Dynamics of Viscous Dissipative Plane Symmetric Gravitational Collapse

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Abstract

We present dynamical description of gravitational collapse in view of Misner and Sharp's formalism. Matter under consideration is a complicated fluid consistent with plane symmetry which we assume to undergo dissipation in the form of heat flow, radiation, shear and bulk viscosity. Junction conditions are studied for a general spacetime in the interior and Vaidya spacetime in the exterior regions. Dynamical equations are obtained and coupled with causal transport equations derived in context of Müller Israel Stewart theory. The role of dissipative quantities over collapse is investigated.

Keywords: Gravitational collapse; Dissipation; Junction conditions; Dynamical equations; Transport equations.

1 Introduction

The ultimate fate of the star (when it undergoes catastrophic phase of collapse) is one of the most important questions in gravitation theory today. When a star has exhausted all of its nuclear fuel, it collapses under the influence of its own gravity and releases large amount of energy. In fact, it is

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a highly dissipative process, i.e., energy is not conserved in it, rather due to various forces and with the passage of time, it becomes lesser. Dissipative process plays dominant role in the formation and evolution of stars.

The initial discussion over this problem was given by Oppenheimer and Snyder [1] who assumed a spherically symmetric distribution of matter. They took the most simplest form of matter, i.e., dust and the flow is considered to be adiabatic. It is somewhat unrealistic to ignore the pressure as it cannot be overlooked in the formation of singularity. Misner and Sharp [2] adopted a better approach by considering an ideal fluid which gave a more realistic analysis of gravitational collapse. Both of them assumed vacuum in the exterior region. Vaidya [3] introduced a non-vacuum exterior by giving the idea of outgoing radiation in collapse. It was physically a quite reasonable assumption as radiation is a confirmation that dissipative processes are occurring, causing loss of thermal energy of the system which is an effective way of decreasing internal pressure.

The Darmois junction conditions [4] gave a way to obtain exact models of an interior spacetime with heat flux to match with exterior Vaidya spacetime. Sharif and Ahmad [5] considered the perfect fluid with positive cosmological constant to discuss the junction conditions with spherical symmetry. The same authors [6] also worked on junction conditions for plane symmetric spacetimes.

Goswami [7] made an attempt in search of a more physical model of collapse. He considered dust like matter with heat flux to conclude that dissipation causes a bounce in collapse before the formation of singularity. Nath at el. [8] investigated dissipation in the form of heat flow and formulated junction conditions between charged Vaidya spacetime in exterior and quasi-spherical Szekeres spacetime in interior regions. They also discussed apparent horizons and singularity formation. Ghosh and Deshkar [9] studied gravitational collapse of radiating star with plane symmetry and pointed out some useful results. A lot of work is being done over gravitational collapse by considering shear free motion of the fluid. Although, it leads to simplification in obtaining exact solutions of the field equations, yet it is an unrealistic approach. Shear viscosity is a source of dissipating energy and plays an important role in collapse. Chan [10] investigated gravitational collapse, with radial heat flow, radiation and shear viscosity. He showed how the pressure became anisotropic due to shear viscosity.

Herrera and Santos [11] discussed the dynamics of gravitational collapse which undergoes dissipation in the form of heat flow and radiation. Di Prisco et al. [12] extended this work by adding charge and dissipation in the form of shear viscosity. Herrera [13] provided comprehensive details of inertia of heat and how it plays an effective role in dynamics of dissipative collapse. Herrera and Martinez [14] presented relativistic model of heat conducting collapsing object and debated over the effect of a parameter which occurs in dynamical equation on collapse. Herrera and collaborators [15]-[16] proposed a model of shear free conformally flat collapse and focused on the role of relaxation process, local anisotropy and relation between dissipation and density inhomogeneity.

Recently, Herrera et al. [17] threw light on behavior of non-equilibrium massive object which lost energy due to heat flow, radiation, shear and bulk viscosity. Matter under consideration was distributed with spherical symmetry. It has become quite clear that when mass and energy densities involved in the physical phenomenon are sufficiently high as in gravitational collapse, gravitational field plays an important and dominant role. The gravitational dynamics then must be taken into account for a meaningful description of such ultra high energy objects. This fact motivated us to elaborate the above mentioned paper in the context of plane symmetries. Matter under consideration is a complicated fluid which suffers through dissipation. Misner and Sharp's prescription is used to work out dynamical equations. Transport equations are obtained in the context of M*ü*ller Israel Stewart theory [18], [19] which is a causal theory for dissipative fluids. Thermodynamic viscous/heat coupling coefficients are taken to be non-vanishing which is expected to be quite plausible in non-uniform stellar models of universe. One of the dynamical equations is then coupled to transport equations in order to figure out the influence of dissipation over collapse.

