# Self-gravitating system made of axions

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We show that the inclusion of an axion-like effective potential in the construction of a self-gravitating system made of scalar fields leads to a decrease on its compactness when the value of the self-interaction coupling constant is increased. By including the current values for the axion mass $m$ and decay constant $f_a$, we have computed the mass and the radius for self-gravitating systems made of axion particles. It is found that such objects will have asteroid-size masses and radius of few meters, then, the self-gravitating system made of axions could play the role of scalar mini-machos that are mimicking a cold dark matter model for the galactic halo.

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The necessity of introducing the dark matter (DM) as the main component of the galactic matter has become a solid fact due to its observational support [1]. Nevertheless, the DM nature is one of the most intriguing mysteries in physics. A large variety of particles have been considered as the main component of DM in the universe and only a few of them are still considered as good prospects since they must fulfill several requirements [2]. Among the survivors, the neutralino and the axion are leading candidates [3]. The question we want to address in the present work is the following: If the DM is mainly composed by axions, what type of astrophysical objects will the axions form?

In order to answer this question, we have solved the Einstein-Klein-Gordon (EKG) equations in the semiclassical limit, where the source is the mean value of the energy-momentum tensor operator $\langle \hat{T}^{\mu\nu} \rangle$ of a real, quantized scalar field constructed with the potential energy density which is given by [4]

$$V(\phi) = m^2 f_a^2 \left[1 - \cos\left(\frac{\phi}{f_a}\right)\right] . \qquad (1)$$

It is found that the resulting self-gravitating system, the axion star, will have asteroid-size mass ($M \sim 10^{-16} M_\odot$) and radius of few meters. These result differs from previous estimates where the effect of the potential energy density was either neglected [5] or it was taken into account but with a wrong sign in the self-interacting term of the potential [6, 8]. In the first case, it is known that there is a maximum mass for such self-gravitating system given by $M_{max} = 0.633\ m_p^2/m$, where $m$ is the mass associated with the scalar field and $m_p$ is the Planck's mass. For the allowed values of the axion mass, $10^{-5}$ eV $< m < 10^{-3}$ eV, [4, 7] the maximum mass for a self-gravitating system made of axion where the potential energy density (1) is neglected lies in the range $10^{-8}\ M\odot < M_{max}^{\text{axion star}} < 10^{-5}\ M\odot$. When the axion is considered to have a repulsive self-interacting term, the maximum could be as big as $M \sim 10^4\ M\odot$.

Here we solve the EKG system including a Taylor expansion of the potential energy density (1) and we observe that its inclusion tends to decrease the mass, and consequently the compactness of the self-gravitating system made of axions. Due to the smallness of the axion star's masses they could play the role of scalar field mini-machos [9] and they will be the final state of the axion miniclusters [10] originated in the early universe at the QCD epoch [11]. Assuming that the axion is the main component of the DM, the galactic halo will be a collisionless ensemble of axion stars and will be indistinguishable to the standard CDM scenario since $N$-body simulations of CDM with ultra-high resolution are insensitive to particle mass granularity smaller than $10^5 M_\odot - 10^3 M_\odot$ [12, 13].

The paper is organized as follows: in section I, the EKG equations for a real, quantized scalar field with a Taylor expansion of the potential energy density (1) are obtained and they are solved for arbitrary values of the axion mass, $m$, and the decay constant, $f_a$. In Sec. II we include the current values of $m$ and $f_a$ and we obtain the mass and radius of the axion-stars. We finish the section II by commenting some consequences derived in the case that the axion-star has the properties calculated here.

