

Supersymmetric Intersecting Branes on the Waves

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Abstract

We construct a general family of supersymmetric solutions in time- and space-dependent wave backgrounds in general supergravity theories describing single and intersecting p -branes embedded into time-dependent dilaton-gravity plane waves of an arbitrary (isotropic) profile, with the brane world-volume aligned parallel to the propagation direction of the wave. We discuss how many degrees of freedom we have in the solutions. We also propose that these solutions can be used to describe higher-dimensional time-dependent “black holes”, and discuss their property briefly.

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1 Introduction

The understanding of the fundamental nature and quantum properties of spacetime is one of the most important questions in theoretical physics. An example of such problems is the space-time singularities that general relativity predicts especially in time-dependent setting. However, time-dependent solutions are rather difficult subject in the effective string theories [1]-[6]. Dilaton-gravity plane waves provide a rare example of tractable strongly curved (possibly singular) time-dependent space-time backgrounds. They also allow a formulation of (time-dependent) matrix theories of quantum gravity [7]-[36]. Other time-dependent brane solutions, though non-supersymmetric, are discussed in [37, 38].

Brane supergravity solutions also plays important role [39]-[43], since they lead to the formulation of the AdS/CFT correspondence. Hence, it is important to derive supergravity p -branes embedded into dilaton-gravity plane waves.

In a recent paper [44], the simplest of these solutions which are supersymmetric configurations corresponding to time-dependent extremal p -branes aligned along the propagation direction of the plane wave were obtained generalizing earlier work [6, 29]. These solutions are restricted to single brane solutions. It has been known that this class of solutions can be extended to much more general intersecting brane configurations, and it would be interesting to see if the solutions can be generalized to such general configurations.

In our previous paper [36], we have given closely related solutions for intersecting branes with wave, but we gave supersymmetric solutions without wave as examples of our solutions. This actually restrict possible solutions considerably. In this paper we revisit this class of solutions, and show that our previous solutions actually give a very general family of solutions which are wider than those for single brane in Ref. [44]. Our solutions involve many arbitrary functions and we examine how many degrees of freedom we have. We also discuss some physical applications of these solutions to black hole physics. In particular, we consider intersecting D1-D5 brane system, and show that we can effectively compactify it to five dimensions though close to the possible horizon six-dimensional nature of the solution reappears.

This paper is organized as follows. In the next section, we briefly summarize our solutions derived in our previous paper [36], where we set wave profile to zero. This gave a strong restriction on the solutions. However, here we show that relaxing this condition we can obtain much more general solutions including those in Ref. [44]. In sect. 3, we discuss examples of single brane and two intersecting brane solutions, and use coordinate reparametrization to count the number of arbitrary functions involved in the solutions. This shows that our solutions are quite general one. In sect. 4, we discuss D1-D5 system as an example. We show that this can be effectively compactified to five dimensions by using periodic functions in one of the light-like coordinates. Still we show that we can avoid closed time-like curve by choosing suitable parameters in the compactification. This produces a black hole system which looks like a fluctuating black hole. Close to the horizon, the solution exhibits six-dimensional nature as is usual for any compactified theory. We calculate Ricci scalar curvature and Kretschmann invariant of this black hole and find that there is no curvature singularity at the “horizon” in six dimensions. There remain several interesting questions in this kind of solutions, but we only mention some of these, leaving detailed study for future. The final section is devoted to concluding remarks.

2 Time-dependent brane system in supergravity

The low-energy effective action for the supergravity system coupled to dilaton and n_A - form field strength is given by

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[R - \frac{1}{2} (\partial\Phi)^2 - \sum_{A=1}^m \frac{1}{2n_A!} e^{a_A \Phi} F_{n_A}^2 \right], \quad (2.1)$$

where G_D is the Newton constant in D dimensions and g is the determinant of the metric. The last term includes both RR and NS-NS field strengths, and $a_A = \frac{1}{2}(5 - n_A)$ for RR field strength and $a_A = -1$ for NS-NS 3-form. In the eleven-dimensional supergravity, there is a four-form and no dilaton. We put fermions and other background fields to be zero.

