The Physical Properties of the Cosmic Acceleration

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The observed late-time acceleration of the cosmic expansion constitutes a fundamental problem in modern theoretical physics and cosmology. In an attempt to weight the validity of a large number of dark energy models, I use the recent measurements of the expansion rate of the Universe, the clustering of galaxies the CMB fluctuations as well as the large scale structure formation, to put tight constraints on the different models.

1. INTRODUCTION

Recent studies in observational cosmology lead to the conclusion that the available high quality cosmological data (Type Ia supernovae, CMB, etc.) are well fitted by an emerging "standard model". This standard model, assuming flatness, is described by the Friedman equation:

$$H^2(a) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\rho_m(a) + \rho_X(a)\right] , \qquad (1)$$

where $a(t)$ is the scale factor of the universe, $\rho_m(a)$ is the density corresponding to the sum of baryonic and cold dark matter, with the latter needed to explain clustering, and an extra component $\rho_X(a)$ with negative pressure, called dark energy, needed to explain the observed accelerated cosmic expansion (eg., Davis et al. 2007; Kowalski et al. 2008; Komatsu et al. 2009 and references therein).

The nature of the dark energy is one of the most fundamental and difficult problems in physics and cosmology. Indeed, during the last decade there has been a theoretical debate among the cosmologists regarding the nature of the exotic "dark energy". Various candidates have been proposed in the literature, such as a cosmological constant Λ (vacuum), time-varying $\Lambda(t)$ cosmologies, quintessence, $k-$essence, vector fields, phantom, tachyons, Chaplygin gas and the list goes on (for recent reviews see Peebles & Ratra 2003; Copeland, Sami & Tsujikawa 2006; Caldwell & Kamionkowski 2009). Within this framework, high energy field theories generically indicate that the equation of state of such a dark energy is a function of cosmic time. To identify this type of evolution of the equation of state, a detailed form of the observed $H(a)$ is required which may be obtained by a combination of multiple dark energy observational probes.

In this cosmological framework, a serious issue is how (and when) the large scale structures form. Galaxies and large-scale structure grew gravitationally from tiny, nearly scale-invariant adiabatic Gaussian fluctuations. In this paper we focus also on galaxy clusters that occupy an eminent position in the structure hierarchy, being the most massive virialized systems known and therefore they appear to be ideal tools for testing theories of structure formation and extracting cosmological information.

The cluster distribution basically traces scales that have not yet undergone the non-linear phase of gravitationally clustering, thus simplifying their connections to the initial conditions of cosmic structure formation.

The structure of the paper is as follows. In section 2 we briefly discuss the dark energy issue. In sections 3 and 4 we present the various dark energy models and we use a joint statistical analysis, in order to place constraints on the main cosmological parameters. In section 5 we present the corresponding theoretical predictions regarding the formation of the galaxy clusters. Finally, we draw our conclusions in section 6. Throughout the paper we will use $H_0 \simeq 71 \text{km/sec/Mpc}$.

2. THE DARK ENERGY EQUATION OF STATE

In the context of general relativity it is well known that for homogeneous and isotropic flat cosmologies, driven by non-relativistic matter and dark energy with equation of state $p_X = w(a)\rho_X$, the expansion rate of the Universe can be written as (see eq.1)

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_m a^{-3} + \Omega_X e^{3\int_a^1 d\ln y[1+w(y)]} . \quad (2)$$

Note, that $E(a)$ is the normalized Hubble flow, Ω_m is the dimensionless matter density at the present epoch, $\Omega_X = 1 - \Omega_m$ is the corresponding dark energy density parameter and $w(a)$ is the dark energy equation of state. Inverting the above equation we simply derive

$$w(a) = \frac{-1 - \frac{2}{3}a\frac{d\ln E}{da}}{1 - \Omega_m a^{-3}E^{-2}(a)} . \qquad (3)$$

The exact nature of the dark energy has yet to be found and thus the dark energy equation of state parameter includes our ignorance regarding the physical mechanism which leads to a late cosmic acceleration.