The paper is written in the following manner. The next section is about the matter distribution in the interior region and some physical quantities relevant to matter under consideration. The Einstein field equation are worked out in section **3** and junction conditions are discussed in section **4**. Dynamical equations are formulated in section **5** and are coupled to transport equations in section **6**. The last section discusses and concludes the main results of the paper.

2 Interior matter distribution and some physical quantities

A 4-dimensional spacetime is split into two regions: interior V^- and exterior V^+ through a hypersurface Σ which is the boundary of both regions. We assume the matter distribution in the interior region to be consistent with plane symmetry. The interior region V^- admits the following line element

$$ds_-^2 = -f(t,z)dt^2 + g(t,z)(dx^2+dy^2) + h(t,z)dz^2, \tag{1}$$

where $\{\chi^{-\mu}\} \equiv \{t,x,y,z\}$ $(\mu = 0,1,2,3)$. The fluid is presumed to dissipate energy in terms of heat flow, radiation, shearing and bulk viscosity.

The energy-momentum tensor for such a fluid is defined as

$$T_{ab} = (\mu + p + \Pi)V_aV_b + (p+\Pi)g_{ab} + q_aV_b + q_bV_a + \epsilon l_a l_b + \pi_{ab}, \tag{2}$$

where $\mu,\ p,\ \Pi,\ q_a,\ l_a$ and π_{ab} are the energy density, pressure, bulk viscosity, heat flow, null four-vector in z-direction and shear viscosity tensor respectively. Heat flow q_a is taken to be orthogonal to velocity V^a, i.e., $q_aV^a = 0$. Moreover, we have

$$V^aV_a = -1, \quad l^aV_a = -1, \quad \pi_{ab}V^b = 0, \quad \pi_{[ab]} = 0, \quad \pi^a_a = 0, \quad l^al_a = 0. \tag{3}$$

In the standard irreversible thermodynamics by Eckart, we have the following relation [20]

$$\pi_{ab} = -2\eta\sigma_{ab}, \quad \Pi = -\zeta\Theta, \tag{4}$$

where η and ζ stand for coefficients of shear and bulk viscosity, σ_{ab} is the shear tensor and Θ is the expansion. The algebraic nature of Eckart constitutive equations causes several problems but we are concerned with the causal approach of dissipative variables. Thus we would not assume (4) rather we shall resort to transport equations of M*ü*ller-Israel-Stewart theory.

The shear tensor σ_{ab} is defined as

$$\sigma_{ab} = V_{(a;b)} + a_{(a}V_{b)} - \frac{1}{3}\Theta h_{ab}, \tag{5}$$

where the acceleration a_a and the expansion Θ are given by

$$a_a = V_{a;b}V^b, \quad \Theta = V^a_{;a} \tag{6}$$

and $h_{ab} = g_{ab} + V_a V_b$ is the projection tensor. The shear tensor σ_{ab} satisfies

$$V_a \sigma^{ab} = 0, \quad \sigma^{ab} = \sigma^{ba}, \quad \sigma^a_a = 0. \tag{7}$$

In co-moving coordinates, one can take

$$V^a = \frac{1}{\sqrt{f}}\delta^a_0, \quad q^a = \frac{q}{\sqrt{h}}\delta^a_3, \quad l^a = \frac{1}{\sqrt{f}}\delta^a_0 + \frac{1}{\sqrt{h}}\delta^a_3, \tag{8}$$

here q is a function of t and z.

Using Eq.(8), the non-vanishing components of the shear tensor σ_{ab} turn out to be

$$\sigma_{11} = -\frac{g}{3}\sigma = \sigma_{22}, \quad \sigma_{33} = \frac{2h}{3}\sigma, \tag{9}$$

where

$$\sigma = \frac{1}{2\sqrt{f}}\left(\frac{\dot{h}}{h} - \frac{\dot{g}}{g}\right). \tag{10}$$

Thus we have

$$\sigma_{ab}\sigma^{ab} = \frac{2}{3}\sigma^2. \tag{11}$$

Also, in view of Eqs.(3) and (4), it yields

$$\pi_{0a} = 0, \pi^3_3 = -2\pi^2_2 = -2\pi^1_1. \tag{12}$$

In compact form, it can be written as

$$\pi_{ab} = \Omega(\chi_a\chi_b - \frac{1}{3}h_{ab}), \tag{13}$$

where $\Omega = \frac{3}{2}\pi^3_3$ and χ^a is a unit four-vector in z-direction satisfying

$$\chi^a\chi_a = 1, \quad \chi^a V_a = 0, \quad \chi^a = \frac{1}{\sqrt{h}}\delta^a_3. \tag{14}$$

In view of Eqs.(6) and (8), it follows that

$$a_3 = \frac{f'}{2f}, \quad \Theta = \frac{1}{\sqrt{f}}\left(\frac{\dot{g}}{g} + \frac{\dot{h}}{2h}\right), \tag{15}$$

where dot and prime represent derivative with respect to time t and z respectively.