We take the following metric:

$$ds_D^2 = e^{2\Xi(u,r)} [-2dudv + K(u,r)du^2] + \sum_{\alpha=1}^{d-2} e^{2Z_\alpha(u,r)} (dy^\alpha)^2 + e^{2B(u,r)} (dr^2 + r^2 d\Omega_{\tilde{d}+1}^2), \quad (2.2)$$

where $D = d + \tilde{d} + 2$, the coordinates u, v and y^α , ($\alpha = 1, \dots, d-2$) parameterize the d -dimensional worldvolume where the branes belong, and the remaining $\tilde{d} + 2$ coordinates r and angles are transverse to the brane worldvolume, $d\Omega_{\tilde{d}+1}^2$ is the line element of the $(\tilde{d} + 1)$ -dimensional sphere. Note that u and v are null coordinates. The metric components Ξ, Z_α, B and the dilaton Φ and K are assumed to be functions of u and r . In our previous paper [36], we took K depending on y_α as well, but this dependence is dropped for simplicity. For the field strength backgrounds, we take

$$F_{n_A} = E'_A(u, r) du \wedge dv \wedge dy^{\alpha_1} \wedge \dots \wedge dy^{\alpha_{q_A-1}} \wedge dr, \quad (2.3)$$

where $n_A = q_A + 2$. Throughout this paper, the dot and prime denote derivatives with respect to u and r , respectively. The ansatz (2.3) means that we have an electric background. We could, however, also include magnetic background in the same form as the electric one.

In our previous paper [36], we have shown that the solutions to the field equations are given by

$$ds_D^2 = \prod_B H_B^{\frac{2(q_B+1)}{\Delta_B}} \left[e^{2\xi(u)} \prod_A H_A^{-\frac{2(D-2)}{\Delta_A}} (-2dudv + K(u,r)du^2) + \sum_{\alpha=1}^{d-2} \prod_A H_A^{-\frac{2\gamma_A^{(\alpha)}}{\Delta_A}} e^{2\zeta_\alpha(u)} (dy^\alpha)^2 + e^{2\beta(u)} (dr^2 + r^2 d\Omega_{\tilde{d}+1}^2) \right],$$

$$E_A = \sqrt{\frac{2(D-2)}{\Delta_A}} H_A^{-1}, \quad \Phi = \sum_A \frac{\epsilon_A a_A (D-2)}{\Delta_A} \ln H_A + \phi(u), \quad (2.4)$$

where H_A is a harmonic function

$$H_A = h_A(u) + \frac{Q_A}{r^{\tilde{d}}}, \quad (2.5)$$

with h_A being an arbitrary function of u and Q_A a constant, $\epsilon_A = +1(-1)$ is for electric (magnetic) backgrounds and

$$\gamma_A^{(\alpha)} = \begin{cases} D-2 \\ 0 \end{cases} \quad \text{for} \quad \begin{cases} y^\alpha \text{ belonging to } q_A\text{-brane} \\ \text{otherwise} \end{cases}. \quad (2.6)$$

We have two constraints still to be satisfied:

$$\epsilon_A a_A \phi + 2 \sum_{\alpha \notin q_A} \zeta_\alpha + 2\tilde{d}\beta = 0, \quad (2.7)$$

$$\left(r^{\tilde{d}+1} K'\right)' = -2e^{-2(\xi-\beta)r^{\tilde{d}+1}} \prod_A H_A^{2(D-2)/\Delta_A} [W(u, r) + V(u)], \quad (2.8)$$

where

$$W(u, r) \equiv \sum_{A,B} \frac{(D-2)^2}{\Delta_A \Delta_B} \left(\frac{\Delta_A}{D-2} \delta_{AB} + 2 \right) (\ln H_A)' (\ln H_B)' + 2 \sum_A \frac{D-2}{\Delta_A} (\ln H_A)'' + 4(D-2)(\dot{\beta} - \dot{\xi}) \sum_A \frac{(\ln H_A)'}{\Delta_A}, \quad (2.9)$$