## I. EINSTEIN-KLEIN-GORDON WITH AN AXION-LIKE POTENTIAL

Since axions are real scalar particles, it is very useful to remember how self-gravitating systems made of spin zero particles are constructed. We will follow the method developed by Ruffini and Bonazzola [14]. The self-gravitating system arise as a solution of the EKG equations:

$$G_{\mu\nu} = 8\pi G < \hat{T}_{\mu\nu} > , \qquad (2)$$

$$\left(\Box - \frac{dV(\Phi)}{d\Phi^2}\right)\Phi = 0 , \qquad (3)$$

where $\Box = (1/\sqrt{-g})\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu]$ and $V(\phi)$ is the scalar field potential. Here $< \cdots >$ denotes average over the ground state of the system of many particles. Its presence refers to the fact that we are working in the semi-classical limit of the Einstein's equations. We will work with units

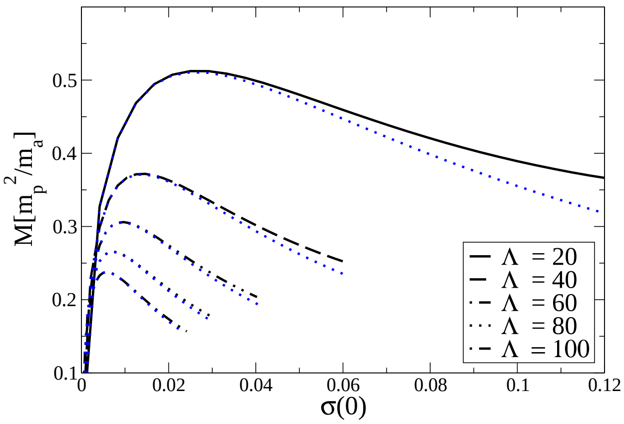


FIG. 1: Gravitational mass as a function of the central value of the scalar field $\sigma(0)$ for different values of $\Lambda$. Dotted lines include only an expansion of the axion potential up to the term $\Phi^4$.

where $c = \hbar = 1$. In the case of a spherically symmetric, static space-time described by

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(\sin^2\theta d\phi^2 + d\theta)\,, \tag{4}$$

it has been shown that such self-gravitating systems are fully characterized by the scalar field properties, i.e. the mass $m$ of the scalar field and its energy density potential $V(\Phi)$ [8, 15]. The total mass of the resulting object and the typical radii depends mainly on these two properties of the scalar field. The axion will not be the exception. To deal with the quantum nature of the axion field, we have to compute the average $\langle \hat{T}^{\mu\nu}\rangle$ in eq. 2. What it is usually done is to quantize the scalar field $\Phi \to \hat{\Phi} = \hat{\Phi}^+ + \hat{\Phi}^-$ where

$$\begin{aligned}
\hat{\Phi}^+ &= \sum_{nlm} \mu^+_{nlm} R_{nl}(r) Y^l_m(\theta,\psi) e^{-iE_n t} \\
\hat{\Phi}^- &= \sum_{nlm} \mu^-_{nlm} R_{nl}(r) Y^{l*}_m(\theta,\psi) e^{+iE_n t}
\end{aligned} \tag{5}$$

and $\mu^{+(-)}_{nlm}$ are the usual creation (annihilation) operators for a particle with angular momentum $l$, azimuthal momentum $m$ and energy $E_n$. These operators satisfy the usual commutation relations $[\mu^+_{nlm}, \mu^+_{n'l'm'}] = [\mu^-_{nlm}, \mu^-_{n'l'm'}] = 0$ and $[\mu^+_{nlm}, \mu^-_{n'l'm'}] = \delta_{nn'}\delta_{ll'}\delta_{mm'}$. With the operator $\hat{\Phi}$, it is now possible to construct the energy-momentum tensor operator $\hat{T}_{\mu\nu}$ just by inserting the operator $\hat{\phi}$ into the classical expression for the energy-momentum tensor

$$T^\mu_\nu = g^{\mu\sigma}\partial_\sigma\phi\partial_\nu\phi - \frac{1}{2}\delta^\mu_\nu g^{\lambda\sigma}\partial_\lambda\phi\partial_\sigma\phi - \delta^\mu_\nu V(\phi)\,. \tag{6}$$

The average $\langle Q|\hat{T}_{\mu\nu}|Q\rangle$ is done by considering an state $|Q\rangle$ for which all the $N$ particles are in the ground state ($l = m = 0$, $n = 1$). The ground state satisfies $\mu^-_{100}|Q\rangle = 0$. This procedure, as was already pointed out in [14], cancels all time dependence on the vacuum expectation value $\langle Q|\hat{T}_{\mu\nu}|Q\rangle$ and, for the case of a free scalar field ($V(\Phi) = \frac{m^2}{2}\Phi^2$), the real quantized scalar field yields to the same field equations as those obtained by using a classical complex scalar field. At these level, the self-gravitating system for a real quantized scalar field doesn't differ from a complex classical, hence, a real quantized scalar field doesn't produce the so call "oscillatons" [16], which are time-dependent. In our case we are interested the axion potential (1). In order to compute $\langle \hat{T}^{\mu\nu}\rangle$, we will do a Taylor's expansion of (1), i.e.

