Spacetime rejects superposition due to discreteness

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The existence of a minimum size of spacetime is generally considered as an indispensable element in a complete theory of quantum gravity. In this essay we will analyze one of its implications for quantum gravity in terms of a minimum explanation. It is shown that the discreteness of spacetime may result in the collapse of the wave function and prohibit the superposition of different spacetimes. As a result, quantum and gravity may be combined with the help of quantum collapse in discrete spacetime.

It is a key issue in the foundations of quantum gravity whether the quantum superposition of different spacetimes exists, and if it exists, what is its precise mathematical description and physical meaning [1-2]. This problem originates from the fundamental conflict between the superposition principle of quantum mechanics (QM) and the general covariance principle of general relativity (GR) [3][1]. Although it is still unclear how to solve this conflict, the orthodox approaches to quantum gravity (e.g. string theory and loop quantum gravity) all assume the existence of quantum superposition of spacetimes. However, there are at least some worries about this implicit assumption. First, this assumption is implied by neither QM nor GR. It is actually a new conjecture beyond these two theories [5]. Next, no experience indicates that the gravitational field must be quantized; on the contrary, as Feynman pointed out [6], the extreme weakness of quantum gravitational effects may suggest the non-existence of quantized gravity or superposition of spacetimes. Thirdly, when the superposition of different spacetimes is treated in a physical way, not merely in a formal way, GR must be compromised to some extent and thus cannot be considered fundamental, as QM is assumed to be universal here [3,7]. But it still needs to explain why QM is fundamental while GR is not.

Contrary to the uncertainty of the assumption of quantum superposition of spacetimes, the existence of a minimum size of spacetime has been widely argued and acknowledged as a model-independent result of the proper combination of QM and GR (see, e.g. [8] for a review]. Moreover, it is generally considered as an indispensable element in a complete theory of quantum gravity. In this essay, we will mainly analyze one implication of the discreteness of spacetime for quantum gravity. It will be shown that the discreteness of spacetime may result in the dynamical collapse of the wave function, and thus the quantum

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[1] This conflict between QM and GR can be regarded as a different form of the problem of time in quantum gravity. It is widely acknowledged that QM and GR contain drastically different concepts of time (and spacetime), and thus they are incompatible in nature. In QM, time is an external (absolute) element (e.g. the role of absolute time is played by the external Minkowski spacetime in quantum field theory). In contrast, spacetime is a dynamical object in GR. This then leads to the

superposition of different spacetimes will be prohibited. As a consequence, the existence of a minimal size of spacetime may permit no quantization of spacetime.

Although both QM and GR are based on the assumption of continuous spacetime, the appearance of infinity in quantum field theory and singularity in general relativity already suggests that spacetime may be not continuous but discrete. It is generally expected that these problems will be finally solved with the help of the discreteness of spacetime [9]. In fact, it has been widely argued that a proper combination of QM and GR, two firm results of which are the formula of black hole entropy and the generalized uncertainty principle [10], may inevitably result in the discreteness of spacetime. In particular, the argument for the discreteness of spacetime is model-independent, and it turns out to be a general feature in most approaches to quantum gravity (e.g. string theory, loop quantum gravity, and quantum geometry etc). Besides, the argument also implies that the minimum time interval and the minimum length are respectively of the order of Planck time (T_p) and Planck length (L_p). For example, the minimum length can be derived from the following generalized uncertainty principle [10]:

$$\Delta x = \Delta x_{QM} + \Delta x_{GR} \geq \frac{\hbar}{2\Delta p} + \frac{2{L_p}^2 \Delta p}{\hbar} \tag{1}$$

Since the formulations and meanings of discrete spacetime are different in the existing theories and arguments, here we only resort to its minimum explanation, namely that a spacetime interval shorter than the minimum size of spacetime (i.e. Planck scale) is physically meaningless, and it cannot be measured in principle either[2]. For example, a physical process can only happen during a time interval not shorter than the minimum time interval, namely Planck time.

In order to see how the discreteness of spacetime may have a limitation on the superposition of spacetimes, we consider a quantum superposition of two different energy eigenstates. Each eigenstate has a well-defined static mass distribution in the same spatial region with radius *R* (cf. [3]). For example, they are rigid balls of radius *R* with different uniform mass density. The initial state is

$$\psi(x,0) = \frac{1}{\sqrt{2}} [\varphi_1(x) + \varphi_2(x)] \tag{2}$$

where $\varphi_1(x)$ and $\varphi_2(x)$ are two energy eigenstates with energy eigenvalues E_1 and E_2 respectively. According to the linear Schrödinger evolution, we have:

$$\psi(x,t) = \frac{1}{\sqrt{2}} [e^{-iE_1 t/\hbar} \varphi_1(x) + e^{-iE_2 t/\hbar} \varphi_2(x)] \tag{3}$$

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notorious problem of time in quantum gravity [4].