$$V(u) \equiv \sum_{\alpha=1}^{d-2} \left(\ddot{\zeta}_\alpha + \dot{\zeta}_\alpha^2 \right) + (\tilde{d}+2) \left(\ddot{\beta} + \dot{\beta}^2 \right) - 2\dot{\xi} \left[\sum_{\alpha=1}^{d-2} \dot{\zeta}_\alpha + (\tilde{d}+2)\dot{\beta} \right] + \frac{1}{2}(\dot{\phi})^2, \quad (2.10)$$

In our previous paper [36], we first chose K and then solved Eq. (2.8). This gave rather strong constraints on possible solutions. However we can obtain more general solutions if Eq. (2.8) is regarded as the equation for K when other metric functions are given, which is an elliptic type differential equation with respect to r . Here we generalize our previous solutions [36] in this viewpoint. This approach has recently been taken in Ref. [44] for a single brane. Our solutions here include single and intersecting brane solutions as well as wider solutions including more arbitrary functions than those in [44].

3 Solutions with time-dependent harmonic functions

In this section, we present nontrivial solutions with both r - and u -dependent harmonic functions H_A in (2.5).

Before presenting our solutions, we discuss gauge freedom of null-coordinate transformation. Under the coordinate transformation

$$u = X(\tilde{u}), \quad v = \tilde{v} + Y(\tilde{u}), \quad (3.1)$$

where X and Y are arbitrary functions of u , we recover the same solution (2.4), (2.5), (2.7), and (2.8), by replacing K and ξ with

$$\tilde{K}(\tilde{u}, r) = K(X(\tilde{u}), r) \frac{dX}{d\tilde{u}} - 2 \frac{dY(\tilde{u})}{d\tilde{u}}, \quad (3.2)$$

$$\tilde{\xi}(\tilde{u}) = \xi(X(\tilde{u})) + \frac{1}{2} \ln \left(\frac{dX}{d\tilde{u}} \right), \quad (3.3)$$

respectively. Using X and Y , we can gauge away ξ and a function of u in K in our solutions. We will discuss more details of this procedure in the concrete examples shortly.

For all branes in M-theory and superstring theories, we have the following relation

$$\frac{2(D-2)}{\Delta_A} = 1. \quad (3.4)$$

In what follows, we assume this relation. In the following subsections, we present concrete examples of a single-brane and two intersecting-brane systems.

3.1 Single brane

We first consider a single A -brane. In this case, (2.8) gives

$$\left(r^{\tilde{d}+1}K'\right)' = -2e^{-2(\xi-\beta)} \left[r^{\tilde{d}+1} \left(\ddot{h}_A + 2(\dot{\beta} - \dot{\xi})\dot{h}_A + V(u)h_A \right) + Q_A V(u)r \right], \quad (3.5)$$

upon substituting (2.5) and sorting out the terms in the orders of r . We can integrate (3.5) to obtain

$$K(u, r) = \frac{A_1(u)}{2(\tilde{d}+2)}r^2 - \frac{B_1(u)}{2(\tilde{d}-2)}r^{-(\tilde{d}-2)} - \frac{C_1(u)}{\tilde{d}}r^{-\tilde{d}} + D_1(u), \quad (3.6)$$

where

$$A_1(u) = -2e^{-2(\xi-\beta)} \left(\ddot{h}_A + 2(\dot{\beta} - \dot{\xi})\dot{h}_A + V(u)h_A \right) \quad (3.7)$$

$$B_1(u) = -2e^{-2(\xi-\beta)}Q_A V(u) \quad (3.8)$$

and $C_1(u)$ and $D_1(u)$ are arbitrary functions of u .