$$V(\Phi) \sim \frac{m^2}{2}\Phi^2 - \frac{1}{4!}\frac{m^2}{f_a^2}\Phi^4 + \frac{1}{6!}\frac{m^2}{f_a^4}\Phi^6 - \ldots \tag{7}$$

We will show that the final result doesn't depend strongly on the number of terms considered in the Taylor expansion of (1). The relevant term that should be considered is the self-interacting term $\Phi^4$ and the sign it carries with itself, which differs from the one considered in boson stars (BS) [6]. With the potential (7), now it is possible to compute $\langle \hat{T}^\mu_\nu\rangle$ by doing the quantization and average procedure previously discussed. The computed average of the stress energy tensor is

$$\begin{aligned}
\langle T^0_0\rangle &= -\frac{E^2R^2}{2B} - \frac{R'^2}{2A} - \frac{m^2R^2}{2} + \frac{m^2R^4}{12f_a^2} - \frac{m^2R^6}{144f_a^4} + \ldots\,, \\
\langle T^1_1\rangle &= \frac{E^2R^2}{2B} + \frac{R'^2}{2A} - \frac{m^2R^2}{2} + \frac{m^2R^4}{12f_a^2} - \frac{m^2R^6}{144f_a^4} + \ldots\,, \\
\langle T^2_2\rangle &= \frac{E^2R^2}{2B} - \frac{R'^2}{2A} - \frac{m^2R^2}{2} + \frac{m^2R^4}{12f_a^2} - \frac{m^2R^6}{144f_a^4} + \ldots
\end{aligned} \tag{8}$$

We have dropped all sub-indexes since, as we have already pointed out, we will assume that the axion is on its ground state. We can observe that there is not time dependence in (8). Furthermore, there are new numerical factors that appear due to the average performed in $\hat{T}_{\mu\nu}$ [17]. For instance $\langle\Phi^4\rangle = 2R^4$ and $\langle\Phi^6\rangle = 5R^6$, in such a way that we can not recover the original $\cos(\Phi/f_a)$ from which we departed. Following a similar procedure but now applied to the scalar wave equation (3), with a potential (7) and the spherically symmetric metric (4), it is obtained the Einstein-Klein-Gordon system:

$$\begin{aligned}
\frac{A'}{A^2r} + \frac{1}{r^2}\left(1 - \frac{1}{A}\right) &= -8\pi G\langle T^0_0\rangle\,, \\
\frac{B'}{ABr} - \frac{1}{r^2}\left(1 - \frac{1}{A}\right) &= 8\pi G\langle T^1_1\rangle\,, \\
R'' + \left(\frac{2}{r} + \frac{B'}{2B} - \frac{A'}{2A}\right)R' + A\Big[\left(\frac{E^2}{B} - \frac{m^2}{2}\right)R + & \\
+ \frac{m^2R^3}{6f_a^2} - \frac{m^2R^5}{48f_a^4}\Big] &= 0\,.
\end{aligned} \tag{9}$$

Following standard definitions [6], we rewrite the system (9) in dimensionless variables: $x = rm$, $R = \sigma/\sqrt{4\pi G}$

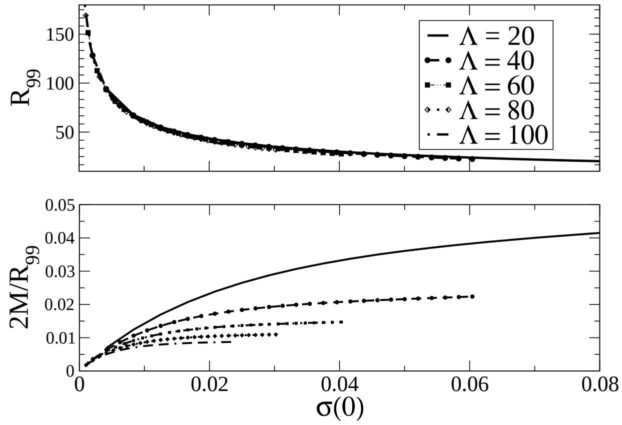


FIG. 2: $R_{99}$ and compactness for different configurations. The dependence of $R_{99}$ on $\Lambda$ is negligible. For a given value of $\sigma(0)$, the compactness decreases as $\Lambda$ increases.

