Production of tidal-charged black holes at the Large Hadron Collider

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Abstract

Tidal-charged black hole solutions localized on a three-brane in the five-dimensional gravity scenario of Randall and Sundrum have been known for some time. The solutions have been used to study the decay, and growth, of black holes with initial mass of about 10 TeV. These studies are interesting in that certain black holes, if produced at the Large Hadron Collider, could live long enough to leave the detectors. I examine the production of tidal-charged black holes at the Large Hadron Collider and show that it is very unlikely that they will be produced during the lifetime of the accelerator.

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I. INTRODUCTION

Models of low-scale gravity could allow particle physics experiments to study the strong-gravity regime [1–5]. One consequence of these models is the possibility of black hole production at the Large Hadron Collider (LHC) if the fundamental scale of gravity M_D is about a TeV [6–9].

Black hole solutions in higher dimensions have a complicated dependence on both the gravitational field of the brane and the geometry of the extra dimensions. The models with large extra dimensions in a flat spacetime (ADD) [1, 2] are popular. One reason for this interest is that if the geometrical scales of the extra dimensions are all large compared to $1/M_D$, then there is a wide regime in which the geometry of the extra dimensions plays no essential role. In ADD this means considering black holes with horizon radius much smaller than the size of the extra dimensions. For large flat compactified extra dimensions, the topology of the black hole can be assumed to be spherically symmetric in the $(n+3)$-spatial dimensions and the boundary conditions from the compactification can be neglected. This allows the simple Myers-Perry metric [10] to be used. This metric is also useful in the Randall-Sundrum scenario [3] if the warping scale or (AdS radius) is large compared to $1/M_D$ [9].

In a regime in which the brane can not be neglected, the so called brane world scenario, there are very few physical solutions to the higher-dimensional black hole problem (see Ref. [11, 12] for a review). One of the earliest and most widely discussed solutions is a black string solution [13], but this solution is far from physical. Actually, it is unknown if physical black hole solutions exist at all in the Randall-Sundrum model.

A class of Randall-Sundrum solutions have been found in which the solution is a Reissner-Nördstrom metric with the electric charge replaced by a tidal charge [14]. If the effects of this tidal-charge term are negligible, the solution becomes an effective four-dimensional solution and the effects of low-scale gravity are unlikely to be observed. If the extra term is significant, the effects of low-scale gravity may be observable if the fundamental scale of gravity is low enough.

The decays of tidal-charged black holes in the contexts of the microcanonical picture have been studied [15–18]. The microcanonical corrections may be significant when the object reaches the Planck size and the classical black hole description fails (For the case of

the microcanonical treatment of ADD black hole decays at the LHC, also see Ref. [19].). The microcanonical corrections to the canonical decay treatment are larger in the Randall-Sundrum scenario for Planck-sized objects, and in certain cases they may live long enough to be considered quasi-stable. The possibility that black holes produced at the LHC might be quasi-stable has raised safety concerns (see for e.g. Ref [20]), which have been addressed in general [21–23], and in the context of tidal charged black holes [16–18]. I will have nothing new to add to the decay discussion.

Based on recent phenomenological constraints on the tidal charge [18], I estimate the production cross section at LHC energies and show that it is tiny over most of the allowed parameter space. The probability of producing these black holes during the lifetime of the LHC accelerator is vanishingly small. Hence, a scenario of high missing-energy signatures of black holes at the LHC, because of a long lifetime, is disfavoured.

II.   PRODUCTION CROSS SECTION

A static solution for a mass M localized on a three-brane in the five-dimensional gravity scenario of Randall-Sundrum is [14]

$$-g_{tt} = (g_{rr})^{-1} = 1 - \frac{2M}{M_p^2}\frac{1}{r} + \frac{q}{M_5^2}\frac{1}{r^2}\,, \tag{1}$$

where q is a dimensionless tidal charge, $M_p = 1.2 \times 10^{16}$ TeV is the effective Planck scale on the brane, and M_5 is the fundamental Planck scale in the five-dimensional bulk. Equation (1) is an exact solution of the effective Einstein equations on the brane and represents the induced metric on the brane in the strong-gravity regime. Attempts to extend the metric off the brane have failed [11, 12] so it is hoped that the brane characteristics of the black hole dominate.

