Strong gravitational lensing across dilaton anti-de Sitter black hole

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In this work we investigate gravitational lensing effect in strong field region around a dilaton black holes in an anti de Sitter (ADS) space. We also analyse the dependence of the radius of the photon sphere and deflection angle on dilaton coupling and cosmological constant in this black hole space time. Finally the values of minimum impact parameter , the separation between the first and the other images as well as the ratio between the flux of the first image and the flux coming from all the other images are determined to characterize some possible distinct signatures of such black holes.

1. Introduction :

One of the key signature of astrophysical objects like black holes is the gravitational lensing effect. Gravitational lensing is the deflection of electromagnetic radiation in a gravitational field. Intense research on the phenomena of gravitational lensing is being carried out in recent times. The results of these studies also have their relevance in the detection of extrasolar planets and in compact dark matter to estimate the value of the cosmological constant. When the gravitational field is strong,i.e light rays pass very close to a black hole, observer can detect infinite series of images formed very close to black hole. An incoming photon with an impact parameter, deviates as it approaches a minimum distance towards the black hole. In such a case deflection angle increases with decrease of the impact parameter.The photon will traverse a complete loop across the black hole when the deflection angle exceeds 2π. For further lowering of the value of the impact parameter,it will execute several windings around the black hole. It is shown [1] that the deflection angle diverges as light rays approach towards the photon sphere. This phenomena is known as gravitational lensing in strong deflection region. Very long baseline interferometry may be able to detect relativistic images and the effects of strong fields within these objects[2]. Recently various works have been done on gravitational lensing in both strong and weak field region[3–21]. Strong gravitational lensing can provide informations about the nature of spacetime around various kinds of black holes. Our aim in this work is to study the strong field gravitational lensing phenomena near a dilaton coupled black hole in presence of cosmological constant [22]. Here we have explored the role of cosmological constant on deflection angle in the limit of small dilaton coupling and have obtained the values of minimum impact parameter ,radius of the photon sphere , the separation between the first and the other images as well as the ratio between the flux of the first image and the flux coming from all the other images. One of the key motivation of the present study is that in presence of dilaton field, black holes have distinct features which are different from other black holes. Moreover such black holes are interesting to study because the fate of dark energy dominated universe may be influenced when dynamical modulus or dilaton fields coupled to string curvature are taken into account [23]and it is suggested that our accelerating universe may be dominated by the energy density of dilaton field[24].
The gravitational action for dilaton black holes in anti-de Sitter spacetime can be described as,[22]:

$$S = \int d^4x\sqrt{-g}[R - 2\partial_\mu\varphi\partial^\mu\varphi - e^{-2a\varphi}F^2 - V(\varphi)] \tag{1}$$

where R is the Ricci scalar, $V(\varphi) = \frac{-2a}{3(1+a^2)^2}[a^2(3a^2-1)e^{\frac{-2\varphi}{a}} + (3-a^2)e^{2a\varphi} + 8a^2e^{a\varphi-\frac{\varphi}{a}}]$ is the potential term in presence of cosmological constant λ for dilaton anti de Sitter black hole ($\lambda < 0$), φ is the dilaton field and 'a' represents coupling parameter of the dilaton with Maxwell field $F_{\mu\nu}$.

2.Strong Field Lensing By Dilaton Anti-de Sitter Black hole :

Considering dilaton scalar filed φ coupled to electromagnetic field of charge Q with arbitrary coupling parameter 'a' in presence of a Liouville type potential containing the cosmological constant λ, Gao and Zhang obtained the dilaton

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anti-de Sitter black hole solutions. To study strong field lensing occurring around such dilaton anti-de Sitter black holes, we consider the following spacetime[22]