The Taub's mass for plane symmetric spacetime is defined by [21]

$$m(t,z) = \frac{(g)^{3/2}}{2}R^{12}_{12} = \frac{1}{8\sqrt{g}}\left(\frac{\dot{g}^2}{f} - \frac{g'^2}{h}\right). \tag{16}$$

3 The Einstein field equations

The Einstein field equations for the metric (1) yield the following set of equations

$$\frac{\dot{g}}{2g}\left(\frac{\dot{g}}{2g}+\frac{\dot{h}}{h}\right)+\frac{fg'}{2gh}\left(\frac{h'}{h}+\frac{g'}{2g}\right)-\frac{fg''}{gh}=8\pi(\mu+\epsilon)f, \quad (17)$$

$$\frac{\dot{g}}{2f}\left(\frac{\dot{f}}{2f}+\frac{\dot{g}}{2g}-\frac{\dot{h}}{2h}\right)+\frac{g'}{4h}\left(\frac{f'}{f}-\frac{h'}{h}-\frac{g'}{g}\right)-\frac{f'g}{4fh}\left(\frac{h'}{h}+\frac{f'}{f}\right)$$
$$+\frac{\dot{h}g}{4fh}\left(\frac{\dot{h}}{h}+\frac{\dot{f}}{f}\right)-\frac{\ddot{g}}{2f}+\frac{g''}{2h}+\frac{g}{2fh}(f''-\ddot{h})=8\pi(p+\Pi-\frac{1}{3}\Omega)g,$$
$$(18)$$

$$\frac{g'}{2g}\left(\frac{g'}{2g}+\frac{f'}{f}\right)+\frac{\dot{g}h}{2fg}\left(\frac{\dot{g}}{2g}+\frac{\dot{f}}{f}\right)-\frac{\ddot{g}h}{fg}=8\pi(p+\Pi+\epsilon+\frac{2}{3}\Omega)h, \quad (19)$$

$$\frac{\dot{g}}{2g}\left(\frac{g'}{g}+\frac{f'}{f}\right)+\frac{g'\dot{h}}{2gh}-\frac{\dot{g}'}{g}=-8\pi(q+\epsilon)\sqrt{fh}. \quad (20)$$

After some manipulation, we can also write Eq.(20) in the following form

$$4\pi(q+\epsilon)\sqrt{h}=\frac{1}{3}(\Theta-\sigma)'-\sigma\frac{\sqrt{g}\,'}{\sqrt{g}}. \quad (21)$$

4 Junction conditions

We discuss junction conditions for the interior region V^- given by Eq.(1) and the exterior region V^+ which is taken as plane symmetric Vaidya spacetime ansatz given by the line element [22]

$$ds_+^2=\frac{2m(\nu)}{Z}d\nu^2-2d\nu dZ+Z^2(dX^2+dY^2), \quad (22)$$

where $\chi^{+\mu}\equiv\{\nu,X,Y,Z\}$ $(\mu=0,1,2,3)$, ν is the retarded time and $m(\nu)$ represents total mass inside Σ. The line element for the hypersurface Σ is defined as

$$(ds^2)_\Sigma=-d\tau^2+A^2(\tau)(dx^2+dy^2), \quad (23)$$

where $\xi^i \equiv (\tau, x, y)$ $(i = 0, 1, 2)$ are the intrinsic coordinates of Σ.

The Darmois junction conditions [4] are

- The continuity of the line elements over the hypersurface Σ gives
$$(ds^2)_\Sigma = (ds^2_-)_\Sigma = (ds^2_+)_\Sigma. \tag{24}$$
This is called continuity of the first fundamental form.

- The continuity of the extrinsic curvature K_{ab} over the hypersurface Σ yields
$$[K_{ij}] = K^+_{ij} - K^-_{ij} = 0, \quad (a, b = 0, 1, 2). \tag{25}$$
This is known as continuity of the second fundamental form.

Here $K^\pm_{ij}$ is the extrinsic curvature defined as

$$K^\pm_{ij} = -n^\pm_\sigma(\frac{\partial^2\chi^\sigma_\pm}{\partial\xi^i\partial\xi^j} + \Gamma^\sigma_{\mu\nu}\frac{\partial\chi^\mu_\pm\partial\chi^\nu_\pm}{\partial\xi^i\partial\xi^j}), \quad (\sigma, \mu, \nu = 0, 1, 2, 3). \tag{26}$$

where $n^\pm_\sigma$ are the components of outward unit normal to hypersurface Σ in the coordinates $\chi^{\pm\mu}$.