The tidal term arises due the gravitational affects from the fifth dimension and the dimensionless parameter q can be thought of as a tidal charge arising from the projection onto the brane of free gravitational field effects in the bulk. The parameter q is determined by both the brane source, i.e. the mass M, and any Coulomb part of the bulk Weyl tensor that survives when M is set to zero [14]. It may be positive or negative. Ref. [14] argues that negative q is more natural. In this case, there will be one horizon that is outside the Schwarzschild horizon $R_S = 2M/M_p^2$. The bulk effects tend to strengthen the gravitational

field.

The tidal charge affects the geodesics and the gravitational potential, and hence indirect limits may be placed on it by observations. The effects of the tidal term die off quickly with increasing r. At astrophysical distances, the weak-field effects are being probed and the correction term is much less than the Schwarzschild term in the effective potential. However, in the strong-gravity regime q can be large, and the horizon radius would become

$$R_H \sim \frac{\sqrt{-q}}{M_5} \, . \tag{2}$$

The tidal term will dominate the Schwarzschild term, which reflects the fact that gravity becomes effectively five-dimensional at high energies. Current observations place only weak limits on the tidal charge [14], but in principle significant tidal modifications could arise in the strong-field regime. Thus it is useful to investigate the phenomenology of these black holes at the LHC.

The tidal-charge parameter can only be deduced on dimensional grounds. Ref. [14] suggests a value of $q = -M/M_5$. Ref. [15] generalized this to $q = -(M_p/M_5)^\alpha (M/M_5)$, where α is a real parameter. For the tidal term to dominate, $\alpha \gg -4$. Ref. [15–17] took as a typical choice $\alpha \geq 0$. A recent more general parameterization is given by [18]

$$q = - \left(\frac{M_p}{M_5} \right)^\alpha \left(\frac{M}{M_5} \right)^\beta , \tag{3}$$

where $\beta > 0$ is a real parameter. Thus the original calculations used the special case of $\alpha = 0$ and $\beta = 1$. For the tidal-charged black hole to be semiclassical, $\alpha \gtrsim -2$ [18].

A feature of black hole production in particle collisions is that its cross section is essentially the horizon area of the forming black hole and grows with the centre of mass energy of the colliding particles as some power. In the ADD scenario, this cross section is rather large and given by

$$\sigma_{\text{ADD}} \sim \frac{1}{M_D^2} \sim 1 \text{ nb} \, , \tag{4}$$

where $M_D \sim 1$ TeV is the fundamental Planck scale in D spacetime dimensions. For the tidal-charged black hole case,

$$\sigma \sim \frac{-q}{M_5^2} \sim \frac{1}{M_5^2} \left(\frac{M_p}{M_5} \right)^\alpha \left(\frac{M}{M_5} \right)^\beta . \tag{5}$$

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For this cross section to be comparable to the ADD case, $|q|$ must be of $\mathcal{O}(1)$. Recent calculations [15, 17] used $\alpha \geq 0$ and $\beta = 1$, which could give a huge cross section and exceed the ADD case. However, α can be restricted from above by precision measurements of the deviation of Newton's law [24] and thus $\alpha > 0$ may not be a valid choice.