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)d\Omega^2 \tag{2}$$

where $A(r) = [(1-\frac{r_+}{r})(1-\frac{r_-}{r})^{\frac{(1-a^2)}{(1+a^2)}} - \frac{\lambda}{3}r^2((1-\frac{r_-}{r})^{\frac{2a^2}{(1+a^2)}}]$,$B(r) = [(1-\frac{r_+}{r})(1-\frac{r_-}{r})^{\frac{(1-a^2)}{(1+a^2)}} - \frac{\lambda}{3}r^2((1-\frac{r_-}{r})^{\frac{2a^2}{(1+a^2)}}]^{-1}$, $C(r) = r^2(1-\frac{r_-}{r})^{\frac{2a^2}{(1+a^2)}}$, $r_-r_+ = (1+a^2)Q^2e^{2a\varphi_0}$, the expression of dilaton field with asymptotic value φ_0 is $e^{2a\varphi} = e^{2a\varphi_0}(1-\frac{r_-}{r})^{\frac{2a}{(a^2+1)}}$ and the only non-vanishing component of $F_{\mu\nu}$ is $F_{01} = \frac{Q^2e^{2a\varphi}}{C}$

In order to explain strong field lensing effect around black holes,we need to obtain the expressions of radius of the photon sphere and also the deflection angle. Using these two variables one can then determine other observables such as separation between the first and other images, the ratio between the flux of the first image and the flux coming from all the other images.

Radius of the photon sphere:

Two different methods has been used [1, 25] and obtained the following condition to determine the radius of the photon sphere as,

$$\frac{C'}{C} = \frac{A'}{A} \tag{3}$$

After some algebraic calculation and taking leading order terms in $\frac{1}{r}$,one can obtain the radius of the photon sphere in the strong field region as,

$$r_{ps} = \frac{k_2 + \sqrt{k_2^2 - 4k_1k_3}}{2k_1} \tag{4}$$

where $k_1 = [2(1+a^2) - \frac{4a^2r_-^2\lambda}{3(1+a^2)} + \frac{4\lambda}{3}r_-^2a^2 - 2a^2r_-^2\frac{\lambda}{3} - \frac{(a^2-1)}{(1+a^2)}2a^2r_-^2\frac{\lambda}{3}]$,
$k_2 = [2r_+(1+a^2) + 2r_-(1-a^2) + 2r_- + (1+a^2)r_+ + (1-a^2)r_- - \frac{2a^2\lambda}{3}r_-^3\frac{(a^2-1)}{(1+a^2)}]$,
$k_3 = [2(1-a^2)r_-r_+ + 2r_-r_+ + 2r_-^2\frac{(1-a^2)}{((1+a^2)} + (1+a^2)r_-r_+ + 2(1-a^2)r_-r_+ + (1-a^2)r_-^2 - \frac{2a^2(1-a^2)}{(1+a^2)}r_-^2]$.
Equation (4) exhibits the dependence of r_{ps} on cosmological constant λ as well as dilaton coupling 'a'.Variation of r_{ps} with dilaton coupling 'a' is shown in the following figure.

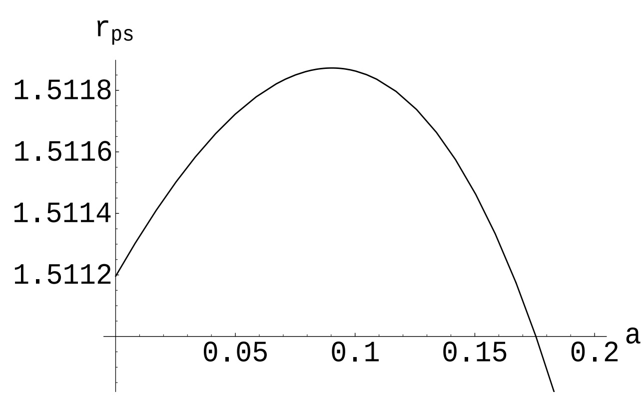

FIG. 1: Graph of the dilaton coupling 'a' versus the radius of the photon sphere r_{ps} for Q=0.3 ,$\varphi_0 = 1$ and $\lambda = 0.1$