[2] This minimum explanation only means that spacetime is not continuous or infinitely divisible, which can be regarded as the essence of spacetime discreteness. As we think, the model-independence of the argument for the discreteness of spacetime strongly implies that discreteness is probably a more basic feature of spacetime, and it may have a deeper basis beyond QM and GR, which are still based on continuous spacetime. For instance, as the holographic principle implies [11-13], the information inside a finite spatial region should be finite. As a consequence, space cannot be infinitely divisible and must be discrete. Therefore, it may be appropriate to analyze the implications of this minimum explanation of spacetime discreteness for quantum gravity. Indeed, the existence of a minimum size of spacetime, beyond which the very concept of spacetime loses their meaning, has a very similar meaning to the speed limit defined by the speed of light in special relativity [8].

and

$$\rho(x,t)=|\psi(x,\mathrm{t})|^2=\frac{1}{2}[\varphi_1{}^2(\mathrm{x})+\varphi_2{}^2(\mathrm{x})+2\varphi_1(\mathrm{x})\varphi_2(\mathrm{x})\cos(\Delta E/\hbar\cdot t)] \tag{4}$$

This result indicates that the probability density $\rho(x,t)$ will oscillate with a period $T=h/\Delta E$ in each position of space, where $\Delta E=E_2-E_1$ is the energy difference. This has no problem when the energy difference is very small as in usual situations. But when the energy difference ΔE exceeds the Planck energy E_p [3], $\rho(x,t)$ will oscillate with a period shorter than the minimum time interval T_p [4], while this is inconsistent with the requirement of the discreteness of spacetime. According to its minimum explanation, the minimum time interval T_p is the minimum distinguishable size of time, and no change can happen during a time interval shorter than T_p. Therefore, when considering the restriction of the minimum time interval, the superposition of two energy eigenstates with an energy difference larger than the Planck energy, which results in an oscillation with a period shorter than the minimum time interval, cannot hold and must collapse into one of its energy eigenstates[5], which has no such oscillation.

We can also reach this conclusion from a different aspect. The existence of a minimum time interval demands that no oscillation or interference effect between the two energy branches in superposition can exist when their energy difference exceeds the Planck energy. Once there is no such oscillation, the above probability density will not change with time and the corresponding superposition state will become an energy eigenstate. Since the measurement result of this state can only be one of the initial energy eigenstates, this resulting energy eigenstate should be also one of the initial energy eigenstates. This again means that the superposition state has collapsed into one of its energy branches[6].

Since there is one kind of equivalence between the difference of energy distribution and the difference of spacetimes according to GR, the above result also implies that the quantum superposition of two different spacetimes cannot exist and should collapse into one of the definite spacetimes in the superposition. In order to make this argument more precise, we need to define the difference between two spacetimes here. As suggested by the generalized uncertainty principle denoted by Eq. (1), the difference of

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[3] There is no limitation on the maximum of the energy of each eigenstate in principle. For example, the energy of a macroscopic object in a stationary state can be larger than the Planck energy. On the other hand, if the energy of a microscopic particle cannot be larger than the Planck energy and QM indeed fails at the energy scale larger than the Planck energy, then there is no quantum superposition of different spacetimes (as defined later) either, which is also consistent with the conclusion of this paper.

[4] Here we ignore the gravitational fields in the superposition, as their existence does not influence the conclusion of our analysis. When the energy difference is very tiny such as for a microscopic particle, the corresponding gravitational fields in the superposition are almost the same and not orthogonal, and the interference effect between the two branches or the oscillation can be detected in experiment, while when the energy difference become larger and larger such as approaching the Planck energy, the gravitational fields in the superposition are not orthogonal either, and thus the oscillation can also be detected in continuous spacetime in principle.

[5] Different from Penrose's gravity-induced collapse argument [3], which strongly depends on the assumption that gravity is not emergent but fundamental and the general covariance principle of GR is universally valid (and thus does not influence the validity of other theories without quantum collapse such as string theory that rejects this assumption), the argument here only depends on the existence of a minimum size of spacetime, though which seems to already imply that gravity as a geometric property of spacetime described by GR is indeed fundamental.

