

Solar System tests of Hořava-Lifshitz black holes

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(Dated: January 20, 2010)

In the present paper we consider the possibility of observationally testing Hořava gravity at the scale of the Solar System, by considering the classical tests of general relativity (perihelion precession of the planet Mercury, deflection of light by the Sun and the radar echo delay) for the Kehagias-Sfetsos asymptotically flat black hole solution. All these gravitational effects can be fully explained in the framework of the vacuum solution of Hořava gravity, and it is shown that the analysis of the classical general relativistic tests severely constrain the free parameter of the solution.

I. INTRODUCTION

Recently, Hořava proposed a renormalizable gravity theory in four dimensions which reduces to Einstein gravity with a non-vanishing cosmological constant in IR but with improved UV behaviors [1, 2]. The latter theory admits a Lifshitz scale-invariance in time and space, $t \rightarrow l^z t$ and $x^i \rightarrow l x^i$ (in particular, $z = 3$ for the present case), exhibiting a broken Lorentz symmetry at short scales, while at large distances higher derivative terms do not contribute, and the theory reduces to standard general relativity (GR). Since one of the most important aspects of the theory is a Lifshitz-type anisotropic scaling, it is often called Hořava-Lifshitz gravity. The Hořava theory has received a great deal of attention and since its formulation various properties and characteristics have been extensively analyzed, ranging from formal developments [3], cosmology [4], dark energy [5] and dark matter [6], and spherically symmetric or rotating solutions [7, 10–13].

The natural setting of Hořava-Lifshitz gravity is the ADM formalism, where the 4-dim metric is parameterized by the following

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt) . \quad (1)$$

N is the lapse function and N_i the shift function, respectively.

The IR-modified Hořava action is given by

$$S = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda_g K^2) - \frac{\kappa^2}{2\nu_g^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2\nu_g^2} \epsilon^{ijk} R_{il}^{(3)} \nabla_j R^{(3)l}_k \right. \\ \left. - \frac{\kappa^2 \mu^2}{8} R_{ij}^{(3)} R^{(3)ij} + \frac{\kappa^2 \mu^2}{8(3\lambda_g - 1)} \left(\frac{4\lambda_g - 1}{4} (R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2 \right) + \frac{\kappa^2 \mu^2 \omega}{8(3\lambda_g - 1)} R^{(3)} \right], \quad (2)$$

where κ , λ_g , ν_g , μ , ω and Λ_W are constant parameters. $R^{(3)}$ is the three-dimensional curvature scalar for g_{ij} ; K_{ij} is the extrinsic curvature given by

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i) , \quad (3)$$

and C^{ij} is the Cotton-York tensor, defined as

$$C^{ij} = \epsilon^{ikl} \nabla_k \left(R^{(3)j}_l - \frac{1}{4} R^{(3)} \delta_l^j \right) . \quad (4)$$

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The fundamental constants of the speed of light c , Newton's constant G , and the cosmological constant Λ are provided by the parameters of the theory and are defined as

$$c^2 = \frac{\kappa^2 \mu^2 |\Lambda_W|}{8(3\lambda_g - 1)^2}, \quad G = \frac{\kappa^2 c^2}{16\pi(3\lambda_g - 1)}, \quad \Lambda = \frac{3}{2} \Lambda_W c^2, \quad (5)$$

respectively.

There are basically four versions of Hořava gravity, namely, those with or without the ‘‘detailed balance condition’’, and with or without the ‘‘projectability condition’’. The ‘‘detailed balance condition’’ restricts the form of the potential in the 4-dim Lorentzian action to a specific form in terms of a 3-dim Euclidean theory. In a cosmological context, this condition leads to obstacles, and thus must be abandoned. In this context, the last term in Eq. (2) represents a ‘soft’ violation of the ‘detailed balance’ condition, which modifies the IR behavior. This IR modification term, $\mu^4 R^{(3)}$, with an arbitrary cosmological constant, represent the analogs of the standard Schwarzschild-(A)dS solutions, which were absent in the original Hořava model. The ‘‘projectability condition’’ essentially stems from the fundamental symmetry of the theory, i.e., the foliation-preserving diffeomorphism invariance, and must be respected. Foliation-preserving diffeomorphism consists of a 3-dim spatial and the space-independent time reparametrization. Note that the lapse function is essentially the gauge degree of freedom associated with the time reparametrization, so that it is natural to restrict it to be space-independent, i.e., $N = N(t)$.