In the present general solutions, we have $(d+4)$ arbitrary functions: the metric functions $\xi(u), \zeta_\alpha(u), \beta(u)$, the dilaton field $\phi(u)$, $h_A(u)$, $C_1(u)$ and $D_1(u)$. We also have one constraint (2.7) for those functions. For a single brane, there is no $\zeta_\alpha (\alpha \notin q_A)$, so (2.7) gives

$$\beta(u) = -\frac{\epsilon^{AA}a_A}{2\tilde{d}}\phi(u). \quad (3.9)$$

Using the function Y in the above coordinate transformation, D_1 can be gauged away. This was also noted in Ref. [44]. On the other hand, $X(\tilde{u})$ can be used to gauge away ξ , or choose any function of u as ξ . Hence there are $(d+1)$ degrees of freedom in the present single brane system. We note that the term C_1 was not considered in Ref. [44]. For a single brane, it is natural to take ξ and all ζ_α equal. Still we have four arbitrary functions.

If we set $A_1(u) = B_1(u) = 0$, we recover our previous solutions [36], while, if we set $C_1(u) = 0$, we find the solutions by Craps et al. [44]. Indeed, one can check that their solution corresponds to the specific choice

$$h_A(u) = e^{-f(u)}, \quad \xi(u) = \zeta_\alpha = -\frac{11-p}{8}f(u), \quad \beta(u) = \frac{p-3}{8}f(u), \quad \phi(u) = \frac{7-p}{4}f(u) \quad (3.10)$$

with the correspondence (left is our notation)

$$q_A \leftrightarrow p, \quad d-1 \leftrightarrow p, \quad \tilde{d} \leftrightarrow 7-p, \quad (3.11)$$

with only one arbitrary function $f(u)$, whereas ours have four.

As an example, let us consider D3-brane ($d=4$). In this case, $a_A=0$ and we have five arbitrary functions of u ; $\zeta_1, \zeta_2, \phi, h_3$ and C_1 . If we set $\xi = \zeta_1 = \zeta_2 = f(u)/2$, we find the similar solution in [20, 30], although h_3 depends on u .

3.2 Intersecting two branes

Let us consider two intersecting branes A and B . In this case, (2.8) gives

$$\left(r^{\tilde{d}+1}K'\right)' = -2e^{-2(\xi-\beta)}r^{\tilde{d}+1}H_AH_B \left[\frac{\dot{h}_A\dot{h}_B}{H_AH_B} + \frac{\ddot{h}_A}{H_A} + \frac{\ddot{h}_B}{H_B} + 2(\dot{\beta} - \dot{\xi}) \left(\frac{\dot{h}_A}{H_A} + \frac{\dot{h}_B}{H_B} \right) + V(u) \right]. \quad (3.12)$$

Substituting (2.5) and sorting out the terms in the orders of r , we find

$$\left(r^{\tilde{d}+1}K'\right)' = A_2(u)r^{\tilde{d}+1} + B_2(u)r + \frac{C_2(u)}{r^{\tilde{d}-1}}, \quad (3.13)$$

where

$$\begin{aligned} A_2(u) &= -2e^{-2(\xi-\beta)} \left[\dot{h}_A \dot{h}_B + \ddot{h}_A h_B + \ddot{h}_B h_A + 2(\dot{\beta} - \dot{\xi})(\dot{h}_A h_B + \dot{h}_B h_A) + V h_A h_B \right] \\ B_2(u) &= -2e^{-2(\xi-\beta)} \left[\ddot{h}_A Q_B + \ddot{h}_B Q_A + 2(\dot{\beta} - \dot{\xi})(\dot{h}_A Q_B + \dot{h}_B Q_A) + V(h_A Q_B + h_B Q_A) \right] \\ C_2(u) &= -2e^{-2(\xi-\beta)} V Q_A Q_B \end{aligned} \quad (3.14)$$