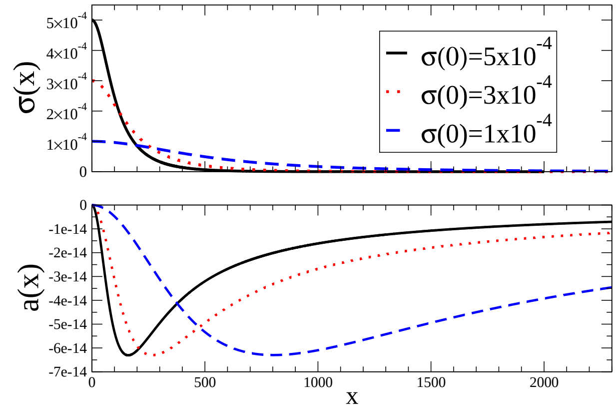


FIG. 3: Scalar field and potential for a typical axion star with different values of $\sigma(0)$ and an axion mass of $m_a = 1.0 \times 10^{-5}$eV.

and $\tilde{B} = m^2 B/E^2$, and we found convenient to define the dimensionless self-interaction term

$$\Lambda = \frac{1}{24\pi}\left(\frac{m_p}{f_a}\right)^2 . \tag{10}$$

The system (9) is solved by standard numerical methods by demanding regular function at the origin and flatness at infinity using a shooting method. Even though the set of equations (9) is very similar as the case for typical BS, [6], the behavior we found for the family of solutions with zero-nodes is completely different. A full set of equilibrium configurations is shown in Fig. 1, where it is plotted the gravitational mass for different values of $\sigma(0)$ and $\Lambda$. The equilibrium configurations have a maximum mass $M_{max}$ at some $\sigma(0) = \sigma_c$ for each value of $\Lambda$. But the switch in the potential sign of the term $\Phi^4$ produces a significant change in the behavior of $M_{max}$ in comparison with standard BS [6]. The relation $M_{max} \sim \Lambda^{1/2}$ is not satisfied anymore. Instead of increasing $M_{max}$ as we increase the value of $\Lambda$, we found a decreasing $M_{max}$. These effect is expected since instead of adding a repulsive interaction between the particles of the system, the change in the sign due to the cosine-like potential (1) implies an attractive potential and then, the total number of particles needed to form an equilibrium configuration that balance the gravitational collapse against the quantum pressure is lower than the case of a repulsive potential. One can think that these effect is apparent and as soon as the complete potential (1) will be implemented in the EKG system, a different behavior will be seen. But the decrease in the mass of the equilibrium configurations is a robust behavior. This is shown in Fig. 1 too, where it is plotted, in dotted lines, the masses of the equilibrium configurations when only is taken into account a Taylor expansion of (1) up to the fourth power on $\Phi$. The bigger the value of $\Lambda$, the lower the differences on the masses. This is because when $\Lambda$ is increased, $\sigma_c$ decreases (and equivalently $\Phi(r)$ where we are interested). Then, the true expansion parameter of (1) is $\Lambda\Phi$, an it is always an small parameter . Another interesting issue is that the radius $R_{99}$, defined as the radius where 99% of the total gravitational mass is reached, has not a strong dependence on the value of $\Lambda$ as it is shown in upper panel of Fig. 2. Combining the invariance of the radius as $\Lambda$ increases, with the decrease in the mass, it results that the self-gravitating system made of scalar field that has an axion-like potential has a lower compactness ($2M/R_{99}$) as the self-interaction term increases. This "newtonization" of the system is shown on the lower panel of Fig. 2.