IR-modified Hořava gravity seems to be consistent with the current observational data, but in order to test its viability more observational constraints are necessary [14, 15]. Thus, it is the purpose of the present paper to consider the classical tests (perihelion precession, light bending, and the radar echo delay, respectively) of general relativity for static gravitational fields in the framework of Hořava-Lifshitz gravity. To do this we shall adopt for the geometry outside a compact, stellar type object (the Sun), the static and spherically symmetric metric obtained by Kehagias and Sfetsos [12]. This work is essentially based on the paper [14].

II. STATIC AND SPHERICALLY SYMMETRIC BLACK HOLE SOLUTIONS

Consider a static and spherically symmetric solution, with the metric ansatz:

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (6)$$

By substituting the metric into the action, the resulting reduced Lagrangian, after angular integration, is given by

$$\mathcal{L} = \frac{\kappa^2 \mu^2}{8(1 - 3\lambda_g)} e^{(\nu+\lambda)/2} \left[(2\lambda_g - 1) \frac{(e^{-\lambda} - 1)^2}{r^2} + 2\lambda_g \frac{e^{-\lambda} - 1}{r} (\lambda' e^{-\lambda}) + \frac{\lambda_g - 1}{2} (\lambda' e^{-\lambda})^2 - 2(\omega - \Lambda_W)(1 - e^{-\lambda}(1 - r\lambda')) - 3\Lambda_W^2 r^2 \right]. \quad (7)$$

The above reduced Lagrangian yields the following equations of motions

$$(2\lambda_g - 1) \frac{(e^{-\lambda} - 1)^2}{r^2} - 2\lambda_g \frac{e^{-\lambda} - 1}{r} (-\lambda' e^{-\lambda}) + \frac{\lambda_g - 1}{2} (\lambda' e^{-\lambda})^2 - 2(\omega - \Lambda_W)[1 - e^{-\lambda}(1 - r\lambda')] - 3\Lambda_W^2 r^2 = 0, \quad (8)$$

$$\frac{(\nu' + \lambda')}{2} \left[(\lambda_g - 1)(\lambda' e^{-\lambda}) - 2\lambda_g \frac{e^{-\lambda} - 1}{r} + 2(\omega - \Lambda_W)r \right] + (\lambda_g - 1) \left[(-\lambda'' + (\lambda)') e^{-\lambda} - \frac{2(e^{-\lambda} - 1)}{r^2} \right] = 0, \quad (9)$$

by varying the functions ν and λ , respectively.

Now, imposing $\lambda_g = 1$, which reduces to the Einstein-Hilbert action in the IR limit, one obtains the following solution of the vacuum field equations in Hořava gravity,

$$e^{\nu(r)} = e^{-\lambda(r)} = 1 + (\omega - \Lambda_W)r^2 - \sqrt{r[\omega(\omega - 2\Lambda_W)r^3 + \beta]}, \quad (10)$$

where β is an integration constant. Throughout this work, we consider the Kehagias-Sfetsos (KS) asymptotically flat solution [12], i.e., $\beta = 4\omega M$ and $\Lambda_W = 0$:

$$e^{\nu(r)} = 1 + \omega r^2 - \omega r^2 \sqrt{1 + \frac{4M}{\omega r^3}}. \quad (11)$$

If the limit $4M/\omega r^3 \ll 1$, from Eq. (11) we obtain the standard Schwarzschild metric of general relativity, $e^{\nu(r)} = 1 - 2M/r$. There are two event horizons at $r_{\pm} = M \left[1 \pm \sqrt{1 - 1/(2\omega M^2)} \right]$. To avoid a naked singularity at the origin, impose the condition $\omega M^2 \geq \frac{1}{2}$. Note that in the GR regime, i.e., $\omega M^2 \gg 1$, the outer horizon approaches the Schwarzschild horizon, $r_+ \simeq 2M$, and the inner horizon approaches the central singularity, $r_- \simeq 0$.

III. SOLAR SYSTEM TESTS FOR HOŘAVA-LIFSHITZ BLACK HOLES

There are three fundamental tests which provide observational evidence for GR and its generalizations, namely, the precession of the perihelion of Mercury, the deflection of light by the Sun, and the radar echo delay observations. The latter have been used to successfully test the Schwarzschild solution of general relativity and some of its generalizations. In this Section, we consider these standard Solar System tests of general relativity in the case of the KS asymptotically flat solution [12] of Hořava-Lifshitz gravity. Throughout this Section we use the natural system of units with $G = c = 1$. The analysis outlined below is essentially based on the paper [14], and a similar analysis has been carried out in the context of braneworld models [16].