The equations of hypersurface Σ in terms of coordinates $\chi^{\mp\mu}$ are given as

$$k_-(t, z) = z - z_\Sigma = 0, \tag{27}$$
$$k_+(\nu, Z) = Z - Z_\Sigma(\nu) = 0, \tag{28}$$

where z_Σ is taken to be an arbitrary constant. Using Eqs.(27) and (28), the interior and exterior metrics take the following form over hypersurface Σ

$$(ds^2_-)_\Sigma = -f(t, z_\Sigma)dt^2 + g(t, z_\Sigma)(dx^2 + dy^2), \tag{29}$$
$$(ds^2_+)_\Sigma = 2\left(\frac{m(\nu)}{Z_\Sigma} - \frac{dZ_\Sigma}{d\nu}\right)d\nu^2 + Z^2_\Sigma(dX^2 + dY^2). \tag{30}$$

In view of junction condition (24), we get

$$Z^2_\Sigma = g(t, z_\Sigma), \tag{31}$$
$$\frac{dt}{d\tau} = \frac{1}{\sqrt{f}}, \tag{32}$$
$$\frac{d\nu}{d\tau} = \left(2\frac{dZ_\Sigma}{d\nu} - \frac{2m(\nu)}{Z_\Sigma}\right)^{-1/2}. \tag{33}$$

Using Eqs.(27) and (28), the unit normals in V^- and V^+ respectively, turn out to be

$$n_\mu^- = \sqrt{h}(0,0,0,1), \tag{34}$$

$$n_\mu^+ = \left[2\left(\frac{dZ}{d\nu} - \frac{m(\nu)}{Z}\right)\right]^{-1/2}\left(-\frac{dZ}{d\nu},0,0,1\right). \tag{35}$$

The non-zero components of the extrinsic curvature $K_{ij}^\pm$ are

$$K_{00}^- = -\left(\frac{f'}{2f\sqrt{h}}\right)_\Sigma, \tag{36}$$

$$K_{00}^+ = \left[\frac{d^2\nu}{d\tau^2}\left(\frac{d\nu}{d\tau}\right)^{-1} - \frac{m}{Z^2}\frac{d\nu}{d\tau}\right]_\Sigma, \tag{37}$$

$$K_{11}^- = K_{22}^- = \left(\frac{g'}{2\sqrt{h}}\right)_\Sigma, \tag{38}$$

$$K_{11}^+ = K_{22}^+ = \left[Z\frac{dZ}{d\tau} - 2m\frac{d\nu}{d\tau}\right]_\Sigma. \tag{39}$$

Now, by the junction condition (25), i.e., continuity of extrinsic curvatures, it follows that

$$\left[\frac{d^2\nu}{d\tau^2}\left(\frac{d\nu}{d\tau}\right)^{-1} - \frac{m}{Z^2}\frac{d\nu}{d\tau}\right]_\Sigma = -\left(\frac{f'}{2f\sqrt{h}}\right)_\Sigma, \tag{40}$$

$$2m\frac{d\nu}{d\tau} = \frac{\dot{g}}{2\sqrt{f}} - \frac{g'}{2\sqrt{h}}. \tag{41}$$

Using Eqs.(33) and (41), we obtain

$$\left(\frac{d\nu}{d\tau}\right)^{-1} = \frac{1}{\sqrt{g}}\left[\frac{\dot{g}}{2\sqrt{f}} + \frac{g'}{2\sqrt{h}}\right]. \tag{42}$$

Inserting Eq.(42) in (41), it follows that

$$m(\nu) = \frac{1}{8\sqrt{g}}\left(\frac{\dot{g}^2}{f} - \frac{g'^2}{h}\right) \tag{43}$$

and hence

$$m(t,z) \overset{\Sigma}{=} m(\nu). \tag{44}$$

Differentiating Eq.(42) with respect to τ, and making use of Eqs.(43) and (42), we can write Eq.(40) as

$$\frac{1}{2\sqrt{fhg}}\left[\frac{-\dot{g}'}{\sqrt{g}}+\frac{g'\dot{h}}{2h\sqrt{g}}+\frac{f'\dot{g}}{2f\sqrt{g}}+\frac{\sqrt{h}}{\sqrt{f}}\left\{\frac{-\ddot{g}}{\sqrt{g}}+\frac{\dot{g}\dot{f}}{2f\sqrt{g}}+\sqrt{g}\left(\frac{\dot{g}}{2g}\right)^2\right.\right.$$
$$\left.\left.+\frac{f}{4g^{3/2}}\left(\frac{g'}{\sqrt{h}}\right)^2+\frac{f'g'}{2h\sqrt{g}}+\frac{\sqrt{f}}{\sqrt{h}}\left(\frac{g'\dot{g}}{2g^{3/2}}\right)\right\}\right]\overset{\Sigma}{=}0. \quad (45)$$