On the other hand, it is possible to extent the previous methodology in the framework of modified gravity (see Linder & Jenkins 2003; Linder 2004). In this scenario, it is assumed that the dark energy may be an illusion, indicating the need to revise the general relativity and thus also the Friedmann equation. From the mathematical

point of view, it can be shown that instead of using the exact Hubble flow through a modification of the Friedmann equation we can utilize a Hubble flow that looks like the nominal one (see eq.1). The key point here is to consider that the accelerated expansion of the universe can be attributed to a "geometrical" dark energy component. Due to the fact that the matter density in the universe (baryonic+dark) can not accelerate the cosmic expansion, we perform the following parametrization (Linder & Jenkins 2003; Linder 2004):

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_m a^{-3} + \delta H^2 \ . \tag{4}$$

Obviously, with the aid of the latter approach we include any modification to the Friedmann equation of general relativity in the last term of eq.(4). Now from eqs.(3, 4) we can derive, after some algebra, the "geometrical" dark energy equation of state

$$w(a) = -1 - \frac{1}{3} \frac{d\ln\delta H^2}{d\ln a} \ . \tag{5}$$

From now on, for the modified cosmological models we will use the above formulation.

3. LIKELIHOOD ANALYSIS

We will use various cosmological observations in order to constrain the dark energy models described in section 4. First of all, we use the Baryonic Acoustic Oscillations (BAOs). BAOs are produced by pressure (acoustic) waves in the photon-baryon plasma in the early universe, generated by dark matter overdensities. Evidence of this excess was recently found in the clustering properties of the luminous SDSS red-galaxies (Eisenstein et al. 2005; Padmanabhan et al. 2007) and it can provide a "standard ruler" with which we can constrain the dark energy models. In particular, we use the following estimator: $A(\mathbf{p}) = \frac{\sqrt{\Omega_m}}{[z_s^2 E(a_s)]^{1/3}} \left[\int_{a_s}^1 \frac{da}{a^2 E(a)} \right]^{2/3}$, measured from the SDSS data to be $A = 0.469 \pm 0.017$, where $z_s = 0.35$ [or $a_s = (1 + z_s)^{-1} \simeq 0.75$]. Therefore, the corresponding χ^2_{BAO} function is simply written

$$\chi^2_{\mathrm{BAO}}(\mathbf{p}) = \frac{[A(\mathbf{p}) - 0.469]^2}{0.017^2} \tag{6}$$

where $\mathbf{p}$ is a vector containing the cosmological parameters that we want to fit.

On the other hand, a very accurate and deep geometrical probe of dark energy is the angular scale of the sound horizon at the last scattering surface as encoded in the location l_1^{TT} of the first peak of the Cosmic Microwave Background (CMB) temperature perturbation spectrum. This probe is described by the CMB shift parameter (Bond, Efstathiou & Tegmark, 1997; Nesseris & Perivolaropoulos 2007) and is defined as $R = \sqrt{\Omega_m} \int_{a_{ls}}^1 \frac{da}{a^2 E(a)}$. The shift parameter measured

from the WMAP 5-years data (Komatsu et al. 2009) is $R = 1.71 \pm 0.019$ at $z_{ls} = 1090$ [or $a_{ls} = (1 + z_{ls})^{-1} \simeq 9.17 \times 10^{-4}$]. In this case, the χ^2_{cmb} function is given

$$\chi^2_{\mathrm{cmb}}(\mathbf{p}) = \frac{[R(\mathbf{p}) - 1.71]^2}{0.019^2} \tag{7}$$

