eq. (3) for explaining the dark energy. Cohen *et al* argued that when choosing an IR cutoff comparable to the current horizon size of the universe, the corresponding UV cutoff from Eq. (3) is $\sim 10^{-2.5} ev$ and the resulting quantum energy density of Λ^4 requires no cancellation to be consistent with current observations. Therefore, Eq. (3) can solve the fine-tuning problem of the cosmological constant. However, there may exist a loophole in Cohen *et al*'s deduction of UV cutoff. According to the above UV/IR connection argument, an effective local quantum field theory should be able to describe the standard models particles ($m \geq 100 Gev$) when Eq. (3) is satisfied. But when $\Lambda < m$ the energy density should be not Λ^4 but $m\Lambda^3$, and thus we have $m\Lambda^3 \sim 10^{-10} ev^4$ and $\Lambda \sim 10^{-7} ev$ (see also [9]). Consequently, the present-day UV cutoff is actually much smaller than $10^{-2.5} ev$ according to Eq. (3). As a result, the theory cannot describe the cosmic microwave background (CMB) radiation since the present temperature of the universe is $T_0 \sim 10^{-4} ev$ [9]. This inconsistency shows that the UV/IR connection argument denoted by Eq. (3) may have serious problems when being used to explain the dark energy of the universe, which might imply that dark energy probably does not originate from the quantum zero-point energy predicted by an effective QFT.

This conclusion has more support when applying Eq. (3) to other epochs of the universe. It has been argued that, when assuming most dark energy comes from the quantum zero-point energy satisfying Eq. (3), the matter-dominated epoch of the universe cannot be consistently described [8]. In response to this problem, nonsaturated HDE models are proposed. In these models, Eq. (3) is not saturated during the epochs not dominated by the dark energy. However, it is found that even such nonsaturated HDE models cannot account for the radiation-dominated epoch of the universe either [9]. The results are generic in that they do not depend on the choice of the IR cutoff. In conclusion, an effective QFT, with a relationship between UV and IR cutoffs denoted by Eq. (3), cannot consistently describe all epochs of the universe, and thus cannot explain the dark energy of the universe [9][1].

In fact, the above conclusion is quite understandable. When considering the success of local quantum field theory for describing high-energy particles with a UV cutoff Λ much larger than $10^{-2.5} ev$, the

---

[1] In Ref. [9], the author rightly said: "Our overall conclusion is therefore that the basic framework underlying all HDE models seems too *ad hoc* to have any real explanatory value, which still keeps us in need of firmer theoretical background."

theory should be unable to consistently describe a very large system such as the whole universe, as the IR cutoff L will be much smaller than the size of the universe according to Eq. (3). Therefore, an inverse application of Eq. (3), namely using L to limit Λ as Cohen *et al* did, will have problems when explaining the dark energy of the universe. In addition, there is also another worry, namely whether we can take the left side of Eq. (3) as the actual quantum zero-point energy. There are some reasons against such direct equivalence. They are as follows. First, the energy is only predicted by an effective local quantum field theory which eliminates those states that cannot be described by it. But such a theory is surely an incomplete description of actual situations. Moreover, the states that cannot be described by the theory do exist and may also have corresponding quantum zero-point energy. Obviously this part of energy is not included in Eq. (3). Next, the density of quantum zero-point energy in Eq. (3) is still local and extensive, which seems inconsistent with the spirit of the holographic principle, although the total energy satisfies a restriction. Besides, it is not evident how to calculate the energy density in an effective quantum field theory when the total energy is restricted. The left side of Eq. (3) implicitly assumes that the energy density integral is continuous from the IR cutoff to the UV cutoff. However, since the holographic principle requires that the number of degrees of freedom of any system is finite, it seems more natural that the integral is discrete and sparse in some sense, but still from the IR cutoff to the possible maximum UV cutoff such as Planck mass M_P. Lastly, the revision of convention QFT must be radical due to the existence of the holographic principle, and thus it is very likely that we should re-understand the quantum zero-point energy predicted by conventional QFT. They may not exist in a fundamental theory (see, e.g. Refs. [14,15]).

