

COSMOLOGICAL SINGULARITIES IN FLRW SPACETIMES

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In this talk we review the appearance of new types of singularities (big rip, sudden singularities...) in FLRW cosmological models that have arisen on considering explanations for accelerated expansion of our universe.

Keywords: Cosmological singularities, dark energy, geodesics.

1. Introduction

During the last decade the observational evidence in favour of accelerated expansion of our universe has increased considerably.¹ In order to cope with this fact two lines of development have been proposed: either general relativity is to be modified² or our universe is pervaded by an exotic source named dark energy,³ which is responsible for the accelerated expansion. Violation of classical energy conditions by this fluid has made feasible a number of new scenarios⁴ for the final fate of our universe (big rip,⁵ sudden singularities,⁶ big brake,⁷ big freeze,⁸ inaccessible singularities,⁹ directional singularities,¹⁰ w -singularities,¹¹ in braneworld models¹²...), which had not been considered previously. In this talk we deal with the appearance and strength of these singular behaviours in FLRW cosmological spacetimes from the point of view of geodesic incompleteness.

2. Behaviour of singularities

We choose coordinates for which the FLRW metric is written as

$$ds^2 = -dt^2 + a^2(t) \{ f^2(r) dr^2 + r^2 d\Omega^2 \}, \quad f^2(r) = \frac{1}{1 - kr^2}, \quad k = 0, \pm 1,$$

and we assume that the scale factor $a(t)$ admits a generalised power expansion¹³ in the vicinity of an event at t_0 ,

$$a(t) = c_0 |t - t_0|^{\eta_0} + c_1 |t - t_1|^{\eta_1} + \dots,$$

where $\eta_0 < \eta_1 < \dots$, $c_0 > 0$. Models with oscillatory behaviour¹⁴ might drop out of this scheme. At lowest order three different qualitative behaviours arise:

- $\eta_0 > 0$: a vanishing scale factor at t_0 means a Big Bang or Big Crunch.

- $\eta_0 = 0$: a finite scale factor at t_0 means that either $a(t)$ is analytical and the event is regular or on the other hand a weak or sudden singularity appears.
- $\eta_0 < 0$: a diverging scale factor at t_0 means a Big Rip singularity.

Geodesics are parametrised by their proper time τ , $(t(\tau), r(\tau), \theta(\tau), \phi(\tau))$,

$$t' = \sqrt{\delta + \frac{P^2}{a^2(t)}}, \quad r' = \pm \frac{P}{a^2(t)f(r)},$$

may be simplified in terms of a constant of geodesic motion P and $\delta = 0, 1$ for null and timelike geodesics. The null case $\delta = 0$ is straightforwardly solved

$$a(t)t' = P \Rightarrow \int_{t_0}^t a(t') dt' = P(\tau - \tau_0),$$

and the geodesic is complete if and only if the scale factor is an integrable function.

Dealing with finite bodies, a singularity is strong¹⁵ if tidal forces are capable of disrupt them. Following Tipler, a singularity is strong if their volume tends to zero. Following Królak, just a negative derivative of the volume is required. The results are consigned in Table 1.¹⁶ Conformal diagrams for these models can be found in.¹⁷

Table 1. Strength of singularities in FLRW models.

η_0	η_1	k	c_0	Tipler	Królak
$(-\infty, 0)$	(η_0, ∞)	$0, \pm 1$	$(0, \infty)$	Strong	Strong
0	$(0, 1)$	$0, \pm 1$	$(0, \infty)$	Weak	Strong
0	$[1, \infty)$	$0, \pm 1$	$(0, \infty)$	Weak	Weak
$(0, 1)$	(η_0, ∞)	$0, \pm 1$	$(0, \infty)$	Strong	Strong
1	$(1, \infty)$	0, 1	$(0, \infty)$	Strong	Strong
1	$(1, \infty)$	-1	$(0, 1) \cup (1, \infty)$	Strong	Strong
1	$(1, 3)$	-1	1	Weak	Strong
1	$[3, \infty)$	-1	1	Weak	Weak
$(1, \infty)$	(η_0, ∞)	$0, \pm 1$	$(0, \infty)$	Strong	Strong

3. Conclusions

We have obtained a classification of singular events in FLRW models in terms of their strength and the exponents of an expansion of the scale factor around an event at t_0 . In models with $\eta_0 \leq -1$, null geodesics avoid the Big Rip singularity. The only models which allow regular behaviour close to t_0 are those with $\eta_0 = 0$ (de Sitter, sudden. . .) and with $\eta_0 = 1$, $k = -1$, $c_0 = 1$ (Milne).

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