It is evident that the radius of the photon sphere at first increases with increase in the dilaton coupling parameter 'a', becomes maximum and then decreases with coupling parameter 'a'. It has also been found that r_{ps} becomes imaginary for coupling '$a \sim 0.8$'. Similar situation arises in lensing effect near a Reissner-Nordstrom (RN) black hole when the charge $Q > \frac{1}{2}$ [25]. From equation (4),one can find that the dependence of r_{ps} on λ is nearly flat. Setting $r_- = 0$,'a' $= 0$ and $\lambda = 0$, in the above expression, we can retrieve the Schwarzschild value of r_{ps}. We consider the strong field expansion in presence of a photon sphere. A photon, coming from infinity has an impact parameter

u and when it approaches towards the black hole and reaches the distance of closest approach r_0, it deviates away from the black hole .
Utilising the principle of conservation of angular momentum, the impact parameter u can be expressed in terms of the distance of closest approach r_0[26] as

$$u = \sqrt{\frac{C_0}{A_0}} \tag{5}$$

Deflection Angle:

Let us now calculate the deflection angle for the above dilaton-anti de Sitter black hole. We have seen from[1] that the deflection angle can be increased by decreasing the impact parameter where the authors have shown the variation of deflection angle w.r.t impact parameter for various values of ν of JNW black holes[27].The deflection angle becomes enourmously large when the impact parameter approaches towards the value of the impact parameter of the corresponding photon spehere. However at some point it exceeds 2π. At this point it will make a complete loop enclosing the black hole. Finally for $r_0 = r_{ps}$, deflection angle will diverge and the circulating photon can be captured. To find out analytical behaviour of deflection angle in strong field region [28]in terms of black hole parameters as well as cosmological constant,let us at first introduce a regularised integral $I_R(r_0)$ for this dilaton-anti de Sitter black hole spacetime as,

$$I_R(r_0) = \int_0^1 [R(z,r_0)f(z,r_0) - R(0,r_{ps})f_0(z,r_0))] \tag{6}$$

$$= \int_0^1 \frac{2dz}{r_0\sqrt{d}(1-\frac{r_-(1-z)}{r_0})^{\frac{2a^2}{(a^2+1)}}\sqrt{a_1 z + bz^2}} - \int_0^1 \frac{2dz}{r_0\sqrt{d}(1-\frac{r_-}{r_0})^{\frac{2a^2}{(a^2+1)}}\sqrt{a_1 z + bz^2}} \tag{7}$$

where

$$d = \frac{(1-\frac{r_+}{r_0})(1-\frac{r_-}{r_0})^{\frac{(1-a^2)}{(1+a^2)}} - \frac{\lambda}{3}r_0^2((1-\frac{r_-}{r_0})^{\frac{2a^2}{(1+a^2)}}}{r_0^2(1-\frac{r_-}{r_0})^{\frac{2a^2}{(1+a^2)}}} \tag{8}$$

$$R(z,r_0) = \frac{2}{r_0\sqrt{d}(1-\frac{r_-(1-z)}{r_0})^{\frac{2a^2}{(a^2+1)}}} \tag{9}$$

$$R(0,r_0) = \frac{2}{r_0\sqrt{d}(1-\frac{r_-}{r_0})^{\frac{2a^2}{(a^2+1)}}} \tag{10}$$

$$f(z,r_0) \approx f_0(z,r_0) = \frac{1}{\sqrt{a_1 z + bz^2}} \tag{11}$$

$$l = (\frac{2}{dr_0^2} + 2\frac{4a^4\lambda r_-^2}{3dr_0^2(a^2+1)^2}) \tag{12}$$

$$b = -\frac{1}{dr_0^2} - \frac{4a^4\lambda r_-^2}{3dr_0^2(a^2+1)^2}) \tag{13}$$

$$c = 1 - \frac{1}{dr_0^2} + \frac{\lambda}{3d} - \frac{4a^4\lambda r_-^2}{3dr_0^2(a^2+1)^2}) \tag{14}$$