To sum up, the dark energy of the universe cannot be accounted for by the quantum zero-point energy predicted by an effective QFT satisfying the UV/IR restriction of Eq. (3). Therefore, the popular explanation of the HDE model, namely that HDE comes from quantum zero-point energy predicted by an effective QFT, is wrong. Since the HDE denoted by Eq. (1) is indeed favored by observations, it probably comes from elsewhere. What we need to do is to re-explain Eq. (1) as a model of dark energy. There is already an alternative in the literature [2]. Let's turn to it now.

3. Thomas's bulk holography argument

Soon after Cohen *et al*'s UV/IR connection argument, Thomas proposed a bulk holography argument to solve the cosmological constant problem [2]. According to his argument, the dark energy of the universe originates from holographic quantum contributions of zero-point energy, which comes from both the holographic reduction in the number of independent degrees of freedom and the holographic energy per degree of freedom. It is generally considered that this argument provides another explanation of the HDE model denoted by Eq. (1) (see, e.g. Ref. [4]).

The argument of Thomas can be formulated as follows. In order to calculate a global quantum effect on the background geometry of the universe, it is natural to postulate that uniformly volume distributed

---

Unfortunately, it seems that most authors ignore this important paper.

bulk holographic degrees of freedom are delocalized on the scale of the background radius of curvature, denoted by L, since this is the relevant holographic length scale. The Heisenberg quantum energy of each delocalized holographic degree of freedom is $E \sim 1/L$. According to the holographic principle, the total number of the holographic degrees of freedom is $N \le L^2 M_P^2$. Then the quantum contribution to the global vacuum energy density, $\rho_V \sim NE/L^3$, is:

$$\rho_V \le M_P^2 L^{-2} \qquad (4)$$

Such quantum contributions to the vacuum energy also satisfy the gravitational mass bound $NE \le M_P^2 L$.

It seems that Thomas's argument can indeed provide a plausible explanation of the HDE model denoted by Eq. (1). However, a further analysis shows that it cannot. In fact, the above argument and the resulting Eq. (4) only require that the vacuum energy density is not larger than the bound $M_P^2 L^{-2}$. This is consistent with Thomas's original conclusion, namely gravitational holography can render the cosmological constant stable against divergent quantum corrections. If the vacuum energy density is smaller than the bound, then it is obvious that Eq. (4) cannot determine the concrete form of vacuum energy density, and thus cannot deduce Eq. (1) and explain the HDE model[2]. But if the vacuum energy density just equals to the bound, then it appears that Eq. (1) can be deduced. In the following, we will show that the saturated form of Eq. (4) is not consistent with the observational data of dark energy.

When the Bekenstein-Hawking entropy bound is saturated, the total number of the holographic degrees of freedom is $N \equiv A/4L_P{}^2 = \pi L^2 / L_P^2$, where L is the horizon size of the current universe, A is the area of horizon, and L_P is the Planck length. For clarity, we write down all parameters and constants explicitly. According to Thomas's argument, the Heisenberg quantum energy of each degree of freedom is $E \approx \frac{\hbar}{L} c = \frac{\hbar c}{L}$. Then the quantum contribution to the global vacuum energy density is:

$$\rho_V \approx \frac{NE}{4\pi L^3/3} = \frac{3c^4}{4GL^2} \qquad (5)$$

If taking L as the apparent horizon of the universe or the Hubble scale (i.e. $L = H^{-1}c$), then the resulting energy density is obviously larger than the present-day dark energy. In fact, it is also larger than the critical energy density, $\rho_c = 3H^2c^2/8\pi G$. On the other hand, taking L as the particle horizon cannot account for the present accelerating universe (see, e.g. Ref [4]). The left alternative is taking L as the event horizon of the universe. By using the definition of event horizon $L = a(t)\int_t^\infty dt'/a(t')$, we can solve the Friedmann equation for a spatially flat universe. The evolution equation of Ω_V is:

[2] In fact, Eq. (4) cannot assume a saturated form as some degrees of freedom are also occupied by other substances in the universe such as matter and radiation etc. In particular, during the epochs other than those dominated by dark energy, Eq. (4) is purely an inequality and thus cannot determine a definite form of dark energy.