Finally, we use the Union08 sample of 307 supernovae of Kowalski et al. (2008). The corresponding χ^2_{SNIa} function becomes:

$$\chi^2_{\mathrm{SNIa}}(\mathbf{p}) = \sum_{i=1}^{307} \left[\frac{\mu^{\mathrm{th}}(a_i, \mathbf{p}) - \mu^{\mathrm{obs}}(a_i)}{\sigma_i} \right]^2 \ . \tag{8}$$

where $a_i = (1 + z_i)^{-1}$ is the observed scale factor of the Universe, z_i is the observed redshift, μ is the distance modulus $\mu = m - M = 5\log d_L + 25$ and $d_L(a, \mathbf{p})$ is the luminosity distance $d_L(a, \mathbf{p}) = \frac{c}{a} \int_a^1 \frac{dy}{y^2 H(y)}$ where c is the speed of light. We can combine the above probes by using a joint likelihood analysis: $\mathcal{L}_{tot}(\mathbf{p}) = \mathcal{L}_{\mathrm{BAO}} \times \mathcal{L}_{\mathrm{cmb}} \times \mathcal{L}_{\mathrm{SNIa}}$ or $\chi^2_{tot}(\mathbf{p}) = \chi^2_{\mathrm{BAO}} + \chi^2_{\mathrm{cmb}} + \chi^2_{\mathrm{SNIa}}$ in order to put even further constraints on the parameter space used. Note, that we define the likelihood estimator [1] as: $\mathcal{L}_j \propto \exp[-\chi^2_j/2]$.

4. CONSTRAINTS ON THE FLAT DARK ENERGY MODELS

In this section, we consider a large family of flat dark energy models and with the aid of the above cosmologically relevant observational data, we attempt to put tight constraints on their free parameters. In the following subsections, we briefly present these cosmological models which trace differently the nature of the dark energy.

4.1. Constant equation of state - QP model

In this case the equation of state is constant (see for a review, Peebles & Ratra 2003; hereafter QP-models). Such dark energy models do not have much physical motivation. In particular, a constant equation of state parameter requires a fine tuning of the potential and kinetic energies of the real scalar field. Despite the latter problem, these dark energy models have been used in the literature due to their simplicity. Notice that dark energy models with a canonical kinetic term have $-1 \le w < -1/3$. On the other hand, models of phantom dark energy ($w < -1$) require exotic nature, such as a

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scalar field with a negative kinetic energy. Now using eq.(2) the normalized Hubble parameter becomes

$$E^2(a) = \Omega_m a^{-3} + (1 - \Omega_m)a^{-3(1+w)} \ . \qquad (9)$$

Comparing the QP-models with the observational data (we sample $\Omega_m \in [0.1, 1]$ and $w \in [-2, -0.4]$) we find that the best fit values are $\Omega_m = 0.28 \pm 0.02$ and $w = -1.02 \pm 0.06$ with $\chi^2_{tot}(\Omega_m, w)/dof \simeq 309.2/307$ in very good agreement with the 5 years WMAP data Komatsu et al. (2009). Also Davis et al. (2007) utilizing the Essence-SNIa+BAO+CMB and a Bayesian statistics found $\Omega_m = 0.27 \pm 0.04$, while Kowalski et al. (2008) using the Union08-SNIa+BAO+CMB obtained $\Omega_m \simeq 0.285^{+0.03}_{-0.03}$. Obviously, our results coincide within 1σ errors. It is worth noting that the concordance Λ-cosmology can be described by QP models with w strictly equal to -1. In this case we find: $\Omega_m = 0.28 \pm 0.02$ with $\chi^2_{tot}(\Omega_m)/dof \simeq 309.3/308$.

4.2. The Braneworld Gravity - BRG model

In the context of a braneworld cosmology (hereafter BRG) the accelerated expansion of the universe can be explained by a modification of gravity in which gravity itself becomes weak at very large distances (close to the Hubble scale) due to the fact that our four dimensional brane survives into an extra dimensional manifold (Deffayet, Dvali, & Cabadadze 2002). The interesting point in this scenario is that the corresponding functional form of the normalized Hubble flow, eq. (4), is affected only by one free parameter (Ω_m). Notice, that the quantity δH^2 is given by

$$\delta H^2 = 2\Omega_{bw} + 2\sqrt{\Omega_{bw}}\sqrt{\Omega_m a^{-3} + \Omega_{bw}} , \qquad (10)$$

where $\Omega_{bw} = (1 - \Omega_m)^2/4$. The geometrical dark energy equation of state parameter (see eq.5) reduces to

$$w(a) = -\frac{1}{1 + \Omega_m(a)} , \qquad (11)$$

where $\Omega_m(a) \equiv \Omega_m a^{-3}/E^2(a)$. The joint likelihood analysis provides a best fit value of $\Omega_m = 0.24 \pm 0.02$, but the fit is rather poor $\chi^2_{tot}(\Omega_m)/dof \simeq 369/308$.