$$a_1 = l + 2b\beta \tag{15}$$

$$\beta == \frac{-l+\sqrt{l^2-4bc}}{2b} \tag{16}$$

Integrating equation (7) we get

$$I_R(r_0) = -\frac{2}{r_0\sqrt{d\alpha_1}}[log[(\frac{1}{p(1-\beta)+q}+\frac{\beta_1}{2\alpha_1})+\sqrt{(\frac{1}{p(1-\beta)+q}+\frac{\beta_1}{2\alpha_1})^2+(b-\frac{\beta_1^2}{4\alpha_1^2})}]-log[(\frac{1}{-p\beta+q}+\frac{\beta_1}{2\alpha_1})+\sqrt{(\frac{1}{-p\beta+q}+\frac{\beta_1}{2\alpha_1})^2+(b-\frac{\beta_1^2}{4\alpha_1^2})}]]$$
$$-\frac{2}{r_0\sqrt{bd}(1-\frac{2a^2r_-}{(\alpha^2+1)r_0})}[log((1-\beta+\frac{a_1}{2b})+\sqrt{(1-\beta+\frac{a_1}{2b})^2+(\frac{a_1}{2b})^2})$$
$$-log((-\beta+\frac{a_1}{2b})+\sqrt{(-\beta+\frac{a_1}{2b})^2+(\frac{a_1}{2b})^2})] \quad (17)$$

where $p=\frac{2a^2r_-}{(a^2+1)r_0}$,$q=(1-\frac{2a^2r_-}{(a^2+1)r_0}+\frac{2a^2r_-\beta}{(a^2+1)r_0})$,$\alpha_1=(bq^2-(l+2b\beta)pq)=(bq^2-a_1pq)$,$\beta_1=(a_1p-2bq)$.
The deflection angle $\alpha(\theta)$ in the strong field region in terms of impact parameter u_{ps},the angular separation of the image from the lens θ and the distance between the lens and the observer D_{0L}, can be written as[26]:

$$\alpha(\theta) = -\bar{a}log(\frac{\theta D_{0L}}{u_{ps}}-1)+\bar{b} \quad (18)$$

Using the expression of $I_R(r_0)$, at $r_0 = r_{ps}$, one can easily find

$$\bar{a} = \frac{1}{r_{ps}\sqrt{d}(1-p)\sqrt{b}} \quad (19)$$

and

$$\bar{b} = -\pi + b_R + \bar{a}log(\frac{2b_{ps}}{A_{ps}}) \quad (20)$$

where b_R can be expressed through the regularised integral as

$$b_R = I_R(r_{ps}) \quad (21)$$

Substituting $\bar{a}$,$\bar{b}$,u_{ps},one arrives at the general expression for deflection angle for any value of 'a' in the strong field region. Considering dilaton coupling parameter 'a'<< 1, we have already found the radius of the photon sphere where in the leading order the contribution due to λ turned out to be negligible. We then apply it in determining the deflection angle as a function of cosmological constant λ in strong field region. The expression of deflection angle in this approximation can be written as,

$$\begin{aligned}\alpha(\theta) \;=\; & \frac{1}{(1-\frac{2a^2r_-}{(a^2+1)r_{ps}})}log[\frac{2}{dr_{ps}^2n}]-3.14-\frac{1}{(1-\frac{2a^2r_-}{(a^2+1)r_{ps}})}log[\frac{0.003\sqrt{n}}{r_{ps}(1-\frac{r_-}{r_{ps}})^{\frac{a^2}{(1+a^2)}}}] \\ & -\frac{2}{r_{ps}\sqrt{d\alpha_1}}[log(-\frac{4a^4r_-^2(1-\beta)^2}{r_{ps}^2[1-\frac{4a^4r_-^2(1-\beta)^2}{r_{ps}^2}]}+\frac{2a^2r_-(1-\beta)}{r_{ps}[1-\frac{4a^4r_-^2(1-\beta)^2}{r_{ps}^2}]}) \\ & -log(\frac{2}{(1-\frac{2a^2r_-}{r_{ps}})}-\frac{2}{[1-\frac{4a^4r_-^2(1-\beta)^2}{r_{ps}^2}]})]-\frac{2}{r_{ps}\sqrt{bd}(1-\frac{2a^2r_-}{r_{ps}})}[log(1-\beta)-log(\sqrt{1+(1-\beta)^2}-1)]\end{aligned} \quad (22)$$

where n and β in this approximation reduce to

$$n = (1-\frac{1}{r_{ps}})(1-\frac{r_-}{r_{ps}})-\frac{\lambda}{3}r_{ps}^2(1-\frac{r_-}{r_{ps}})^2 \quad (23)$$

and

$$\beta = \frac{\frac{2(1-\frac{r_-}{r_{ps}})^2}{n}-\frac{2(1-\frac{r_-}{r_{ps}})}{\sqrt{n}}}{\frac{2(1-\frac{r_-}{r_{ps}})^2}{n}} \quad (24)$$