$$\frac{d\Omega_V}{d\ln a} = \Omega_V(1-\Omega_V)(1+\frac{2}{\sqrt{2\pi}}\sqrt{\Omega_V}) \qquad (6)$$

where $\Omega_V \equiv \rho_V / \rho_c$, $\rho_c = 3H^2c^2/8\pi G$ is the critical energy density. Then the equation of state up to the first order is:

$$w_V \approx -\frac{1}{3}\frac{d\ln\rho_V}{d\ln a} - 1 \qquad (7)$$

By inputting the current value $\Omega_V \approx 0.72$, we can obtain $w_0 \approx -\frac{1}{3}(1+\frac{2}{\sqrt{2\pi}}\sqrt{\Omega_V}) \approx -0.56$. This obviously contradicts the latest observations of dark energy which indicates that $w_0 < -0.79$ (see, e.g. Refs. [16,17]).

In fact, we can directly examine the possibility of taking L as the event horizon of the universe in Thomas's model, by invoking the observational restriction of c in Eq. (1). Eq. (5) indicates $c = \sqrt{2\pi} \approx 2.5$. This value is too larger than the best-fit result $c \approx 0.88$. Are there some remedies then? Considering the Heisenberg uncertainty principle, one may reduce the Heisenberg quantum energy by half, namely $E \approx \frac{\hbar/2}{L}c = \frac{\hbar c}{2L}$. Then the quantum contribution to the global vacuum energy density is:

$$\rho_V \approx \frac{NE}{4\pi L^3/3} = \frac{3c^4}{8GL^2} \qquad (8)$$

This leads to $c = \sqrt{\pi} \approx 1.77$, which is still about two times of the best-fit value. Therefore, the saturated form of Eq. (4) cannot be consistent with the observational data of dark energy.

Besides, it is worth noting that a holographic number of modes with the lowest frequency of quantum zero-point energy also gives more vacuum energy than the observed dark energy, as the quantum zero-point energy of the lowest frequency, $E_1 = \frac{hc}{8L}$, is still larger than the above Heisenberg quantum energy. Inspired by this result, we may provide an argument against the existence of quantum zero-point energy in terms of the holographic principle. If quantum zero-point energy indeed exists, then it seems reasonable that it should exist in all holographic degrees of freedom in a fundamental theory. Note that in conventional QFT, the quantum zero-point energy exists in all degrees of freedom. This can be considered as a universal property of quantum zero-point energy based on its original definition, independent of any concrete theory. Then we can work out the quantum zero-point energy density:

$$\rho_V \geq \frac{NE_1}{4\pi L^3/3} = \frac{3\pi c^4}{16GL^2} > \frac{3c^4}{8\pi GL^2} \qquad (9)$$

It can be seen that the total quantum zero-point energy exceeds the mass of a black hole of the same size. This is not only inconsistent with observations, but also likely prohibited by arguments similar to that of Cohen *et al*.

In conclusion, Thomas's bulk holography argument cannot provide a plausible explanation of the

HDE model denoted by Eq. (1) either. But it might give a clue to the last explanation, as there is only a numerical factor ~1/4 missed in the vacuum energy density denoted by Eq. (8).

4. A conjecture on the origin of dark energy

The failure of the arguments of Cohen *et al* and Thomas may reveal something positive about the nature of dark energy. It is that the dark energy of the universe probably does not originate from the quantum zero-point energy. On the other hand, it has been widely argued that space-time itself, as dynamical entity, should have quantum fluctuations (see, e.g. [18, 19]). Therefore, the quantum fluctuations energy of space-time will contribute to the vacuum energy, and it may be the origin of dark energy. In short, dark energy might come from quantum fluctuations of space-time, not from quantum fluctuations in space-time. This is our conjecture. Let's see it in more detail.