Comparing Eq.(45) with Eqs.(19) and (20), it yields

$$p+\Pi+\frac{2}{3}\Omega=q. \quad (46)$$

5 Dynamical equations

The energy-momentum conservation, $T^{ab}_{;b}=0$, gives

$$\begin{aligned} T^{ab}_{;b}V_a &= \frac{(\dot{\mu}+\dot{\epsilon})}{\sqrt{f}}+\frac{(q'+\epsilon')}{\sqrt{h}}+\frac{\dot{g}}{g\sqrt{f}}(\mu+p+\Pi+\epsilon-\frac{1}{3}\Omega) \\ &+ \frac{\dot{h}}{2h\sqrt{f}}(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega)+\frac{(fg)'}{fg}\frac{(q+\epsilon)}{\sqrt{h}}=0 \end{aligned} \quad (47)$$

and

$$\begin{aligned} T^{ab}_{;b}\chi_a &= \frac{1}{\sqrt{f}}(\dot{q}+\dot{\epsilon})+\frac{1}{\sqrt{f}}(q+\epsilon)\frac{\dot{(hg)}}{hg}+\frac{1}{\sqrt{h}}(p'+\Pi'+\epsilon'+\frac{2}{3}\Omega') \\ &+\frac{f'}{2f\sqrt{h}}(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega)+\frac{g'}{g\sqrt{h}}(\epsilon+\Omega)=0. \end{aligned} \quad (48)$$

Now we investigate the dynamical properties of the system using the Misner and Sharp's [2] perspective. For this purpose, we take the proper time derivative as

$$D_T=\frac{1}{\sqrt{f}}\frac{\partial}{\partial t}, \quad (49)$$

and the proper derivative in z-direction as

$$D_{\tilde{Z}}=\frac{1}{\tilde{Z}'}\frac{\partial}{\partial z}, \quad (50)$$

where

$$\tilde{Z} = \sqrt{g}. \tag{51}$$

The velocity U of the collapsing fluid can be defined as the variation of $\tilde{Z}$ with respect to the proper time

$$U = D_T(\tilde{Z}) = \frac{1}{2\sqrt{g}} D_T g. \tag{52}$$

In the case of collapse, the velocity of the collapsing fluid must be negative. In view of Eq.(52), Eq.(16) can take the following form

$$E = \frac{\sqrt{g}\,'}{\sqrt{h}} = [U^2 - \frac{2}{\sqrt{g}} m(t,z)]^{1/2}. \tag{53}$$

Making use of Eq.(50) in Eq.(21), it follows that

$$4\pi(q+\epsilon) = E\left[\frac{1}{3}D_{\tilde{Z}}(\Theta - \sigma) - \frac{\sigma}{\tilde{Z}}\right]. \tag{54}$$

In case of no dissipation, using Eqs.(10), (15) and (52), the above equation becomes

$$D_{\tilde{Z}}\left(\frac{U}{\tilde{Z}}\right) = 0. \tag{55}$$

This implies that $U \sim \tilde{Z}$ depicting that now collapse will be homologous. The rate of change of Taub's mass, using Eqs.(16), (19), (20) and (49), turn out to be

$$D_T m = -4\pi\tilde{Z}^2[(p + \Pi + \epsilon + \frac{2}{3}\Omega)U + (q+\epsilon)E]. \tag{56}$$

Thus the rate of change of Taub's mass represents variation of total energy inside the collapsing plane surface. Since this variation is negative, it shows that total energy is being dissipated during collapse. The first round brackets on the right hand side stand for energy due to work being done by the effective isotropic pressure $(p + \Pi + \frac{2}{3}\Omega)$ and the radiation pressure ϵ. The second brackets describe energy leaving the system due to heat flux and radiation. Similarly, using Eqs.(16), (17), (20) and (50), we get

$$D_{\tilde{Z}} m = 4\pi\tilde{Z}^2[\mu + \epsilon + (q+\epsilon)\frac{U}{E}]. \tag{57}$$

This equation describes about the variation of energy between adjoining plane surfaces inside the fluid distribution. On the right hand side, $(\mu + \epsilon)$ stands for energy density of the fluid element plus the energy of null fluid showing dissipation due to radiation. Moreover, $(q+\epsilon)\frac{U}{E}$ is negative (as $U<0$), telling that energy is leaving due to outflow of heat and radiation.