Integrating Eq. (3.13), we find

$$K = \frac{A_2(u)}{2(\tilde{d}+2)} r^2 - \frac{B_2(u)}{2(\tilde{d}-2)} r^{-(\tilde{d}-2)} + \frac{C_2(u)}{2(\tilde{d}-2)(\tilde{d}-1)} r^{-2(\tilde{d}-1)} - \frac{D_2(u)}{\tilde{d}} r^{-\tilde{d}} + E_2(u), \quad (3.15)$$

for the case that \tilde{d} is not equal to 1 nor 2, where $D_2(u)$ and $E_2(u)$ are arbitrary functions. In the cases of $\tilde{d} = 1$ and 2, we find

$$K = \frac{A_2(u)}{6} r^2 + \frac{B_2(u)}{2} r + C_2(u) \ln r - \frac{D_2(u)}{r} + E_2(u), \quad (3.16)$$

and

$$K = \frac{A_2(u)}{8} r^2 + \frac{1}{2} \left(B_2(u) - \frac{C_2}{r^2} \right) \ln r - \frac{C_2(u) + 2D_2(u)}{4r^2} + E_2(u), \quad (3.17)$$

respectively. The latter case corresponds to an M2-M5 intersecting brane system. If we set $A_2(u) = 0$, $B_2(u) = 0$ and $C_2(u) = 0$, we again recover our previous solutions [36].

In the similar way to the single brane case, we shall gauge away E_2 and ξ in what follows. Compared with the single brane system, our intersecting brane system has additional functions $h_B(u)$, but there is one additional constraint (2.7) for the additional brane. Hence we are again left with $(d+1)$ arbitrary functions in the present system.

Let us give some concrete examples. The D1-D5-brane is given by

$$\begin{aligned} ds^2 &= H_1^{\frac{1}{4}} H_5^{\frac{3}{4}} \left[H_1^{-1} H_5^{-1} (-2dudv + K(u, r) du^2) + H_5^{-1} \sum_{\alpha=1}^4 e^{2\zeta_\alpha(u)} dy_\alpha^2 \right. \\ &\quad \left. + e^{2\beta(u)} (dr^2 + r^2 d\Omega_3^2) \right], \\ \Phi &= \ln \left(\frac{H_1}{H_5} \right)^{\frac{1}{2}} + \phi(u), \end{aligned} \quad (3.18)$$

where

$$H_A = h_A(u) + \frac{Q_A}{r^2} \quad (A = 1, 5), \quad (3.19)$$

and (3.17) without E_2 . It also follows from (2.7) that

$$\beta(u) = \frac{1}{4} \phi(u) = - \sum_{\alpha=1}^4 \zeta_\alpha(u). \quad (3.20)$$

This solution has seven arbitrary functions $h_1(u), h_5(u), \zeta_\alpha(u)$, and $D_2(u)$ in K , while A_2, B_2 , and C_2 in K are given by Eq. (3.14) with $\xi = 0$.

If we assume $V(u) = 0$, (2.7) together with (2.10) yields $\zeta_\alpha = \beta = \phi = 0$. Then $K(u, r)$ is found to be

$$K(u, r) = \frac{\bar{A}_2(u)}{8}r^2 + \frac{\bar{B}_2(u)}{2}\ln r + \bar{D}_2(u)r^{-2}, \quad (3.21)$$

where

$$\begin{aligned} \bar{A}_2(u) &\equiv -8 \left[h_1^{\frac{1}{2}} \left(\dot{h}_5 h_1^{\frac{1}{2}} \right)' + h_5^{\frac{1}{2}} \left(\dot{h}_1 h_5^{\frac{1}{2}} \right)' \right], \\ \bar{B}_2(u) &\equiv -8 \left(Q_5 \dot{h}_1 + Q_1 \dot{h}_5 \right)', \end{aligned} \quad (3.22)$$

and $\bar{D}_2(u)$ is an arbitrary function of u . In the case of $\bar{B}_2 = 0$, K is called asymptotically Brinkmann form. There remain three arbitrary functions of u ; $h_1(u), h_5(u)$ and $\bar{D}_2(u)$.