A. The perihelion precession of Mercury

The line element, given by Eq. (6), provides the following equation of motion for r

$$\dot{r}^2 + e^{-\lambda} \frac{L^2}{r^2} = e^{-\lambda} (E^2 e^{-\nu} - 1), \quad (12)$$

where $e^\nu \dot{t} = E = \text{constant}$ and $r^2 \dot{\phi} = L = \text{constant}$.

The analysis below is simplified by applying a change of variable $r = 1/u$, and using $d/ds = Lu^2 d/d\phi$. We further formally represent $e^{-\lambda} = 1 - f(u)$, so that the equation of motion (12) takes the following form

$$\frac{d^2 u}{d\phi^2} + u = F(u), \quad (13)$$

where $F(u) = \frac{1}{2} \frac{dG(u)}{du}$, and $G(u)$ is defined as

$$G(u) \equiv f(u)u^2 + \frac{E^2}{L^2} e^{-\nu-\lambda} - \frac{1}{L^2} e^{-\lambda}. \quad (14)$$

A circular orbit $u = u_0$ is given by the root of the equation $u_0 = F(u_0)$. Any deviation $\delta = u - u_0$ from a circular orbit must satisfy the equation

$$\frac{d^2 \delta}{d\phi^2} + \left[1 - \left(\frac{dF}{du} \right)_{u=u_0} \right] \delta = O(\delta^2), \quad (15)$$

which is obtained by substituting $u = u_0 + \delta$ into Eq. (13). Therefore, in the first order in δ , the trajectory is given by

$$\delta = \delta_0 \cos \left(\sqrt{1 - \left(\frac{dF}{du} \right)_{u=u_0}} \phi + \beta \right), \quad (16)$$

where δ_0 and β are constants of integration.

The variation of the orbital angle from one perihelion to the next is $\phi = 2\pi/(1 - \sigma)$, where σ is the perihelion advance, which represents the rate of advance of the perihelion. σ is given by $\sigma = 1 - \sqrt{1 - (dF/du)_{u=u_0}}$, or for small $(dF/du)_{u=u_0}$, by $\sigma = \frac{1}{2} \left(\frac{dF}{du} \right)_{u=u_0}$. As the planet advances ϕ radians in its orbit, its perihelion advances $\sigma\phi$ radians. For a complete rotation we have $\phi \approx 2\pi(1 + \sigma)$, and the advance of the perihelion is $\delta\phi = \phi - 2\pi \approx 2\pi\sigma$.

For the KS solution, the perihelion precession is given by:

$$\delta\phi = \pi \frac{3\sqrt{\omega_0} \left\{ 2(x_0^3 + \omega_0)x_0^5 + b^2 \left[2x_0^6 + \left(6\omega_0 - 4\sqrt{\omega_0(4x_0^3 + \omega_0)} \right) x_0^3 + \omega_0^2 - \sqrt{\omega_0^3(4x_0^3 + \omega_0)} \right] \right\}}{x_0^4 (4x_0^3 + \omega_0)^{3/2}}. \quad (17)$$

with the dimensionless parameters defined as $\omega_0 = M^2\omega$, and $x_0 = u_0 M$. In the ‘‘Post-Newtonian’’ limit $4x_0^3/\omega_0 \ll 1$, we obtain the GR result $\delta\phi_{GR} = 6\pi b^2$, where $b^2 = M/a(1 - e^2)$. The variation of the perihelion precession angle as a function of ω_0 is represented in Fig. 1.

The observed value of the perihelion precession of the planet Mercury is $\delta\phi_{Obs} = 43.11 \pm 0.21$ arcsec per century [17]. Therefore the range of variation of the perihelion precession is $\delta\phi_{Obs} \in (42.90, 43.32)$. This range of observational values fixes the range of ω_0 as

$$\omega_0 \in (6.95431 \times 10^{-16}, 6.98821 \times 10^{-16}). \quad (18)$$

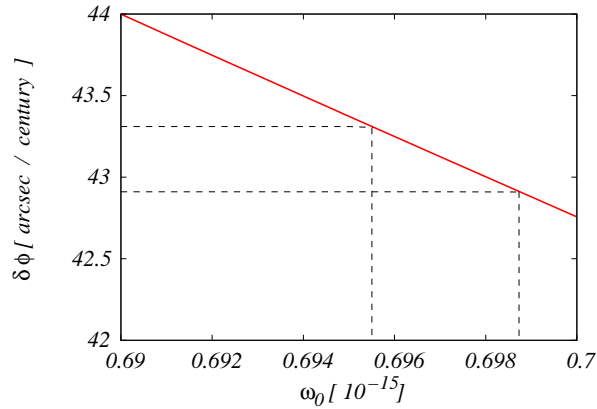


FIG. 1: Variation of the precession angle $\delta\phi$ as a function of ω_0 .