In estimating the upper bound on α, I follow a similar argument to Ref. [18]. The brane has a thickness L, which must be less than the length over which deviations from Newton gravity are not observed: $L \lesssim 44\ \mu\text{m} = 2.2 \times 10^{14}\ \text{TeV}^{-1}$ [24]. For the tidal-charge term to dominate over the Schwarzschild term, the horizon radius must be less than the critical radius r_c, which can be defined to be the radius at which the Schwarzschild term equals the tidal term in the metric:

$$r_c = \frac{1}{2M_p} \left(\frac{M_p}{M_5}\right)^{3+\alpha} \left(\frac{M}{M_5}\right)^{\beta-1}.$$

(6)

In addition, this critical radius must be less than the brane thickness, $r_c \ll L$, else we would have already observed deviations in Newton's law due to the tidal term. Thus, requiring $r_c \ll L$ gives $\alpha \lesssim \alpha_c$, where

$$\alpha_c \equiv \frac{\ln(2LM_p) + (1-\beta)\ln(M/M_5)}{\ln(M_p/M_5)} - 3.$$

(7)

In low-scale gravity, black holes could be produced at the LHC in the range $1 \lesssim M \lesssim 14$ TeV. The upper bound is given by the maximum LHC energy, which would be very unlikely, and the lower bound is given by the lower experimental limit on M_5. For $M_5 = 1$ TeV, $\alpha \lesssim -1.09$ for $M = M_5$, independent of β, and $\alpha \lesssim -1.02$ for $M = 14$ TeV, which occurs at $\beta = 0$. For larger values of M_5, $\alpha \lesssim -0.94$ for the extreme case of $M = M_5 = 14$ TeV, which is highly unlikely. The LHC is a proton-proton collider and thus the probability of producing a parton-parton collision at the maximum LHC energy is vanishingly small. Because of this, the upper bound on α_c is likely to be less than -0.94 for realizable values of high $M = M_5$. Ignoring factors of two in the metric or picking a different definition for the fundamental Planck scale M_5 changes α_c by at most about 0.1, and in no case is α_c larger than about -1.0.

Assuming the largest allowed value of α is -1, gives the upper bound on the parton cross section of

$$\sigma \lesssim \frac{1}{M_5 M_p} \left(\frac{M}{M_5} \right)^{\beta} \lesssim \frac{(M/\text{TeV})^{\beta}}{M_p/\text{TeV}} \text{ TeV}^{-2} \,, \tag{8}$$

where in the last expression I have used $M_5 = 1$ TeV, which gives the maximum cross section. Because of M_p in the denominator, the cross section will be exceedingly low unless β is very large. In terms of numerical values

$$\sigma \lesssim 10^{-16+\beta} \sim 10^{-10+\beta} \text{ fb} \,. \tag{9}$$

To calculate the proton-proton cross section, the parton cross section in Eq. (8) must be convoluted with the parton distribution functions of the proton, summed over all partons that could form the black hole, and integrated over all black hole masses. I will integrate over the full kinematically allowed range although the applicability of Eq. (8) may be questionable near $M \sim M_5$. To ignore this region would significantly reduce the cross section. The results are shown in Fig. 1 as a function of the β parameter for $M_5 = 1$ TeV.

I assume the total integrated luminosity delivered by the LHC will be 1 ab^{-1}. Thus the probability of producing a single tidal-charged black hole event during the lifetime of the accelerator is vanishingly small for reasonable values of β. Only for $\beta \gtrsim 11$ will black hole production become significant. We would expect β to be a few and large values of β are unnatural [18]. The functional form for q in Eq (3) can not be valid over all mass scales and some mass dependence of the parameters α and β is expected.

The πR_H^2 form for the cross section is usually considered a maximum cross section. Possible form factors could increase the cross section by a factor of about three. There is far greater discussion on how the cross section is reduced from the geometrical form. For example, initial-state beam radiation or radiation during a balding phase could reduce the cross section by several orders of magnitude (see Ref. [25] for a review of the ADD case).