Next let us consider D2-D6-brane solution:

$$\begin{aligned} ds^2 &= H_2^{\frac{3}{8}} H_6^{\frac{7}{8}} \left[H_2^{-1} H_6^{-1} (-2dudv + K(u, r)du^2) + H_2^{-1} H_6^{-1} e^{2\zeta_1(u)} (dy^1)^2 \right. \\ &\quad \left. + H_6^{-1} \sum_{\alpha=2}^5 e^{2\zeta_\alpha(u)} dy_\alpha^2 + e^{2\beta(u)} (dr^2 + r^2 d\Omega_2^2) \right], \\ \Phi &= \frac{1}{4} \ln H_2 - \frac{3}{4} \ln H_6 + \phi(u). \end{aligned} \quad (3.23)$$

In this case, there are eight nontrivial u -dependent functions $h_2(u), h_6(u), D_2(u)$ and $\zeta_\alpha (\alpha = 1, \dots, 5)$ with

$$\phi(u) = \frac{4}{3}\beta(u) = - \sum_{\alpha=2}^5 \zeta_\alpha(u). \quad (3.24)$$

4 A fluctuating “black hole”

In the static case, one can construct a black hole solution from the intersecting brane system via compactification. Hence we may find a time-dependent black hole solution by compactifying the present time-dependent intersecting brane systems[38, 45, 46]. We give a simple example of this type.

Let us consider the simple case of D1-D5 intersecting brane system with

$$H_A = 1 + \frac{Q_A}{r^2}, \quad \text{and} \quad K = \frac{2Q_w(u)}{r^2}, \quad (4.1)$$

where Q_w is a function of u . This can be obtained for the choice $h_1 = h_5 = 1$ and $\bar{D}_2 = -2Q_w(u)$. One can check $\bar{A}_2(u) = B_2(u) = 0$ by (3.22) easily. Introducing new function $H_w = 1 + K(u, r)/2$, we find the metric

$$\begin{aligned} ds_{10}^2 &= H_1^{-3/4} H_5^{-1/4} \left[-H_w^{-1} dt^2 + H_w \left(dz + \frac{(H_w - 1)}{H_w} dt \right)^2 \right] \\ &\quad + H_1^{1/4} H_5^{3/4} \left[H_5^{-1} \sum_{\alpha=1}^4 dy_\alpha^2 + dr^2 + r^2 d\Omega_3^2 \right], \end{aligned} \quad (4.2)$$

where $u = (t - z)/\sqrt{2}$ and $v = (t + z)/\sqrt{2}$.

In order to perform a compactification, we write our metric (4.2) as

$$ds_{10}^2 = \left(H_1^{-\frac{1}{12}} H_5^{\frac{5}{12}} H_w^{-\frac{1}{3}} \right) ds_5^2 + H_1^{-3/4} H_5^{-1/4} H_w \left(dz + \frac{(H_w - 1)}{H_w} dt \right)^2 + H_1^{1/4} H_5^{-1/4} \sum_{\alpha=1}^4 dy_\alpha^2, \quad (4.3)$$

where

$$ds_5^2 = -\Xi_5^2 dt^2 + \Xi_5^{-1} (dr^2 + r^2 d\Omega_3^2), \quad (4.4)$$

and $\Xi_5 = (H_1 H_5 H_w)^{-1/3}$ gives the five-dimensional metric in the Einstein frame.

All toroidal y_α -coordinates can be compactified, but the compactification of the z -coordinate is not trivial. We have to impose a periodic condition on the metric functions, which explicitly depend on z through the u -coordinate. Here we assume that the function $H_w(u, r)$ is periodic in the u direction. As a concrete example, we choose a periodic function as

$$Q_w(u) = Q_0 \left[1 + \epsilon \cos \left(\frac{\sqrt{2}u}{R} \right) \right], \quad (4.5)$$

where R is a radius of the z -space and ϵ is a positive constant. In fact, the metric is invariant under the discrete transformation of $z \rightarrow z + 2\pi nR$ ($n \in \mathbf{Z}$). The explicit form of the metric function Ξ_5 is given by

$$\Xi_5 = \left[\left(1 + \frac{Q_1}{r^2} \right) \left(1 + \frac{Q_5}{r^2} \right) \left(1 + \frac{Q_0}{r^2} \left[1 + \epsilon \cos \left(\frac{t-z}{R} \right) \right] \right) \right]^{-1/3}. \quad (4.6)$$