4.3. The parametric Dark Energy model - CPL model

In this model we use the Chevalier-Polarski-Linder (Chevallier & Polarski 2001; Linder 2003, hereafter CPL) parametrization. The dark energy equation of state parameter is defined as a first order Taylor expansion around the present epoch:

$$w(a) = w_0 + w_1(1 - a) . \qquad (12)$$

The normalized Hubble parameter is given by (see eq.2):

$$E^2(a) = \Omega_m a^{-3} + (1 - \Omega_m)a^{-3(1+w_0+w_1)}e^{3w_1(a-1)} , \quad (13)$$

where w_0 and w_1 are constants. We sample the unknown parameters as follows: $w_0 \in [-2, -0.4]$ and $w_1 \in [-2.6, 2.6]$. We find that for the prior of $\Omega_m = 0.28$ the overall likelihood function peaks at $w_0 = -1.1^{+0.22}_{-0.16}$ and $w_1 = 0.60^{+0.62}_{-1.54}$ while the corresponding $\chi^2_{tot}(w_0, w_1)/dof$ is 307.6/307.

4.4. The low Ricci dark energy - LRDE model

In this modified cosmological model, we use a simple parametrization for the Ricci scalar which is based on a Taylor expansion around the present time: $\mathcal{R} = r_0 + r_1(1 - a)$ [for more details see Linder 2004]. It is interesting to point that at the early epochs the cosmic evolution tends asymptotically to be matter dominated. In this framework, we have

$$E^2(a) = \begin{cases} a^{4(r_0+r_1-1)}e^{4r_1(1-a)} & a \geq a_t \\ \Omega_m a^{-3} & a < a_t \end{cases} \qquad (14)$$

where $a_t = 1 - (1 - 4r_0)/4r_1$. The matter density parameter at the present epoch, is related with the above constants via $\Omega_m = \left(\frac{4r_0 - 4r_1 - 1}{4r_1}\right)^{4r_0+4r_1-1} e^{1-4r_0}$. The equation of state parameter that corresponds to the current geometrical dark energy model is given by

$$w(a) = \frac{1 - 4\mathcal{R}}{3}\left[1 - \Omega_m e^{-\int_a^1 (1-4\mathcal{R})(dy/y)}\right]^{-1} . \quad (15)$$

Note, that we sample the unknown parameters as follows: $r_0 \in [0.5, 1.5]$ and $r_1 \in [-2.4, -0.1]$ and here are the results: $r_0 = 0.82 \pm 0.04$ and $r_1 = -0.74^{+0.10}_{-0.08}$ ($\Omega_m \simeq 0.28$) with $\chi^2_{tot}(r_0, r_1)/dof \simeq 309.8/307$.

4.5. The high Ricci dark energy - HRDE model

A different than the previously described geometrical method was defined by Linder & Cahn (2007), in which the Ricci scalar at high redshifts evolves as

$$\mathcal{R} \simeq \frac{1}{4}\left[1 - 3w_1\frac{\delta H^2}{H^2}\right] , \qquad (16)$$

where $\delta H^2 = E^2(a) - \Omega_m a^{-3}$. In this geometrical pattern the normalized Hubble flow becomes:

$$E^2(a) = \Omega_m a^{-3}\left(1 + \beta a^{-3w_1}\right)^{-\ln\Omega_m/\ln(1+\beta)} , \quad (17)$$

where $\beta = \Omega_m^{-1} - 1$. Using the same sampling as in the QP-models, the joint likelihood peaks at $\Omega_m = 0.28 \pm 0.03$ and $w_1 = -1.02 \pm 0.1$ with $\chi^2_{tot}(\Omega_m, w_1)/dof \simeq 309.2/307$. To this end, the effective equation of state parameter is related to $E(a)$ according to eq.(3).