The analysis has been confined to the brane and the bulk space has not been taken into account. Some approximations are involved in the terms that encode the bulk behaviour on the brane. Assuming these simplifications do not alter the underlying phenomenology, it appears very unlikely that tidal-charged black holes could be produced at the LHC.
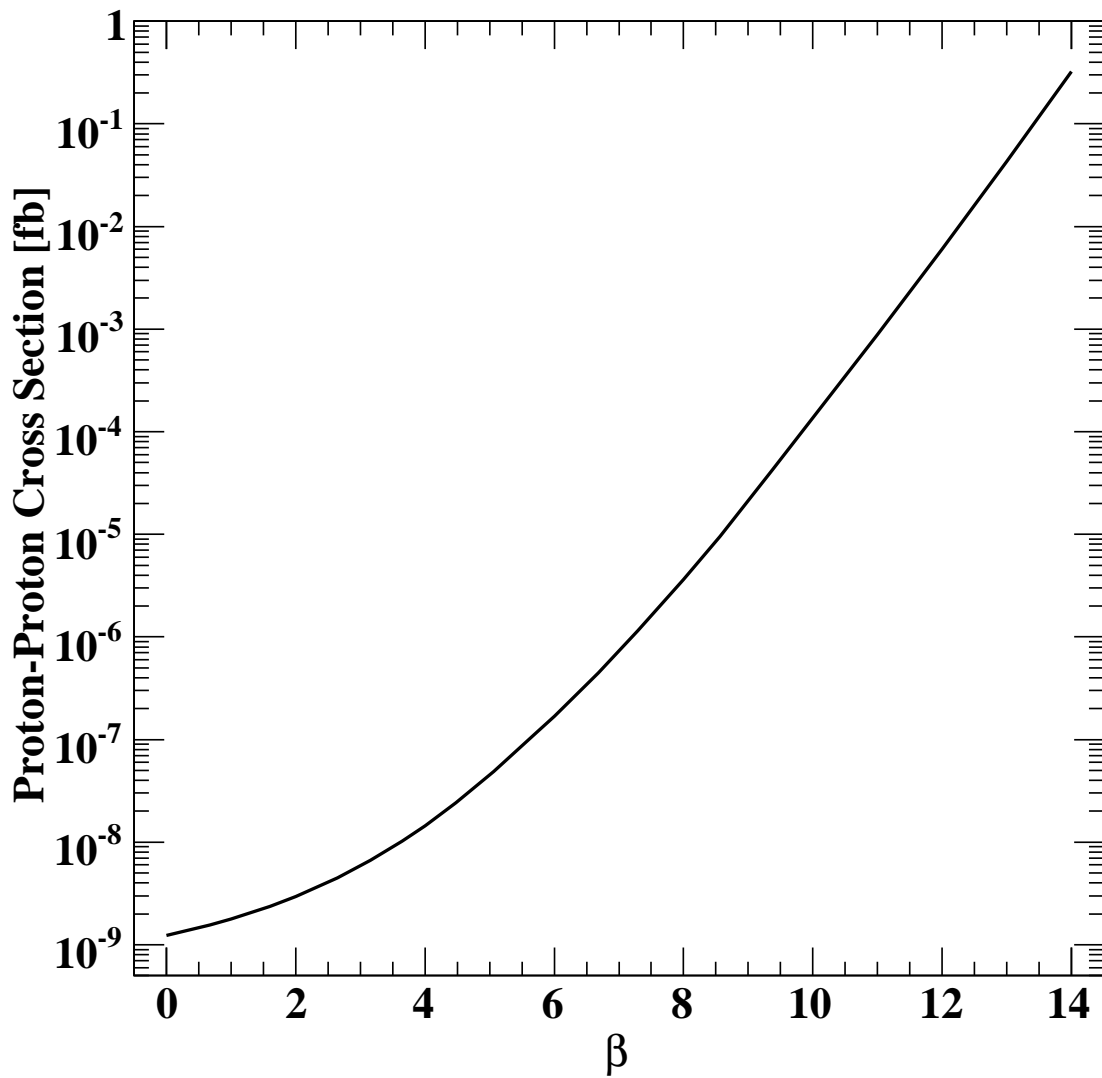
FIG. 1. Proton-proton cross section versus the β parameter for $M_5 = 1$ TeV.