The universe can be considered as a finite system limited by its event horizon in space due to the existence of dark energy. The holographic principle implies that the event horizon contains finite area units, whose number is the Bekenstein-Hawking entropy bound $N \equiv A/4{L_P}^2 = \pi L^2/L_P^2$. We assume that the space-time limited in the event horizon undergoes quantum fluctuations, and its quantum fluctuations energy or Heisenberg quantum energy of one degree of freedom is $\varepsilon \approx \frac{\hbar/2}{2L}c = \frac{\hbar c}{4L}$. Note that the space size limited by the event horizon is 2*L*, not *L*. This is equivalent to introducing one numerical factor 1/2 into Eq. (8) in Thomas's model. On the other hand, since the quantum fluctuations of space-time of one degree of freedom corresponds to two area units at the two ends of the event horizon, the total number of degrees of freedom for such quantum fluctuations is $N/2 = \pi L^2/2L_P^2$. This is equivalent to introducing another numerical factor 1/2 into Eq. (8) in Thomas's model. As thus, the energy density of the quantum fluctuations of space-time in the universe is:

$$\rho_V \approx \frac{\varepsilon N/2}{4\pi L^3/3} = \frac{3c^4}{32GL^2} \qquad (10)$$

Compared with Eq. (8) in Thomas's model, Eq. (10) gains an additional numerical factor 1/4. This additional factor comes not from a mathematical trick, but from a different physical explanation. Eq. (10) indicates $c \approx \sqrt{\pi}/2 \approx 0.886$. This value is consistent with the latest observations [5, 6].

Some comments need to be given before we can reach a definite conclusion. First, it should be stressed that the physical nature and precise mathematical description of the quantum fluctuations of space-time are still unknown, as a complete theory of quantum gravity is not yet available. However, it has been acknowledged that space-time should undergo some kind of quantum fluctuations, and they at least include the fluctuations of space-time metric (see, e.g. [18, 19]). Despite these uncertainties, the above model may be also applicable because it only depends on the total number of degrees of freedom of such fluctuations and the average fluctuation energy of each degree of freedom. Since quantum gravity may finally require a holographic description [10-12], it seems reasonable that the quantum fluctuations of space-time may be essentially nonlocal, and the degrees of freedom of such fluctuations are the delocalized holographic degrees of freedom, which are delocalized between the two ends of the event horizon and which number is one half of the Bekenstein-Hawking entropy bound according to the above analysis. This number is independent of the matter and radiation distribution in the universe and holds true for any epoch of the universe[3]. Moreover, the form of quantum fluctuations energy or Heisenberg quantum energy of one degree of freedom is also fixed by the dimensional relation $\varepsilon \sim \hbar c / 2L$. Therefore, the energy density of the quantum fluctuations of space-time as conceived above will precisely assume the form of Eq. (1). As a result, the above conjecture can uniquely deduce Eq. (1) in the HDE model.

Next, the choice of event horizon also has a physical basis in our conjecture. Since event horizon, contrary to apparent horizon, represents a real boundary of space-time, the quantum fluctuations of space-time should be limited by the event horizon, not by other horizons. Moreover, the event horizon in the context of cosmology as well as in that of a black hole is always defined globally, as the causal structure of space-time is a global thing (see also Ref. [4]). This is also consistent with the nonlocality of the quantum fluctuations of space-time limited by the event horizon. In addition, it is worth noting that the existence of the assumed quantum fluctuations of space-time will result in the existence of a finite event horizon of the universe. This kind of self-consistency may avoid the conceptual paradox concerning the relation between dark energy and event horizon in Cohen *et al*'s argument (see discussions in Ref. [7]).

Thirdly, there is still one undetermined part in the above conjecture, namely the precise relation of the quantum fluctuation energy of one degree of freedom. Although the dimensional relation $\varepsilon \sim \hbar c / 2L$ seems to have a firm basis, the concrete numerical factor in the relation can only be determined by the application of a complete theory of quantum gravity to the universe. The numerical factor 1/2 in the formula $\varepsilon \approx \frac{\hbar / 2}{2L} c$ is only an assumption, which might be an interesting one when considering its consistency with the latest observations. Moreover, it seems consistent with the Heisenberg uncertainty principle when combining with some reasonable assumptions about the fluctuations. For example, we may assume that the fluctuation is Gaussian with $\Delta x \approx L$ and $\Delta p \equiv 2\Delta E / c \approx 2\overline{E} / c$. Then the average quantum fluctuation energy of one degree of freedom is $\varepsilon \equiv \overline{E} \approx \frac{\hbar / 2}{2L} c$, which is just the above formula.