4.6. The tension of cosmological magnetic fields - TCM model

Recently, Contopoulos & Basilakos (2007) proposed a novel idea which is based on the following consideration (hereafter TCM): if the cosmic magnetic field is generated in sources (such as galaxy clusters) whose overall dimensions remain unchanged during the expansion of the Universe, the stretching of this field by the cosmic expansion generates a tension (negative pressure) that could possibly explain a small fraction of the dark energy ($\sim 5 - 10\%$). In this flat cosmological scenario the normalized Hubble flow becomes:

$$E^2(a) = \Omega_m a^{-3} + \Omega_\Lambda + \Omega_B a^{-3+n} , \qquad (18)$$

where Ω_B is the density parameter for the cosmic magnetic fields and $\Omega_\Lambda = 1 - \Omega_m - \Omega_B$. The equation of state parameter which is related to magnetic tension is (see eq.3)

$$w(a) = -\frac{3\Omega_\Lambda + n\Omega_B a^{-3+n}}{3(\Omega_\Lambda + \Omega_B a^{-3+n})} . \qquad (19)$$

To this end, we sample $\Omega_B \in [0, 0.3]$ and $n \in [0, 10]$ and we find that for the prior of $\Omega_m = 0.28$ the best fit values are: $\Omega_B = 0.10 \pm 0.10$ and $n = 3.60^{+4.5}_{-2.6}$ with $\chi^2_{tot}(\Omega_B, n)/dof \simeq 308.9/307$.

4.7. The Pseudo-Nambu Goldstone boson - PNGB model

In this cosmological model the dark energy equation of state parameter is expressed with the aid the potential $V(\phi) \propto [1 + cos(\phi/F)]$ (Abrahamse et al 2008 and references therein):

$$w(a) = -1 + (1 + w_0)a^F , \qquad (20)$$

where $w_0 \in [-2, -0.4]$ and $F \in [0, 8]$. In case of $a \ll 1$ we get $w(a) \longrightarrow -1$. Based on this parametrization the normalized Hubble function is given by

$$E^2(a) = \Omega_m a^{-3} + (1 - \Omega_m)\rho_X(a) . \qquad (21)$$

In this context, the corresponding dark energy density is

$$\rho_X(a) = \exp\left[\frac{3(1 + w_0)}{F}(1 - a^F)\right] . \qquad (22)$$

Notice, that the likelihood function peaks at $w_0 = -1.04 \pm 0.17$ and $F = 5.9 \pm 3.2$ with $\chi^2_{tot}(w_0, F)/dof \simeq 317/307$.

4.8. The early dark energy - EDE model

Another cosmological scenario that we include in our paper is the early dark energy model (hereafter EDE).

The basic assumption here is that at early epochs the amount of dark energy is not negligible (Doran, Stern & Thommes 2006 and references therein). In this model, the total dark energy component is given by:

$$\Omega_X(a) = \frac{1 - \Omega_m - \Omega_e(1 - a^{-3w_0})}{1 - \Omega_m - \Omega_m a^{3w_0}} + \Omega_e(1 - a^{-3w_0}) , \qquad (23)$$

where Ω_e is the early dark energy density and w_0 is the equation of state parameter at the present epoch. Notice, that the EDE model was designed to simultaneously (a) mimic the effects of the late dark energy and (b) provide a decelerated expansion of the universe at high redshifts. The normalized Hubble parameter is written as:

$$E^2(a) = \frac{\Omega_m a^{-3}}{1 - \Omega_X(a)} , \qquad (24)$$

while using eq.(3), we can obtain the equation of state parameter as a function of the scale factor. From the joint likelihood analysis we find that $\Omega_e = 0.05 \pm 0.04$ and $w_0 = -1.14^{+0.18}_{-0.10}$ (for the prior of $\Omega_m = 0.28$) with $\chi^2_{tot}(\Omega_e, w_0)/dof \simeq 308.7/307$.