In order to avoid a closed timelike curve, the z -direction must be spacelike. This condition requires that $\epsilon \leq 1$; otherwise H_w becomes negative at least in the limit of $r \rightarrow 0$, where we expect a horizon. $\epsilon = 1$ must be excluded because the charge Q_w vanishes at $u = \pi R/\sqrt{2}$, when a singularity may appear at $r = 0$. Hence we assume that $0 < \epsilon < 1$.

The metric (4.4) with (4.6) gives effectively a five-dimensional time-dependent spacetime although it also depends on the z -coordinate. It describes an explicit example of a spacetime excited by a pylon which may appear in Kaluza-Klein compactification[47, 48]. Since it is asymptotically flat, one may define the ‘‘mass’’ of this object as

$$M = \frac{\pi[Q_1 + Q_5 + Q_w(u)]}{4G_5}, \quad (4.7)$$

which oscillates in time. The surface of $r = 0$ is a candidate for horizon because it is the case when the spacetime is static ($\epsilon = 0$). Hence one may naively think that this spacetime describes a time-dependent oscillating ‘‘black hole’’. However the ‘‘mass’’ depends not only on time t but also on the inner space coordinate z .

If the compactification radius R is small enough, we may not see z -dependence in a global scale. Taking an average over the internal z -space, we find that the mean mass $\langle M \rangle$ is given by

$$\langle M \rangle = \frac{\pi(Q_1 + Q_5 + Q_0)}{4G_5}. \quad (4.8)$$

We may also find that the ‘‘mass’’ M fluctuates around this average value with the amplitude

$$\frac{\sqrt{\langle (\Delta M)^2 \rangle}}{\langle M \rangle} = \frac{Q_0 \epsilon}{\sqrt{2}(Q_1 + Q_5 + Q_0)}. \quad (4.9)$$

and the typical frequency $\omega = \sqrt{2}/R$.

The Bekenstein-Hawking black hole entropy, which is proportional to the horizon area, may also fluctuate around the averaged value

$$\langle S \rangle = \frac{\langle \mathcal{A} \rangle}{4G_5} = \frac{\pi^2 \sqrt{Q_1 Q_5 Q_0}}{2G_5} \quad (4.10)$$

with the amplitude

$$\frac{\sqrt{\langle (\Delta S)^2 \rangle}}{\langle S \rangle} = \frac{\epsilon}{\sqrt{2}}. \quad (4.11)$$

This spacetime describes a five-dimensional compact object in a global scale, but it shows fluctuations near the ‘‘horizon’’ ($r = 0$). Hence it is not a deterministic five-dimensional spacetime. We can regard it as a fluctuating ‘‘black hole’’, but $r = 0$ may not be a true horizon.

Although this spacetime looks like a fluctuating ‘‘black hole’’ in five dimensions, it is a deterministic spacetime in six dimensions. In fact, when we approach the ‘‘horizon’’, we will see the internal compact z -space as well as the periodic time dependence. Hence the spacetime is essentially six-dimensional, whose metric is given by

$$ds_6^2 = \Xi_6(r) [-2dudv + K(u, r)du^2] + \Xi_6^{-1}(r)(dr^2 + r^2 d\Omega_3^2), \quad \Xi_6(r) \equiv (H_1 H_5)^{-1/2}. \quad (4.12)$$