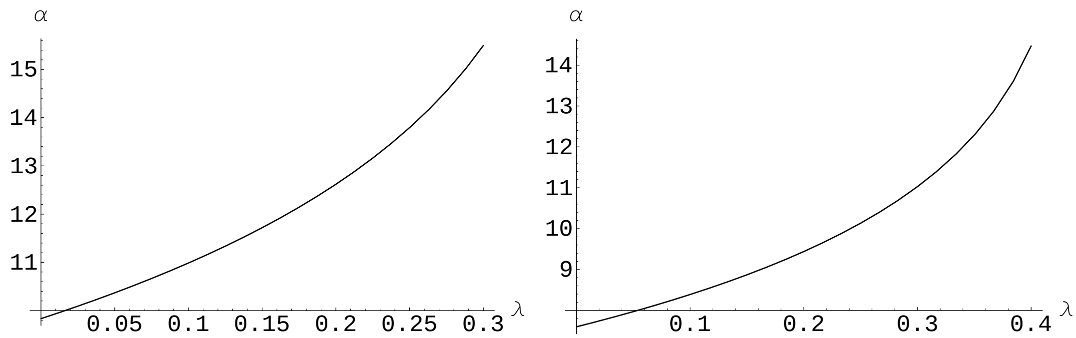

FIG. 2: Graphs of cosmological constant λ versus deflection angle α for coupling constant 'a'$= 0.2$ and charge $Q = 0.1$ and 'a' $= 0.02$ and charge $Q = 0.1$

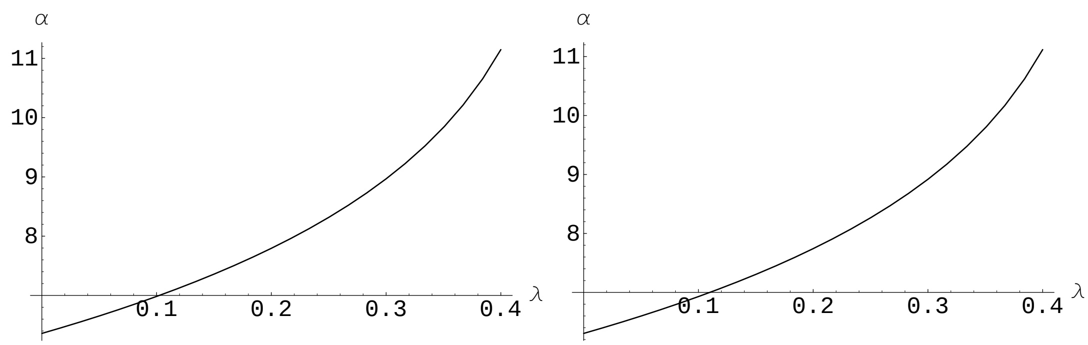

FIG. 3: Graphs of cosmological constant λ versus deflection angle α for coupling constant 'a' $= 0.2$ and charge $Q = 0.3$ and 'a' $= 0.02$ and charge $Q = 0.3$

Variation of $\alpha(\theta)$ with cosmological constant λ is shown in Fig.(2) and Fig.(3)for different dilaton coupling and charge Q taking $\varphi_0 = 0$. For n to be real, equation (23) implies a bound for the cosmological constant λ which depends on the values of the parameters 'a' and 'Q'. For a fixed value of λ the deflection angle increases as dilaton coupling 'a' increases. Comparision of Figures for same coupling parameter, say 'a'=0.02, but different values of charge Q=0.1 and 0.3 reveals the fact that deflection angle decreases as charge Q increases in the approximation 'a'$<< 1$.