4.9. The Variable Chaplygin Gas - VCG model

Let us consider now a completely different model namely the variable Chaplygin gas (hereafter VCG) which corresponds to a Born-Infeld tachyon action (Bento, Bertolami & Sen 2004). Recently, an interesting family of Chaplygin gas models was found to be consistent with the current observational data (Dev, Alcaniz & Jain 2003). In the framework of a spatially flat geometry, it can be shown that the normalized Hubble function takes the following formula:

$$E^2(a) = \Omega_b a^{-3} + (1 - \Omega_b)\sqrt{B_s a^{-6} + (1 - B_s)a^{-n}} , \quad (25)$$

where $\Omega_b \simeq 0.021h^{-2}$ is the density parameter for the baryonic matter (see Kirkman et al. 2003) and $B_s \in [0.01, 0.51]$ and $n \in [-4, 4]$. The effective matter density parameter is: $\Omega_m^{eff} = \Omega_b + (1 - \Omega_b)\sqrt{B_s}$. We find that the best fit parameters are $B_s = 0.05 \pm 0.02$ and $n = 1.58^{+0.35}_{-0.43}$ ($\Omega_m^{eff} \simeq 0.26$) with $\chi^2_{tot}(B_s, n)/dof \simeq 314.7/307$.

5. EVOLUTION OF MATTER PERTURBATIONS

The evolution equation of the growth factor for models where the dark energy fluid has a vanishing anisotropic stress and the matter fluid is not coupled to other matter species is given by (Linder & Jenkins 2003)

$$D'' + \frac{3}{2}\left[1 - \frac{w(a)}{1 + X(a)}\right]\frac{D'}{a} - \frac{3}{2}\frac{X(a)}{1 + X(a)}\frac{D}{a^2} = 0 , \quad (26)$$

where

$$X(a) = \frac{\Omega_m}{1 - \Omega_m}\mathrm{e}^{-3\int_a^1 w(y)d\ln y} = \frac{\Omega_m a^{-3}}{\delta H^2} \ . \qquad (27)$$

Note, that the prime denotes derivatives with respect to the scale factor. Useful expressions of the growth factor can be found for the ΛCDM cosmology in Peebles (1993), for dark energy models with $w = const$ in Silveira, & Waga (1994) for dark energy models with a time varying equation of state in Linder & Cahn (2007) and for the scalar tensor models in Gannouji & Polarski (2008). In this work the growth factor evolution for the current cosmological model is derived by solving numerically eq. (26). Note, that the growth factors are normalized to unity at the present time.

5.1. The formation of galaxy clusters

In this section we briefly investigate the cluster formation processes by generalizing the basic equations which govern the behavior of the matter perturbations within the framework of the current dark energy models. Also we compare our predictions with those found for the traditional Λ cosmology. This can help us understand better the theoretical expectations of the dark energy models as well as the variants from the usual Λ cosmology.
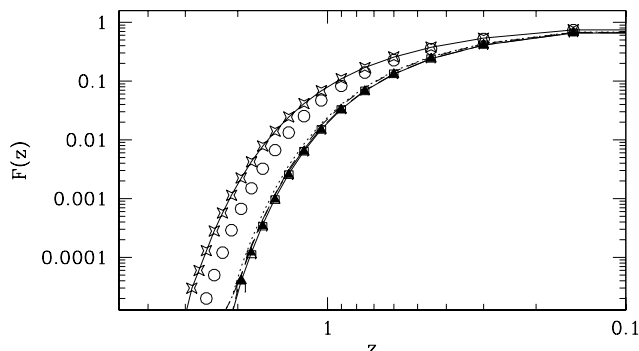

FIG. 1: The predicted fractional rate of cluster formation as a function of redshift for the current cosmological models ($\sigma_8 = 0.80$). The points represent the following cosmological models: (a) BRG (open stars), (b) LRDE (open squares), (c) TCM (open triangles), (d) EDE (open circles) and (e) PNGB (solid triangles). The lines represent: (a) CPL model (long dashed), (b) HRDE model (dot line) and VCG model (dashed line).