Making use of Eqs.(16), (19), (51) and (53), the acceleration D_TU of the collapsing matter inside the hypersurface Σ is given as

$$D_TU=-4\pi(p+\Pi+\epsilon+\frac{2}{3}\Omega)\tilde{Z}-\frac{m}{\tilde{Z}^2}+\frac{Ef'}{2f\sqrt{h}}. \quad (58)$$

Substituting the value of $\frac{f'}{2f}$ from the above equation into Eq.(48), it follows that

$$\begin{aligned}(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega)D_TU &= -(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega)\\ &\times[4\pi\tilde{Z}(p+\Pi+\epsilon+\frac{2}{3}\Omega)+\frac{m}{\tilde{Z}^2}]\\ &-E^2[D_{\tilde{Z}}(p+\Pi+\epsilon+\frac{2}{3}\Omega)+\frac{2}{\tilde{Z}}(\epsilon+\Omega)]\\ &-E[D_Tq+D_T\epsilon+4(q+\epsilon)\frac{U}{\tilde{Z}}+2(q+\epsilon)\sigma].\end{aligned} \quad (59)$$

This equation has the form of Newton's second law, i.e.,

$$Force=Mass\quad density\quad\times\quad Acceleration.$$

The term within the brackets on the left hand side stands for "effective" inertial mass and the remaining term is acceleration. The first term on the right hand side represents gravitational force. Since by the equivalence principle, inertial mass is equivalent to passive gravitational mass and passive gravitational mass is equivalent to active gravitational mass. Thus the factor within round brackets stands for active gravitational mass and the factor within the square brackets shows how dissipation effects active gravitational mass. The second square brackets firstly include gradient of effective pressure which involves radiation pressure and the collective effect of shear and bulk viscosity. The second contribution is of local anisotropy of pressure which is the result of radiation and shear viscosity. The last square brackets entirely depend upon dissipation. The hydrostatic equilibrium can be obtained from the above equation by substituting $U=0,\ q=0,\ \epsilon=0,\ \Pi=0$ and $\Omega=0$.

$$D_{\tilde{Z}}p=-(\mu+p)\frac{h}{\tilde{Z}'^2}\left[\frac{m}{\tilde{Z}^2}+4\pi\tilde{Z}p\right].$$

6 Transport equations

The general expression for entropy 4-current is given as [20]

$$S^\mu = SnV^\mu + \frac{q^\mu}{T} - (\beta_0\Pi^2 + \beta_1 q_\nu q^\nu + \beta_2 \pi_{\nu\kappa}\pi^{\nu\kappa})\frac{V^\mu}{2T} + \frac{\alpha_0 \Pi q^\mu}{T} + \frac{\alpha_1 \pi^{\mu\nu} q_\nu}{T}, \quad (60)$$

where n is particle number density, T is temperature, $\beta_A(\rho, n) \geq 0$ are thermodynamic coefficients for scalar, vector and tensor dissipative contributions to the entropy density and $\alpha_A(\rho, n)$ are thermodynamic viscous/heat coupling coefficients. The divergence of extended current (follows from Gibbs equation and Bianchi identities) is given by

$$\begin{aligned} TS^\alpha_{;\alpha} &= -\Pi\left[V^\alpha_{;\alpha} - \alpha_0 q^\alpha_{;\alpha} + \beta_0 \Pi_{;\alpha} V^\alpha + \frac{T}{2}\left(\frac{\beta_0}{T}V^\alpha\right)_{;\alpha}\Pi\right] \\ &- q^\alpha[h^\mu_\alpha (\ln T)_{,\mu}(1+\alpha_0\Pi) + V_{\alpha;\mu}V^\mu - \alpha_0 \Pi_{;\alpha} - \alpha_1 \pi^\mu_{\alpha;\mu} \\ &+ \alpha_1 \pi^\mu_\alpha h^\beta_\mu (\ln T)_{,\beta} + \beta_1 q_{\alpha;\mu}V^\mu + \frac{T}{2}\left(\frac{\beta_1}{T}V^\mu\right)_{;\mu} q_\alpha] \\ &- \pi^{\alpha\mu}\left[\sigma_{\alpha\mu} - \alpha_1 q_{\mu;\alpha} + \beta_2 \pi_{\alpha\mu;\nu}V^\nu + \frac{T}{2}\left(\frac{\beta_2}{T}V^\nu\right)_{;\nu}\pi_{\alpha\mu}\right]. \quad (61)\end{aligned}$$

