Equipartition of energy and the first law of thermodynamics at the apparent horizon

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Abstract

We apply the holographic principle and the equipartition law of energy to the apparent horizon of a Friedmann-Robertson-Walker universe and derive the Friedmann equation describing the dynamics of the universe. We also show that the equipartition law of energy can be interpreted as the first law of thermodynamics at the apparent horizon.

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I. INTRODUCTION

The entropy of a black hole is 1/4 of the area of the event horizon of the black hole measured in Planck units [1]. This area law of entropy was generalized to more general systems, and later was taken as a general principle: the so called holographic principle [2]. The strongest evidence of the holographic principle was provided by the AdS/CFT correspondence which states that all information about a gravitational system in a spatial region is encoded in its boundary [3]. On the other hand, the thermodynamic laws of black holes and de-Sitter space-time suggest a deep connection between gravitation and thermodynamics [4]. By using the area law of entropy for all local acceleration horizons, Einstein field equation was derived from the first law of thermodynamics [5].

Recently, the thermodynamic relation $S = E/2T$ [6] between the entropy S, active gravitational energy E and temperature T was reinterpreted as the equipartition law of energy $E = NT/2$, where N is the number of bits on the horizon [7]. Combining the equipartition law of energy and the holographic principle, Einstein field equation was derived [8]. Furthermore, gravity was explained as an entropic force caused by changes in the information associated with the positions of material bodies [8].

Motivated by Bekenstein's original thought experiment about black holes, Verlinde considered a small piece of a spherical holographic screen, and a particle with mass m approaching it from the side, the change of entropy near the screen was assumed to be [8]

$$\Delta S = 2\pi \frac{mc}{\hbar} \Delta x. \tag{1}$$

The effective force acting on the particle due to the change of entropy is

$$F \Delta x = T \Delta S. \tag{2}$$

Because an observer in an accelerated frame has the Unruh temperature

$$T = \frac{1}{2\pi} \hbar a, \tag{3}$$

so we get Newton's second law $F = ma$. Although it seems that we derived Newton's second law for any force, actually the derivation is satisfied for gravitational force only. Suppose the closed holographic screen with radius R can be divided into $N = A/(c_1 L_P^2)$ microscopic cells, where $L_P \equiv \sqrt{G\hbar}$ (we use units with $c = k_B = 1$ throughout this paper) is the Planck length and c_1 is a numerical factor whose value will be determined later. If each cell has c_2 microscopic configurations, then the entropy of the screen is [7]

$$S = N \ln c_2 = \frac{4 \ln c_2}{c_1} \frac{A}{4L_P^2}. \tag{4}$$

Once $4 \ln c_2 = c_1$ is chosen, the standard area law, $S = (A/4L_P^2)$ recovers. Then one assumes that each cell of area $c_1 L_p^2$ contributes an energy $\frac{1}{2}T$, according to the equipartition law, we get the total energy

$$\mathcal{E} = \frac{1}{2} N T = \frac{1}{2} \frac{A}{c_1 L_P^2} T = M. \tag{5}$$

so the Unruh temperature

$$T = \frac{2 M c_1 L_p^2}{4\pi R^2} = \frac{1}{2\pi} \hbar a, \tag{6}$$

and the acceleration

$$a = \frac{c_1 G M}{R^2}. \tag{7}$$

If $c_1 = 1$, then we get the Newton's law of gravitation. So we choose $c_1 = 1$.

By generalizing this approach to the relativistic case, Einstein equation can be derived [8]. As we discussed above, Einstein field equation can also be derived from the first law of thermodynamics, and the Friedmann equation for several gravity theories was derived from the first law of thermodynamics at the apparent horizon [9, 10], a natural question raised is whether there is any connection between the first law of thermodynamics and the equipartition law of energy. In this paper, we address this problem by applying this approach to the apparent horizon in cosmology. We first show that the first law of thermodynamics at the apparent horizon is equivalent to the equipartition law of energy at the apparent horizon, then we show that Friedmann equation can be derived from the equipartition law of energy and the holographic principle at the apparent horizon.