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[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998), arXiv:hep-ph/9803315v1.

[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998), arXiv:hep-ph/9804398v1.

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999), arXiv:hep-ph/9905221v1.

[4] G. Dvali, "Black holes and large N species solution to the hierarchy problem," (2007), arXiv:0706.2050v1 [hep-th].

[5] X. Calmet and S. D. H. Hsu, Phys. Lett. B **663**, 95 (2008), arXiv:0711.2306v1 [hep-th].

[6] P. C. Argyres, S. Dimopoulos, and J. March-Russell, Phys. Lett. B **441**, 96 (1998), arXiv:hep-th/9808138v1.

[7] T. Banks and W. Fischler, "A model for high energy scattering in quantum gravity," (1999), arXiv:hep-th/9906038v1.

[8] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001), arXiv:hep-ph/0106295v1.

[9] S. B. Giddings and S. Thomas, Phys. Rev. D **65**, 056010 (2002), arXiv:hep-ph/0106219v4.

[10] R. C. Myers and M. J. Perry, Ann. Phys. (N.Y.) **172**, 304 (1986).

[11] R. Gregory, in *Proceedings of the 4th Aegean Summer School on Black Holes*, Vol. 769 (Lect. Notes Phys., 2009) pp. 259–298, arXiv:0804.2595 [hep-th].

[12] P. Kanti, in *Proceedings of the Recent Developments in Gravity*, Vol. 189 (J. Phys. Conf. Ser., 2009) p. 012020, arXiv:0903.2147v1 [hep-th].

[13] A. Chamblin, S. W. Hawking, and H. R. Reall, Phys. Rev. D **61**, 065007 (2000), arXiv:hep-th/9909205v2.

[14] N. Dadhich, R. Maartens, P. Papadopoulos, and V. Rezania, Phys. Lett. B **487**, 1 (2000), arXiv::hep-th/0003061v3.

[15] R. Casadio and B. Harms, Int. J. Mod. Phys. A **17**, 4635 (2002), arXiv:hep-th/0110255v2.

[16] A. Schelpe, "Randall-Sundrum black holes at colliders," (2008), arXiv:0809.2353v1 [hep-th].

[17] R. Casadio, S. Fabi, and B. Harms, Phys. Rev. D **80**, 084036 (2009), arXiv:0901.2948v3 [hep-ph].

[18] R. Casadio, S. Fabi, B. Harms, and O. Micu, "Theoretical survey of tidal-charged black holes at the LHC," (2009), arXiv:0911.1884v1 [hep-th].

[19] D. M. Gingrich and K. Martell, J. Phys. G: Nucl. Part. Phys. **35**, 035001 (2007), arXiv:0708.0647v3 [hep-ph].

[20] R. Plaga, "On the potential catastrophic risk from metastable quantum-black holes produced at particle colliders," (2008), arXiv:0808.1415v3 [physics.gen-ph].

[21] S. B. Giddings and M. L. Mangano, Phys. Rev. D **78**, 035009 (2008), arXiv:0806.3381v2 [hep-ph].

[22] S. B. Giddings and M. L. Mangano, "Comments on claimed risk from metastable black holes," (2008), arXiv:0808.4087v1 [hep-ph].

[23] B. Koch, M. Bleicher, and H. Stoecker, Phys. Lett. B **672**, 71 (2009), arXiv:0807.3349v2 [hep-ph].

[24] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle, and H. E. Swanson, Phys. Rev. Lett. **98**, 021101 (2007), arXiv:hep-ph/0611184v1.

[25] D. M. Gingrich, Int. J. Mod. Phys. A **21**, 6653 (2006), arXiv:hep-ph/0609055v2.