The 2nd law of thermodynamics requires that $S^\alpha_{;\alpha} \geq 0$. This leads to the following transport equations for our dissipative variables

$$\tau_0 \Pi_{,\alpha}V^\alpha + \Pi = -\zeta\Theta + \alpha_0\zeta q^\alpha_{;\alpha} - \frac{1}{2}\zeta T\left(\frac{\tau_0}{\zeta T}V^\alpha\right)_{;\alpha}\Pi, \quad (62)$$

$$\begin{aligned} \tau_1 h^\beta_\alpha q_{\beta;\mu}V^\mu + q_\alpha &= -k[h^\beta_\alpha T_{,\beta}(1+\alpha_0\Pi) + \alpha_1 \pi^\mu_\alpha h^\beta_\mu T_{,\beta} + T(a_\alpha \\ &- \alpha_0 \Pi_{;\alpha} - \alpha_1 \pi^\mu_{\alpha;\mu})] - \frac{1}{2}kT^2\left(\frac{\tau_1}{kT^2}V^\beta\right)_{;\beta} q_\alpha \quad (63)\end{aligned}$$

and

$$\tau_2 h^\mu_\alpha h^\nu_\beta \pi_{\mu\nu;\rho}V^\rho + \pi_{\alpha\beta} = -2\eta\sigma_{\alpha\beta} + 2\eta\alpha_1 q_{<\beta;\alpha>} - \eta T\left(\frac{\tau_2}{2\eta T}V^\nu\right)_{;\nu}\pi_{\alpha\beta}, \quad (64)$$

where

$$q_{<\beta;\alpha>} = h^\mu_\beta h^\nu_\alpha\left(\frac{1}{2}(q_{\mu;\nu} + q_{\nu;\mu}) - \frac{1}{3}q_{\sigma;\kappa}h^{\sigma\kappa}h_{\mu\nu}\right), \quad (65)$$

with k as the thermal conductivity. The relaxation times are given by

$$\tau_0 = \zeta\beta_0, \quad \tau_1 = kT\beta_1, \quad \tau_2 = 2\eta\beta_2. \tag{66}$$

Notice that if the thermodynamic coupling coefficients are assumed to be zero, Eqs.(62)-(64) turn to be Eqs.(2.21)-(2.23) as given in [20]. The independent components of Eqs.(62)-(64) are calculated as follows.

$$\begin{aligned}
\tau_0\dot{\Pi} &= -\left(\zeta + \frac{\tau_0\Pi}{2}\right)\Theta\sqrt{f} + \alpha_0\zeta\frac{\sqrt{f}}{\sqrt{h}}\left[q' + q\left(\frac{f'}{2f} + \frac{g'}{g}\right)\right] \\
&- \left[\frac{\zeta T}{2}\left(\frac{\tau_0}{\zeta T}\right)^{\cdot} + \sqrt{f}\right]\Pi,
\end{aligned} \tag{67}$$

$$\begin{aligned}
\tau_1\dot{q} &= -k\frac{\sqrt{f}}{\sqrt{h}}\left[T'(1 + \alpha_0\Pi + \frac{2}{3}\alpha_1\Omega) + T\left\{\frac{f'}{2f} - \alpha_0\Pi''\right.\right. \\
&- \left.\left.\alpha_1\left(\frac{2}{3}\Omega' + \frac{f'}{3f}\Omega + \frac{g'}{g}\Omega\right)\right\}\right] \\
&- q\left[\frac{kT^2}{2}\left(\frac{\tau_1}{kT^2}\right)^{\cdot} + \frac{\tau_1}{2}\Theta\sqrt{f} + \sqrt{f}\right],
\end{aligned} \tag{68}$$

$$\begin{aligned}
\tau_2\dot{\Omega} &= -2\sqrt{f}\eta\sigma + \eta\alpha_1\frac{\sqrt{f}}{\sqrt{h}}(2q' - \frac{g'}{g}q) \\
&- \left[\eta T\left(\frac{\tau_2}{2\eta T}\right)^{\cdot}\Omega + \frac{\tau_2}{2}\Theta\sqrt{f}\Omega + \Omega\sqrt{f}\right].
\end{aligned} \tag{69}$$

Now we discuss the action of dissipation over dynamics of collapsing object. We couple these transport equations to dynamical equation (59). Using Eq.(68) in Eq.(59), it follows that

$$\begin{aligned}
&(\mu + p + \Pi + 2\epsilon + \frac{2}{3}\Omega)(1-\Lambda)D_TU = (1-\Lambda)F_{grav} + F_{hyd} \\
&+\frac{kE^2}{\tau_1}\left[D_{\tilde{Z}}T(1 + \alpha_0\Pi + \frac{2}{3}\alpha_1\Omega) - T\left\{\alpha_0 D_{\tilde{Z}}\Pi + \frac{2}{3}\alpha_1\left(D_{\tilde{Z}}\Omega + \frac{3}{\tilde{Z}}\Omega\right)\right\}\right] \\
&+E\left[\frac{kT^2q}{2\tau_1}D_T\left(\frac{\tau_1}{kT^2}\right) - D_T\epsilon\right] - E\left[\left(\frac{3q}{2} + 2\epsilon\right)\Theta - \frac{q}{\tau_1} - 2(q+\epsilon)\frac{U}{\tilde{Z}}\right],
\end{aligned} \tag{70}$$

where F_{grav} and F_{hyd} are given by

$$\begin{aligned} F_{grav} &= -(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega) \\ &\times \left[m+4\pi(p+\Pi+\epsilon+\frac{2}{3}\Omega)\tilde{Z}^3\right]\frac{1}{\tilde{Z}^2}, \end{aligned} \tag{71}$$

$$F_{hyd} = -E^2\left[D_{\tilde{Z}}(p+\Pi+\epsilon+\frac{2}{3}\Omega)+2(\epsilon+\Omega)\frac{1}{\tilde{Z}}\right] \tag{72}$$

and

$$\Lambda = \frac{kT}{\tau_1}\left(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega\right)^{-1}\left(1-\frac{2}{3}\alpha_1\Omega\right). \tag{73}$$