The concept of estimating the fractional rate of cluster formation has been proposed by different authors (eg., Weinberg 1987; Richstone, Loeb & Turner 1992). In particular, these authors introduced a methodology which computes the rate at which mass joins virialized structures, which grow from small initial perturbations in the universe. The basic tool is the Press & Schechter (1974) formalism which considers the fraction of mass in the

universe contained in gravitationally bound structures (such as galaxy clusters) with matter fluctuations greater than a critical value δ_c, which is the linearly extrapolated density threshold above which structures collapse (Eke, Cole & Frenk 1996). Assuming that the density contrast is normally distributed with zero mean and variance $\sigma^2(M, z)$ we have:

$$\mathcal{P}(\delta, z) = \frac{1}{\sqrt{2\pi}\sigma(M, z)}\exp\left[-\frac{\delta^2}{2\sigma^2(M, z)}\right] \ . \qquad (28)$$

In this kind of studies it is common to parametrize the rms mass fluctuation amplitude at 8 h^{-1}Mpc which can be expressed as a function of redshift as $\sigma(M, z) = \sigma_8(z) = D(z)\sigma_8$. The current cosmological models are normalized by the analysis of the WMAP 5 years data $\sigma_8 = 0.80$ (Komatsu et al. 2009). The integration of eq. (28) provides the fraction of the universe, on some specific mass scale, that has already collapsed producing cosmic structures (galaxy clusters) at redshift z and is given by Richstone et al. (1992):

$$P(z) = \int_{\delta_c}^\infty \mathcal{P}(\delta, z)d\delta = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\delta_c}{\sqrt{2}\sigma_8(z)}\right)\right] \ . \qquad (29)$$

Notice, that for the model of modified gravity (BRG) we use $\delta_c \simeq 1.47$ (see Schafer & Koyama 2008), for the EDE model we use $\delta_c \simeq 1.4$ (see Bartelmann, Doran & Wetterich 2006). For the rest of the dark energy models, due to the fact that $w \simeq -1$ close to the present epoch, we utilize a δ_c approximation which is given by Weinberg & Kamionkowski (2003 see their eq.18).

Obviously the above generic form of eq.(29) depends on the choice of the background cosmology. The next step is to normalize the probability to give the number of clusters which have already collapsed by the epoch z (cumulative distribution), divided by the number of clusters which have collapsed at the present epoch ($z = 0$), $F(z) = P(z)/P(0)$. In figure 1 we present, in a logarithmic scale, the behavior of normalized cluster formation rate as a function of redshift for the various dark energy models. In general, prior to $z \sim 0.2$ the cluster formation has terminated due to the fact that the matter fluctuation field, $D(z)$, effectively freezes. For the traditional Λ cosmology we find the known behavior in which galaxy clusters appear to be formed at high redshifts $z \sim 2$ (Basilakos 2003; Basilakos, Sanchez & Perivolaropoulos 2009 and references therein). From figure 1 it becomes also clear, that the vast majority of the dark energy models seem to have a cluster formation rate which is close to that predicted by the usual Λ cosmology (see solid line in figure 1). However, for the BRG and EDE cosmological scenarios we find that galaxy clusters appear to form earlier ($z \sim 3$) than in the CPL, LRDE, HRDE, TCM, PNGB and VCG dark energy models.

6.   CONCLUSIONS

In this work we have studied analytically and numerically the overall dynamics of the universe for a large number of dark energy models beyond the *concrdance* Λ cosmology, by using several parameterizations for the dark energy equation of state. We performed a joint likelihood analysis, using the current high-quality observational data (SNIa, CMB shift parameter and BAOs), and we put tight constraints on the main cosmological parameters. We also find that for the vast majority of the dark energy models, the large scale structures (such as galaxy clusters) start to form prior to $z \sim 1 - 2$.

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