It is obtained by compactification of all toroidal y_α -coordinates in ten-dimensional spacetime as

$$\begin{aligned} ds_{10}^2 &= H_1^{1/4} H_5^{3/4} \left[H_1^{-1} H_5^{-1} (-2dudv + K(u, r)du^2) + H_5^{-1} \sum_{\alpha=1}^4 dy_\alpha^2 + (dr^2 + r^2 d\Omega_3^2) \right] \\ &= H_1^{-1/4} H_5^{1/4} ds_6^2 + H_1^{1/4} H_5^{-1/4} \sum_{\alpha=1}^4 dy_\alpha^2. \end{aligned} \quad (4.13)$$

Although this spacetime is compact in the z -direction as well as in the toroidal y_α -direction, the z -direction is not homogeneous. As a result, the spacetime is time-dependent but it is no longer spherically symmetric, i.e. it depends on z as well as t, r . The inhomogeneity in the z -direction becomes prominent especially in the scale near (or smaller than) the compactification radius R . This spacetime is regular at $r = 0$, which is shown by calculating the curvature invariants. The Ricci scalar curvature and Kretschmann invariant are given by

$$\mathcal{R} = -\frac{r^4(Q_1 - Q_5)^2}{((r^2 + Q_1)(r^2 + Q_5))^{5/2}} \rightarrow 0 \quad \text{as } r \rightarrow 0 \quad (4.14)$$

and

$$\begin{aligned} \mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma} &= \frac{24Q_1^4 Q_5^4 + 96Q_1^3 Q_5^3 (Q_1 + Q_5)r^2 + O(r^4)}{(r^2 + Q_1)^5 (r^2 + Q_5)^5} \\ &\rightarrow \frac{24}{Q_1 Q_5} \quad \text{as } r \rightarrow 0 \end{aligned} \quad (4.15)$$

We thus see that these scalars do not diverge at the ‘‘horizon’’ ($r = 0$).

Hence we conclude that this solution describes a static and spherically symmetric five-dimensional compact object with fluctuations in a large scale, but it becomes a periodically oscillating and non-spherical six-dimensional object in a small scale.

There are several questions with this solution which deserve further consideration. Does this metric really describes a time-dependent black hole or else? Is the horizon, if it exists, time-dependent? How is the mass of the ‘‘black hole’’ defined? When we approach the ‘‘horizon’’, what kind of spacetime structure do we see? Those questions are interesting by themselves and are left for future study.

5 Concluding Remarks

We have constructed a fairly general family of supersymmetric solutions in time- and space-dependent wave backgrounds in supergravity theories. These solutions describe intersecting p -branes embedded into time-dependent dilaton-gravity plane waves of an arbitrary (isotropic) profile, with the brane world-volume aligned parallel to the propagation direction of the wave.

In our previous paper [36], we have derived this class of solutions but restricted the wave profile by setting $K = 0$ in Eq. (2.8) for simplicity, and then solved the resulting equation. However we have shown in this paper that if we regard (2.8) as the equation for K , we can get more general class of solutions and we have solved it explicitly. We have also discussed how many degrees of freedom we are left with, and found that we have $(d + 1)$ arbitrary functions. This approach has recently been taken in [44] for a single brane. Our solutions here include not only single but also intersecting brane solutions as well as wider solutions including more arbitrary functions than those in [44]. To investigate intersecting branes system is very important, because this class of solutions may describe standard model of particle physics, higher-dimensional black holes and so on. Thus we hope that our solutions provide a useful basis to investigate various physical phenomena.

As a simple physical application, we have also used one of the solutions to construct higher-dimensional time-dependent black hole. The example we have considered is the D1-D5 intersecting brane system, and we have proposed an effectively compactified solution in five dimensions. The result is an “oscillating black hole solution” in five dimensions. This black hole looks like five-dimensional “oscillating” black hole whose frequency is $\omega = \sqrt{2}/R$ (R is compact radius) if seen from infinity. On the other hand it looks like six-dimensional black hole near “horizon”. This is presented just as a simple example of possible compactification of our brane configurations. We may consider more complicated brane configurations and also other variants of compactification. It would be very interesting to contemplate further applying our solutions to more interesting (higher-dimensional) black holes. It is also an interesting subject to study non-extreme extension of our solutions.

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