B. The deflection of light

The deflection angle of a light ray by the gravitational field of a massive body in a spherically symmetric geometry is derived in [18], and is given by $\Delta\phi = 2|\phi(r_0) - \phi(\infty)| - \pi$, with

$$\phi(r) = \phi(\infty) + \int_r^\infty \frac{e^{\lambda/2}}{\sqrt{e^{\nu(r_0)-\nu(r)} (r/r_0)^2 - 1}} \frac{dr}{r}, \quad (19)$$

where r_0 is the distance of the closest approach. Here we have taken into account that in the absence of a gravitational field a light ray propagates along a straight line. In the case of the Sun, the deflection angle of a light ray is $\Delta\phi = 1.72752''$.

The deflection angle of light rays passing nearby the Sun in the KS geometry is given by

$$\phi(x_0) = \phi(\infty) + \int_1^\infty \frac{\left[1 + \omega_0 x_0^2 x^2 - \sqrt{x_0 x (\omega_0^2 x_0^3 x^3 + 4\omega_0)}\right]^{-1/2}}{\sqrt{\left[1 + \omega_0 x_0^2 - \sqrt{x_0 (\omega_0^2 x_0^3 + 4\omega_0)}\right] x^2 / \left[1 + \omega_0 x_0^2 x^2 - \sqrt{x_0 x (\omega_0^2 x_0^3 x^3 + 4\omega_0)}\right] - 1}} \frac{dx}{x}, \quad (20)$$

where $\omega = \omega_0/M^2$, $r_0 = x_0 M$, and considering $r = r_0 x$.

For the Sun, by taking $r_0 = R_\odot = 6.955 \times 10^{10}$ cm, we find for x_0 the value $x_0 = 4.71194 \times 10^5$. The variation of the deflection angle $\Delta\phi = 2|\phi(x_0) - \phi(\infty)| - \pi$ is represented, as a function of ω_0 , in Fig. 2.

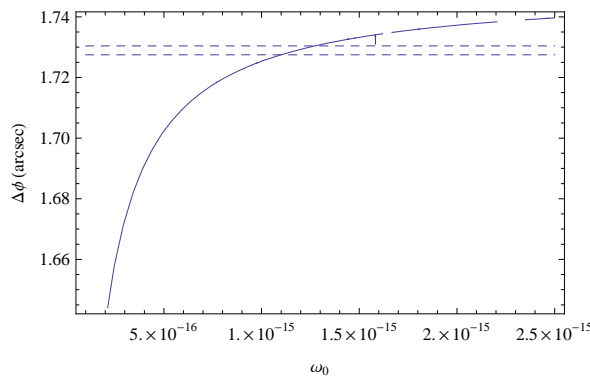


FIG. 2: The light deflection angle $\Delta\phi$ (in arcseconds) as a function of the parameter ω_0 .

The best available data come from long baseline radio interferometry [19], which gives $\delta\phi_{LD} = \delta\phi_{LD}^{(GR)} (1 + \Delta_{LD})$, with $\Delta_{LD} \leq 0.0017$, where $\delta\phi_{LD}^{(GR)} = 1.7275$ arcsec. Thus, the observational constraints of light deflection restricts the value of ω_0 to

$$\omega_0 \in (1.1 \times 10^{-15}, 1.3 \times 10^{-15}) . \quad (21)$$

C. Radar echo delay

The aim of this test is to measure the time required for radar signals to travel to an inner planet or satellite in two circumstances, namely, (i) when the signal passes very near the Sun and, (ii) when the ray does not go near the Sun. The time of travel of light t_0 between two planets, situated far away from the Sun, is given by $t_0 = \int_{-l_1}^{l_2} dy$, where l_1 and l_2 are the distances of the planets to the Sun, respectively.