Observables in strong field region:

We now estimate the values of minimum impact parameter u_{ps}, the radius of the photon sphere r_{ps}, separation between the first and the other images s, the ratio between the flux of the first image and the flux coming from all the other images r and the coefficients $\bar{a}$ and $\bar{b}$. In our case we have considered the coupling parameter 'a'$<< 1$. Using appropriate formula for r and s,

$$r = exp(\frac{2\pi}{\bar{a}}) \tag{25}$$

$$r_m = 2.5 logr \tag{26}$$

and

$$s = \Theta_\infty exp(\frac{\bar{b}}{\bar{a}} - \frac{2\pi}{\bar{a}}) \tag{27}$$

we have obtained the values of observables seting the asymptotic position of a set of images $\Theta_\infty = 16.87$ (Schwarzschild value), Q=0.3 and $\lambda = 0.2$ as follows.

Table: Observables in strong field region taking Q=0.3 and $\lambda = 0.2$:

coupling parameter	u_{ps}	r_{ps}	$\bar{a}$	$\bar{b}$	s μarcsec	r_m (mag)	$\alpha(\theta)$rad
a=0.2	3.3097	1.506	1.0049	0.7534	0.06897	6.785178	7.79
a=0.02	3.3171	1.5081	1.0000478	0.7335	0.06583	6.818087	7.74

From this Table it is observed that for the charged dilaton black hole in presence of cosmological constant the values of impact parameter u_{ps} as well as the radius of the photon sphere r_{ps} are larger than the corresponding Schwarzschild values. Values of $\bar{a}$,$\bar{b}$, s, r and deflection angle $\alpha(\theta)$ are also different. We observe that as we decrease the dilaton coupling parameter 'a' with electromagnetic field ,u_{ps}, r_{ps},as well as r will increase while the separation between the images s, deflection angle $\alpha(\theta)$, $\bar{a}$ and $\bar{b}$ will decrease.
Dependence of n-th image position Θ_n and their n-th magnification μ_n on angular source position β is given qualitatively by the following equations,

$$\Theta_n = \Theta_n^0 + \frac{u_{ps}(\beta - \Theta_n^0) D_{OS} exp(\frac{\bar{b}}{\bar{a}} - \frac{2n\pi}{\bar{a}})}{\bar{a} D_{OL} D_{LS}} \tag{28}$$

and

$$\mu_n = exp(\frac{\bar{b}}{\bar{a}} - \frac{2n\pi}{\bar{a}}) \frac{u_{ps}^2 D_{OS}(1 + exp(\frac{\bar{b}}{\bar{a}} - \frac{2n\pi}{\bar{a}}))}{\bar{a}\beta D_{OL}^2 D_{LS}} \tag{29}$$

where $\Theta_n^0 = \frac{u_{ps}(1+exp(\frac{\bar{b}}{\bar{a}}))}{D_{OL}}$ for $\alpha(\theta) = 2n\pi$,D_{LS},D_{OS} and D_{OL} are the lens-source,observer-source and observer-lens distances respectively.For images on the opposite side of the source,one has to substitute $-\beta$ in equation (28). Sustituting the values of $\bar{a}$,$\bar{b}$ and r_{ps} for coupling parameter a=0.2,Q=0.3,$\lambda = 0.2$,$\varphi_0 = 0$ and $D_{OS} = D_{OL} + D_{Ls} = 2D_{OL}$, we obtain the variation of image position (say for primary and secondary images) on the same side(s) and opposite side (o)of the source and their magnification with respect to angular source position as follows,

$$\Theta_{ps} = \frac{3.3232}{D_{OL}} + \frac{0.02693}{D_{OL}}(\beta - \frac{3.3232}{D_{OL}}) \tag{30}$$

$$\Theta_{ss} = \frac{3.30973}{D_{OL}} + \frac{0.000052}{D_{OL}}(\beta - \frac{3.30973}{D_{OL}}) \tag{31}$$

$$\Theta_{po} = \frac{3.3232}{D_{OL}} - \frac{0.02693}{D_{OL}}(\beta + \frac{3.3232}{D_{OL}}) \tag{32}$$