II. DERIVATION OF THE FRIEDMANN EQUATIONS

Let us focus on a (3+1)-dimensional Friedmann-Robertson-Walker (FRW) universe with the metric

$$ds^2 = h_{ab} dx^a dx^b + \tilde{r}^2 d\Omega_2^2, \tag{8}$$

where $\tilde{r} = a(t) r$ and the 2-dimensional metric $h_{ab} = \mathrm{diag}(-1, a^2/(1 - kr^2))$ with $k = 0, -1, 1$ corresponding to a flat, open and closed universe respectively. The apparent horizon (AH), which is defined by the relation $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r}=0$, turns out to be

$$\tilde{r}_A=\frac{1}{\sqrt{H^2+k/a^2}}, \tag{9}$$

where $H\equiv\dot{a}/a$ denotes the Hubble parameter. The apparent horizon with area $A=4\pi\tilde{r}_A^2$ carries $N=4\pi\tilde{r}_A^2/L_p^2$ bits of information. Suppose during the time interval dt, the radius of the apparent horizon evolves from $\tilde{r}_A$ to $\tilde{r}_A+d\tilde{r}_A$, then the change of the area of the AH is $dA=8\pi\tilde{r}_Ad\tilde{r}_A$. So the number of bit is increased by

$$dN=\frac{8\pi\tilde{r}_A}{L_p^2}d\tilde{r}_A. \tag{10}$$

On the other hand, the change of the Hawking temperature $T_A=\hbar/(2\pi\tilde{r}_A)$ is

$$dT_A=-\frac{\hbar}{2\pi\tilde{r}_A^2}d\tilde{r}_A. \tag{11}$$

From Eq. (5), we get the changes of the total energy

$$d\mathcal{E}=\frac{1}{2}NdT_A+\frac{1}{2}T_AdN=\frac{d\tilde{r}_A}{G}. \tag{12}$$

Note that the entropy of the apparent horizon is $S_A=\pi\tilde{r}_A^2/L_P^2$, so $T_AdS_A=d\tilde{r}_A/G$. This shows the equivalence between the equipartition law of energy and the first law of thermodynamics, $d\mathcal{E}=T_AdS_A$. From the definition of the apparent horizon (9), we get

$$d\tilde{r}_A=-H\tilde{r}_A^3\left(\dot{H}-\frac{k}{a^2}\right)dt. \tag{13}$$

Now we discuss the energy flow through the apparent horizon within a time interval dt. Because the energy-momentum tensor of the matter in the universe is a perfect fluid, $T_{\mu\nu}=(\rho+p)U_\mu U_\nu+pg_{\mu\nu}$, where ρ and p are the energy density and pressure, respectively, the energy flow is

$$d\mathcal{E}=4\pi\tilde{r}_A^2T_{\mu\nu}k^\mu k^\nu dt=4\pi\tilde{r}_A^3(\rho+p)Hdt, \tag{14}$$

where the (approximate) Killing vector or the (approximate) generator of the horizon, the future directed ingoing null vector field $k^\mu=(1,-Hr,0,0)$. Combining Eqs. (12), (13) and (14), we get the following equation

$$\dot{H}-\frac{k}{a^2}=-4\pi G(\rho+p). \tag{15}$$

Using the energy conservation equation

$$\dot{\rho}+3H(\rho+p)=0, \tag{16}$$

we get the Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho. \tag{17}$$

In the above formulae, an integration constant, which can be regarded as a cosmological constant, has been dropped out.

III. CONCLUSIONS

Motivated by the work on the origin of gravity [8], we derived the Friedmann equation from the equipartition law of energy and the holographic principle. We also show that the equipartition law of energy can be interpreted as the first law of thermodynamics at the apparent horizon. This suggests that the equipartition law of energy plays a fundamental role. Although there are many unresolved issues on Verlinde's proposal, it has, at least in some extent, transmitted a message that it is possible for the gravity to have a thermodynamic origin, and our universe in this context is driven by the so called entropic force.

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