If the light travels close to the Sun, the time travel is

$$t = \int_{-l_1}^{l_2} \frac{dy}{v} = \int_{-l_1}^{l_2} e^{[\lambda(r) - \nu(r)]/2} dy, \quad (22)$$

and the time difference is given by

$$\Delta t = t - t_0 = \int_{-l_1}^{l_2} \left\{ e^{[\lambda(r) - \nu(r)]/2} - 1 \right\} dy = \int_{-l_1}^{l_2} \left\{ e^{[\lambda(\sqrt{y^2 + R_\odot^2}) - \nu(\sqrt{y^2 + R_\odot^2})]/2} - 1 \right\} dy, \quad (23)$$

with $r = \sqrt{y^2 + R_\odot^2}$, and R_\odot is the radius of the Sun.

Recent measurements of the frequency shift of radio photons to and from the Cassini spacecraft as they passed near the Sun have greatly improved the observational constraints on the radio echo delay. For the time delay of the signals emitted on Earth, and which graze the Sun, one obtains $\Delta t_{RD} = \Delta t_{RD}^{(GR)} (1 + \Delta_{RD})$, with $\Delta_{RD} \leq (2.1 \pm 2.3) \times 10^{-5}$ [20]. The standard GR radar echo delay value is $\Delta t_{RD}^{(GR)} \approx 4M_\odot \ln(4l_1 l_2 / R_\odot^2) \approx 2.4927 \times 10^{-4}$ s.

For the KS solution, by introducing a new variable ξ defined as $y = 2\xi M_\odot$, and by representing ω as $\omega = \omega_0 / M_\odot^2$, we obtain for the time delay the following expression

$$\Delta t_{RD} = 16M_\odot \omega_0 \int_{-\xi_1}^{\xi_2} \frac{(\xi^2 + a^2) \left[\sqrt{1 + (1/2\omega_0)(\xi^2 + a^2)^{-3/2}} - 1 \right]}{1 - 4\omega_0(\xi^2 + a^2) \left[\sqrt{1 + (1/2\omega_0)(\xi^2 + a^2)^{-3/2}} - 1 \right]} d\xi, \quad (24)$$

where $a^2 = R_\odot^2 / 4M_\odot^2$, $\xi_1 = l_1 / 2M_\odot$, and $\xi_2 = l_2 / 2M_\odot$, respectively. The variation of the time delay as a function of ω_0 is represented in Fig. 3.

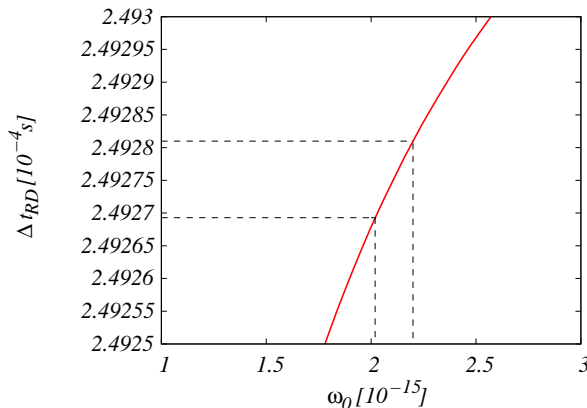


FIG. 3: Variation of the time delay Δt_{RD} as a function of ω_0 .

The observational values of the radar echo delay are consistent with the KS black hole solution in Hořava-Lifshitz gravity if

$$\omega_0 \in (2 \times 10^{-15}, 3 \times 10^{-15}). \quad (25)$$

IV. CONCLUSION

In the present work, we have considered the observational and experimental possibilities for testing, at the level of the Solar System, the Kehagias and Sfetsos solution of the vacuum field equations in Hořava-Lifshitz gravity. In this

context, the parameter ω , having the physical dimensions of length^{-2} , is constrained by the perihelion precession of Mercury to a value of $\omega = 7 \times 10^{-16}/M_{\odot}^2 = 3.212 \times 10^{-26} \text{ cm}^{-2}$. The deflection angle of the light rays by the Sun can be fully explained in Hořava-Lifshitz gravity with the parameter ω having the value $\omega = 10^{-15}/M_{\odot}^2 = 4.5899 \times 10^{-26}$, while the radar echo delay experiment suggests a value of $\omega = 2 \times 10^{-15}/M_{\odot}^2 = 9.1798 \times 10^{-26} \text{ cm}^{-2}$. From these values we can give an estimate of ω as

$$\omega = (5.660 \pm 3.1) \times 10^{-26} \text{ cm}^{-2}. \quad (26)$$

This requires a very precise fine tuning of this constant at the level of the Solar System.

Thus, the study of the classical tests of general relativity provide a very powerful method for constraining the allowed parameter space of Hořava-Lifshitz gravity solutions, and to provide a deeper insight into the physical nature and properties of the corresponding spacetime metrics.

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