To sum up, the above conjecture may provide a plausible physical explanation of the HDE model. Moreover, it will also help to solve some problems plagued by the HDE model, for example, the IR cutoff choice problem, the saturated/ unsaturated problem etc. Although we cannot yet determine the numerical factor in Eq. (1), a theoretical value $c \approx \sqrt{\pi} / 2$ is shown to be perfectly consistent with observations, as well as consistent with Heisenberg's uncertainty principle. In addition, the analysis also implies that the dark energy of the universe may originate from the quantum fluctuations of space-time.

---

[3] Note that this number is one half of the maximum Bekenstein-Hawking entropy bound, and thus there are always another half degrees of freedom left for other substances to occupy.

6. Conclusions

It is generally considered that holographic dark energy comes from the quantum zero-point energy predicted by an effective QFT with UV/IR connection suggested by Cohen *et al*. However, it has been pointed out by Horvat that such a theory cannot consistently describe all epochs of the universe. Moreover, the UV/IR connection argument itself also has some serious drawbacks. Therefore, the well-accepted explanation of the HDE model is actually wrong. Different from the UV/IR connection argument, Thomas presented another bulk holography argument, which is regarded as another support for the HDE model. According to Thomas, the finite quantum zero-point energy consistent with observations comes from both the holographic reduction in the number of independent degrees of freedom and the holographic energy per degree of freedom. However, our calculation shows that this method gives more vacuum energy than the observed dark energy. Therefore, the bulk holography argument cannot provide a plausible explanation of the HDE model either.

The failure of the arguments of Cohen *et al* and Thomas may reveal something positive about the nature of dark energy. Maybe the dark energy of the universe does not originate from the usual quantum zero-point energy. Taking seriously this radical hypothesis, we propose that the dark energy of the universe may originate from the quantum fluctuations of space-time limited in the event horizon of the universe. It is shown that the energy density of such fluctuations assumes the same form as Eq. (1) in the HDE model. Moreover, some primary theoretical considerations suggest that the value of the numerical constant in Eq. (1) is $c \approx \sqrt{\pi}/4$, which is also favored by the latest observations. Therefore, our proposal not only provides a plausible physical basis for the popular HDE model, but also may reveal the origin of dark energy. In short, dark energy probably comes from quantum fluctuations of space-time, not from quantum fluctuations in space-time such as quantum zero-point energy.

References

[1] A. Cohen, D. Kaplan, and A. Nelson, Phys. Rev. Lett. 82 (1999) 4971.
[2] S. Thomas, Phys. Rev. Lett. 89 (2002) 081301.
[3] S. D. Hsu, Phys. Lett. B 594 (2004) 1.
[4] M. Li, Phys. Lett. B 603 (2004) 1.
[5] Q. Wu, Y. G. Gong, A. Z. Wang, and J. S. Alcaniz, Phys. Lett. B 659 (2008) 34.
[6] M. Li, X. D. Li, S. Wang, and X. Zhang, arXiv: 0904.0928.
[7] A. J. M. Medved, Gen. Rel. Grav. 41 (2008) 287.
[8] B. Guberina, R. Horvat, and H. Nikolic, JCAP 0701 (2007) 012.
[9] R. Horvat, JCAP 10 (2008) 022.
[10] J. D. Bekenstein, Phys. Rev. D 23 (1981) 287.
[11] G. ’t Hooft, gr-qc/9311026.

[12] L. Susskind, J. Math. Phys. 36 (1995) 6377.

[13] J. D. Bekenstein, Phys. Rev. D49 (1994) 1912; Int. J. Theor. Phys. 28 (1989) 967.

[14] J. Schwinger, A report on quantum electrodynamics, in J. Mehra (ed.), *The Physicist's Conception of Nature* (Dordrecht: Reidel, 1973), 413 – 429.

[15] S. E. Rugh and H. Zinkernagel, Studies in History and Philosophy of Modern Physics 33 (2002) 663.

[16] A. G. Riess et al, Astrophys. J. 607(2004) 665.

[17] M. Tegmark et al, Phys. Rev. D 69 (2004) 103501.

[18] J. A. Wheeler, *Geometrodynamics*. (New York: Academic Press, 1962).

[19] P. A. Zizzi, Gen. Rel. Grav. 33 (2001) 1305.