[6] In a similar way, the existence of a minimum length demands that no spatial oscillation can exist for the superposition of two momentum bases when their momentum difference exceeds the Planck energy divided by the speed of light. This shows that the existence of a minimum length will also result in the collapse of the wave function.

energy ΔE corresponds to the difference of spacetime $\frac{2L_p{}^2 \Delta E}{\hbar c}$. Then as to the branch states in a quantum superposition with energy difference ΔE, the difference between the spacetimes determined by the branch states may be characterized by the quantity $\frac{2L_p{}^2 \Delta E}{\hbar c}$. The physical meaning of such spacetime difference can be further clarified as follows. Let the two energy eigenstates in the superposition be limited in the regions with the same radius *R* (they may locate in different positions in space). Then the spacetime outside the region can be described by the Schwarzschild metric:

$$ds^2 = (1 - \frac{r_S}{r})^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin\theta^2 d\phi^2 - (1 - \frac{r_S}{r}) c^2 dt^2 \tag{5}$$

where $r_S = \frac{2GE}{c^4}$ is the Schwarzschild radius. By assuming that the metric tensor inside the region *R* is in the same order of that on the boundary, the proper size of the region is

$$L \approx 2\int_0^R (1 - \frac{r_S}{R})^{-1/2} dr \tag{6}$$

Then the space difference of the two spacetimes in the superposition inside the region *R* can be characterized by

$$\Delta L \approx \int_0^R \frac{\Delta r_S}{R} dr = \Delta r_S = \frac{2L_p{}^2 \Delta E}{\hbar c} \tag{7}$$

This result is consistent with the generalized uncertainty principle. Accordingly as to the branch states in a quantum superposition, we can define the difference of their corresponding spacetimes as the difference of the proper spatial sizes of the regions occupied by these states. Such difference represents the fuzziness of the point-by-point identification of the spatial section of the two spacetimes (cf. [4]).

The spacetime difference defined above can be rewritten in the following form:

$$\frac{\Delta L}{L_U} \approx \frac{\Delta E}{E_P} \tag{8}$$

This relation indicates one kind of equivalence between the difference of energy and the difference of spacetimes for the above quantum superposition of two energy eigenstates[7]. Therefore, we can also give a collapse criterion in terms of spacetime difference. If the difference ΔL of the spacetimes in the superposition is close to the minimum size L_U, the superposition state will collapse to one of the definite spacetimes in about a minimum time interval T_p. If the difference ΔL of the spacetimes in the superposition is smaller than L_U, the superposition state will collapse after a finite time interval larger than T_p. As a result, the superposition of spacetimes can only possess a spacetime uncertainty smaller than

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[7] It should be stressed that they are not equivalent in general situations. It is the difference of spacetimes, not the energy difference in the superposition that results in the collapse of the wave function (cf. [14]).

the minimum size in discrete spacetime. If such uncertainty limit is exceeded, the superposition will collapse to one of the definite spacetimes instantaneously. This will ensure that quantum state and its evolution can still be consistently defined during the process of quantum collapse, as the spacetimes with a difference smaller than the minimum size can be regarded as physically identical[8]. It can be shown that this collapse criterion based on the minimum size of discrete spacetime is consistent with the existing experiments and macroscopic experience[9].

Two comments are in order before we conclude this essay. First, it is sometimes claimed that the existence of a minimal length suggests that space should have a quantized structure at the Planck scale, analogous to the quantization of energy in QM (see, e.g. [2]), and thus it will support the assumption of the existence of quantum superposition of spacetimes. However, this claim may go beyond the basic meaning of the minimum size of spacetime, which only indicates that spacetime is not continuous or infinitely divisible. As we have seen, contrary to this claim, the existence of a minimal size may prevent the superposition of different spaces and thus permits no quantization of space.

Next, the above result seems at odds with the most approaches to quantum gravity, which are based on continuous spacetime manifold. However, it may be not against all expectations as we already reject the continuous spacetime manifold in our analysis by resorting to the discreteness of spacetime. Indeed, in view of the existence of an absolute minimum spacetime size one may plausibly question whether any theory based on shorter distances, such as a spacetime continuum, really makes sense [10]. At least, one should worry whether it is appropriate in quantum gravity to assume the same ‘continuum’ (i.e. manifold) structure for spacetime as that employed in both QM and GR [1].

To sum up, the discreteness of spacetime may prohibit the existence of quantum superposition of different spacetimes. As a consequence, quantum and gravity may be reconciled with the help of the quantum collapse in discrete spacetime. In this way, there will be no quantized gravity in its usual meaning. In contrast to the semiclassical theory of quantum gravity, however, the theory will naturally include the backreactions of quantum fluctuations to gravity (e.g. the influence of wavefunction collapse to spacetime), as well as the reactions of gravity to quantum evolution. Therefore, it might provide a consistent framework for a fundamental theory of quantum gravity. Certainly, the details of such quantum collapse and the properties of discrete spacetime need to be further studied. Our analysis implies that spacetime may be not a pure quantum dynamical entity, but it is not wholly classical either.

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[8] Due to the universal existence of quantum fluctuations, there still exist a bit of difference between the space-times whose difference is smaller than the minimum size of discrete space-time. Such difference will generate a very slow collapse of the superposition of these space-times. Thus, strictly speaking, the space-times are almost physically identical.

[9] For example, the superposition of a macroscopic object in an identical physical state (an approximate energy eigenstate) at two separated places will also collapse due to the environmental influence such as particle accretion.

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