Inserting Eq.(67) in Eq.(70), we obtain

$$\begin{aligned} &(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega)(1-\Lambda+\Delta)D_T U = (1-\Lambda+\Delta)F_{grav}+F_{hyd} \\ &+\frac{kE^2}{\tau_1}\left[D_{\tilde{Z}}T\left(1+\alpha_0\Pi+\frac{2}{3}\alpha_1\Omega\right)-T\left\{\alpha_0 D_{\tilde{Z}}\Pi+\frac{2}{3}\alpha_1\left(D_{\tilde{Z}}\Omega+\frac{3}{\tilde{Z}}\Omega\right)\right\}\right] \\ &-E^2\left(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega\right)\Delta\left(\frac{D_{\tilde{Z}}q}{q}+\frac{2}{\tilde{Z}}\right) \\ &+E\left[\frac{kT^2q}{2\tau_1}D_T\left(\frac{\tau_1}{kT^2}\right)-D_T\epsilon\right]+E\left[\frac{q}{\tau_1}+2(q+\epsilon)\frac{U}{\tilde{Z}}\right] \\ &+E\frac{\Delta}{\alpha_0\zeta q}\left(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega\right)\left[\left\{1+\frac{\zeta T}{2}D_T\left(\frac{\tau_0}{\zeta T}\right)\right\}\Pi+\tau_0 D_T\Pi\right], \end{aligned} \tag{74}$$

where Δ is given by

$$\alpha_0\zeta q\left(p+\Pi+\mu+2\epsilon+\frac{2}{3}\Omega\right)^{-1}\left(\frac{3q+4\epsilon}{2\zeta+\tau_0\Pi}\right). \tag{75}$$

Here we see that $(1-\Lambda+\Delta)$ is the major factor that appears in the dynamical equation after coupling it with the transport equations. We would like to mention here that Eq.(74) is the plane symmetric version of Eq.(55) in [17].

7 Summary and Conclusion

Gravitational collapse in a star is an irreversible phenomenon. Dynamics (such as transport processes) of such non-equilibrium objects and connection between their dynamics and thermodynamics are of extensive significance in order to have a better visualization of this problem. Thus we have studied the dynamics of dissipative collapse, i.e., what role does dissipation play with passing time as star collapses under the influence of its own gravity. The most realistic model of matter, i.e, complicated fluid is assumed in the interior region and is taken to be consistent with plane symmetry.

To see how system evolves with time, dynamical equations for the plane symmetric spacetime are obtained using Misner and Sharp formalism. In the dynamical equation (59), we see that the gravitational force represented by the first term on the right hand side is expected to be much effective as compared to non-dissipative fluid and so gravitational collapse is expected to be faster in this case. Moreover, since the pressure gradient is negative in the second term on right hand side of this equation, which combined with the minus sign preceding that term makes a positive contribution, thereby reducing the rate of collapse. The last square brackets entirely depend on dissipation and one cannot expect any such contribution in a dynamical equation for non-dissipative collapse. The third term in this bracket is positive due to negative sign of velocity of collapsing fluid U. It shows that outflow of heat flux $q > 0$ and radiation $\epsilon > 0$ reduces the total energy of the system and hence reduces the rate of collapse.

Transport equations in the context of M$\ddot{u}$ller, Israel and Stewart theory of dissipative fluids are obtained and coupled to dynamical equation in order to see the influence of dissipation over dynamics of a collapsing plane. After this union of dynamical and transport equations, we get equation (74) where the factor $(1 - \Lambda + \Delta)$ appears in the dynamical equation. We see the effect of this factor for different possible values.

- If $0 < (\Lambda - \Delta) < 1$, inertial and gravitational mass densities will be reduced.
- If $(\Lambda - \Delta)$ tends to 1, inertial mass density tends to zero.
- If $(\Lambda - \Delta) > 1$, gravitational force will become positive and it will lead to the reversal of collapse. Another possibility for reversal of collapse is to take $(\Lambda - \Delta) < 1$ such that $(1 - \Lambda + \Delta)$ is sufficiently small. Consequently, it will significantly decrease the gravitational force.

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