$$\Theta_{so} = \frac{3.30973}{D_{OL}} - \frac{0.000052}{D_{OL}}(\beta + \frac{3.30973}{D_{OL}}) \tag{33}$$

$$\mu_p = \frac{0.0895}{\beta D_{OL}^2} \tag{34}$$

$$\mu_s = \frac{0.0001722}{\beta D_{OL}^2} \tag{35}$$

Above equations reveal the fact that the dependence of angular image positions and their magnifications with respect to angular source positions are respectively linear and hyperbolic in nature as was also shown in[29, 30]. From equations (30) and (31) we notice that the angular position on the same side of the source of primary (ps) and secondary(ss) relativistic images increase and decrease respectively with the increment of angular source position β whereas equations(32) and (33) mention the case of decrease of both primary and secondary angular image positions on the opposite side of the source as β increases. Equations (34) and (35) imply the decrease of magnifications of the corresponding images as β increases. As we decrease the dilaton coupling 'a', Θ_p will increase with β,where as μ_p will decrease for decrease of 'a' for a fixed value of β.
Substituting $\beta = 0$ in equation (28),i.e,the source,the observer and the lens are in a line ,Einstein's ring [1, 26, 29, 30]for dilaton coupling $a = 0.2$ may be produced as follows,

$$\Theta_E = \frac{3.3232}{D_{OL}} - \frac{0.02704}{D_{OL}^2} \tag{36}$$

Einstein's ring decreases slowly with a decrease in the value of a. The above qualitative studies of Θ_n , μ_n and Θ_E agree with [29, 30].

Finally the features of strong gravitational lensing by naked singulaities may be observed from r_{ps} vs dilaton coupling curve. For the black holes considered in this work, SNS(strongly naked singularity) is generated approximately above 'a'=0.15 and below 'a'=0.2 where as WNS(weakly naked singularity)region lies approxmately within the region 'a'=0.15 to 0.18. Such classifications of weakly and strongly naked singularities was done by Virbhadra and Ellis [1]. According to them the qualitative features of geodesics are similar for Schwarzschild black hole as well as for WNS. These result into one Einstein ring and no radial critical curve whereas SNS generates two or nil Einstein ring(s)and also one radial critical curve.The authors also mentioned about infinite number of relativistic images for WNS but no relativistic images for SNS. Moreover Virbhadra and Keeton[30] classify WNS and SNS by considering the nature of time delays of relativistic images. The images for Schwarzschild black hole and WNS have positive time delays but SNS generates positive, zero as well as negative time delays. SNS always gives rise to negative time delay of the direct image when the angular source position is large.

3.Conclusion :

In this work we have extensively studied gravitational lensing effect around dilaton anti de Sitter black hole. Such kind of Ads black holes are interesting to study because recently quantum effects on black hole geometries have been incorporated in the context of ADS/CFT correspondence. Moreover five dimensional brane model proposed by Randall and Sundrum with infinite warped extra dimension in a Ads bulk has been considered to be equivalent to four dimensional Einstein gravity [31–34]. In our present work in the context of strong field lensing around dilaton black holes in presence of cosmological constant, the method of gravitational lensing effect has been utilised in determining r_{ps},$\alpha(\theta)$,$\bar{a}$,$\bar{b}$,r and s for some selected values of cosmological constant. From Fig (1)we find that the radius of the photon sphere r_{ps} at first increases with the dilaton coupling 'a', becomes maximum and then again decreases. We have also explored the variation of deflection angle with cosmological constant λ for dilaton coupling parameter 'a'$<< 1$. Different plots in Figs (2) and (3) also reveal that the deflection angle α_θ increases with the dilaton coupling parameter 'a' when λ is kept fixed. We have also explored the variation of the deflection angle with the black hole charge Q. Taking 'a'=0.02, it is shown that α_θ decreases as Q increases. We have calculated several strong field lensing parameters such as separation between the first and other images,the ratio between the flux of the first and other images, minimum impact parameter and constant parametres $\bar{a}$ and $\bar{b}$ taking $\Theta_\infty = 16.87$. The values of u_{ps} ,$\bar{a}$,$\bar{b}$,r and s for such kind of anti-de Sitter black holes are found to be distinct from those obtaned for Schwarzschild and dilaton black holes[15] and therefore can be useful signature to detect such dilaton coupled anti-de Sitter black hole.