## II. AXION-STAR

The previous results where obtained assuming arbitrary values of the mass $m$ of the scalar field associated with the axion as well as free values for the decay constant $f_a$. But the mass of the axion is constrained by astrophysical and cosmological considerations to lie in the range $10^{-5}$ eV $\leq m \leq 10^{-3}$ eV and the decay constant is related to the axion mass by $m = 6\mu\text{eV}\left(\frac{10^{12}\text{GeV}}{f_a}\right)$ [4, 18]. With these two restriction we have $10^{13} < \Lambda < 10^{17}$ and then, the previous selection of dimensionless variables $\{x, \sigma, A.\tilde{B}\}$) is now inadequate in order to solve numerically the system (9). After some frustrated attempts, we found that a more suitable set of variables to numerically solve the system (9) is the following:

$$R = \frac{f_a}{\sqrt{m}}\sigma\,, \quad r = \frac{m_p}{f_a}\sqrt{\frac{m}{4\pi}}x\,, \quad \frac{1}{\tilde{B}} = \frac{E^2}{m^2 B}\,. \tag{11}$$

TABLE I: Masses and $R_{99}$ for the configurations shown in Fig. 3

| $\sigma(0)$ | Mass (Kg) | $R_{99}$ (meters) | density $\rho$ (Kg/m$^3$) |
|---|---|---|---|
| $5\times10^{-4}$ | $3.90\times10^{13}$ | 1.83 | $6.3\times10^{12}$ |
| $3\times10^{-4}$ | $6.48\times10^{13}$ | 2.86 | $2.7\times10^{12}$ |
| $1\times10^{-4}$ | $1.94\times10^{14}$ | 8.54 | $3.1\times10^{11}$ |

Since $\Lambda >> 1$, it is natural to think that the resulting axion star will have a small compactness and low mass. So, besides the change in variables (11), it is convenient to solve for $a(x) = 1 - A(x)$. Solving the system (9) for the new set of variables $\{x, \sigma(x), a(x), \tilde{B}(x)\}$ we obtained typical nodeless configurations for these axion stars. Some of them are shown in Fig. 3. We have taken for definiteness an axion mass $m = 10^{-5}$ eV. The total gravitational mass and the radius $R_{99}$, both in physical units, for those configurations are shown in Table I.

A possible scenario emerges with the hypothesis that DM is mainly composed by axions. As was already pointed out by Kolb and Tkachev [10], nonlinear effects in the evolution of the axion field in the early universe may lead to the formation of "axion miniclusters". These miniclusters may relax, due to the collisional $2a \to 2a$ process or by gravitational cooling [19], and they will evolve to BS. In the present work we have constructed those BS for axion particles by solving the EKG system for a real quantized scalar field who is regulated by a axion-potential (1). These self-gravitating system, the axion stars, have very small masses and radius of meters (Table I) and consequently very low compactness. The resulting densities are not enough to produce stimulated decays of the axion to photons since they occur when $\Gamma_\pi m_p^2 V_e f_\pi/(R m_\pi^4 f_a > 1$ which implies densities $\rho > 10^{15}$ Kg/ m$^3$ for $m = 10^{-5}$ eV [19, 20]. Typical densities for axion stars are shown in table I. The galactic halo will be a ensemble of axion stars and this picture is not in contradiction with observations since the size of axions stars fit into the limits coming from microlensing or gravothermal instability [9]. If DM is distributed as axion stars, their detection will be very hard. The proposed femptolensing to detect axion compact objects [21] is close to its lower detectable limit. Another related issue is the low number of axion star around the earth. Assuming for instance a Navarro-Frenk-White profile for the galactic halo, and a local halo density of 0.3 GeV/cm$^3$ around the Sun, there will be $\sim 1$ axion star in the volume cover between Jupiter and the Sun. Nevertheless, another axion properties can shed light on the axion, such as the conversion of axions in photons in the presence of strong magnetic fields [22]. Collision of axion stars with neutron stars [23] will produce small flashes of light that could be detected by Gamma ray Observatories [24]. A more detailed analysis of those ideas together with a more detailed study of the stability of the axion stars could help us to determine if the DM is mainly composed by scalar field particles as the axions.

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