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[1] K.S.Virbhadra and G.F.R.Ellis,Phys.Rev.**65**,103004,2002.
[2] Constellation-X web page:constellation.gsfc.nasa.gov;maxim web page:maxim.gsfc.nasa.gov;J.S.Ulvestad astro-ph/9901374.
[3] S.Liebes,Jr.,Phys.Rev.**133**,B835,1964.
[4] S.Refsdal,Mon.Not.R.Astron.Soc**128**,295,1964.
[5] R.R.Bourassa and R.Kantowski,Astrophys.J.**195**,13,1975.
[6] P.Schneider,J.Ehlers and E.E.Falco,Gravitational Lenses(Springer-Verlag,Berlin,1992).
[7] S.Refsdal and J.Surdej,Rep.Prog.Phys**57**,117,1994.
[8] J.Wambsganss,Living Rev.Relativity**1**,12,1998.
[9] C.Darwin,Proc.R.Soc.London**249**,180,1959.
[10] J.D.Bekenstein and R.H.Sanders,Astrophys.J**429**,480,1994.
[11] S.Frittelly,T.P.Kling and E.T.Newman,Phys.Rev.D**61**,064021,2000.
[12] K.S.Virbhadra and George F.R.Ellis,Phys.Rev.D**62**,084003,2000.
[13] N.Mukherjee and A.S.Majumdar,G.R.G**39**,583,2007.
[14] E.F.Eiroa,G.E.Romero and D.F.Torres,Phys.Rev.D**66**,024010,2002.
[15] A.Bhadra,Phys.Rev.D**67**,103009,2003.
[16] John Southworth et al,Mon.Not.Roy.Astron.Soc.396:1023-1031,2009.
[17] Eric Morganson, Phil Marshall, Tommaso Treu, Tim Schrabback, Roger D. Blandford, arXiv:0908.0602 [astro-ph.CO]
[18] Song-bai Chen, Ji-liang Jing,Phys.Rev.D**80**,024036,2009.
[19] L.V.E. Koopmans et al. arXiv:0902.3186 [astro-ph.CO]
[20] C. Faure, J.-P. Kneib, S. Hilbert, R. Massey, G. Covone, A. Finoguenov, A. Leauthaud, J.E. Taylor, S. Pires, N. Scoville, Astrophys.J.**695**,1233,2009.
[21] G.S. Bisnovatyi-Kogan, O.Yu. Tsupko,Astrophysics **51**,99,2008.
[22] Gao and Zhang,Phys.Rev.D**70**,124019,2004.
[23] Sami,Toporensky,Tretjakov and Tsujikawa,Phy.Lett.B**619**,193,2005.

[24] M.Gasperini,hep-th/0310293.
[25] C.-M.Claudel,K.S.Virbhadra and G.F.R.Ellis,J.M.P**42**,818,2001.
[26] K.S.Virbhadra,D.Narasimha and S.M.Chitre,Astron.Astrophys**337**,1-8,1998.
[27] A.I.Janis,E.T.Newman and J.Winicour,Phys.Rev.Lett**20**,878,1968.
[28] V.Bozza,Phys.Rev.D**66**,103001,2002.
[29] K.S.Virbhadra,Phys.Rev.D**79**,083004,2009.
[30] K.S.Virbhadra and C.R.Keeton,Phys.Rev.D**77**,124014,2008.
[31] L.Randall and R.Sundrum,Phys.Rev.Lett**83**,4690,1999.
[32] S.W.Hawking,T.Hertog and H.S.Reall,Phys.Rev.D**62**,043501,2000.
[33] Antonino Flachi and Takahiro Tanaka,Phys.Rev.D**78**,064011,2008.
[34] S. Das, D. Maity and S. Sengupta, JHEP **05**, 042 (2008).