Why gravity is fundamental

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It is argued that the existence of a minimum size of spacetime may imply the fundamental existence of gravity as a geometric property of spacetime described by general relativity.

It is still a controversial issue whether gravity is fundamental or emergent. The solution of this problem may have important implications for a complete theory of quantum gravity. On the other hand, independent of the nature of gravity, the existence of a minimum size of spacetime has been widely argued and acknowledged as a model-independent result of the proper combination of quantum mechanics (QM) and general relativity (GR) (see, e.g. [1] for a review). Moreover, the argument implies that the minimum time interval and the minimum length are respectively of the order of Planck time (T_p) and Planck length (L_p). The model-independence of the argument for the discreteness of spacetime strongly suggests that discreteness is probably a more basic feature of spacetime, and it may have a firmer basis beyond QM and GR[1], which are still based on continuous spacetime. Therefore, it may be appropriate to re-examine the relationship between the discreteness of spacetime and the existing fundamental theories from the opposite direction. In this paper, we will take this challenge and analyze the implications of spacetime discreteness for the nature of gravity. Since the formulations and meanings of discrete spacetime are different in the existing theories and arguments, we only resort to its minimum explanation here, namely that a spacetime interval shorter than the minimum size of spacetime (i.e. Planck scale) is physically meaningless, and it cannot be measured in principle either. For instance, a physical process can only happen during a time interval not shorter than the minimum time interval, namely Planck time.

According to the Heisenberg uncertainty principle in QM we have

$$\Delta x \geq \frac{\hbar}{2\Delta p} \tag{1}$$

As a result, the uncertainty of the position of a particle Δx can be arbitrarily small by increasing its momentum uncertainty Δp. This holds true in continuous spacetime. However, the discreteness of spacetime will demand that the uncertainty of the position of a particle should have a minimum value L_U, namely Δx should satisfy the limiting relation

[1] For instance, as the holographic principle implies [2-4], the information inside any finite spatial region should be finite.

$$\Delta x \geq L_U \tag{2}$$

In order to satisfy this relation, the Heisenberg uncertainty principle in discrete spacetime should at least contain another term proportional to the momentum uncertainty, namely in the first order of Δp it should be

$$\Delta x \geq \frac{\hbar}{2\Delta p} + \frac{L_U{}^2 \Delta p}{2\hbar} \tag{3}$$

This generalized uncertainty principle can satisfy the limiting relation imposed by the discreteness of spacetime[2].

It can be seen that the new term, which is required by the discreteness of spacetime, means that the momentum and energy uncertainty of a particle will introduce an inherent spacetime uncertainty for the position of the particle. This further implies that the energy-momentum of a particle will change the geometry of spacetime it moves in (e.g. in each momentum-energy branch of a quantum superposition). Concretely speaking, the energy $E \approx pc$ contained in a region with size L will change the proper size of the region, and the change is

$$\Delta L \approx \frac{L_U T_U E}{2\hbar} \tag{4}$$

where $T_U = L_U / c$. This means that a flat spacetime will be curved by the energy-momentum contained in it.

The above argument may provide a deeper basis for the Einstein equivalence principle in GR. The principle is usually argued with the help of classical mechanics and Newton's law of gravity, along with the experimental evidence of the equivalence of gravitational and inertial mass. The drawback of such an argument is that it may obscure the physical meaning of GR. For example, it suggests that gravity may be merely emergent at the classical level. By comparison, the above argument based on QM and the discreteness of spacetime implies that gravity is essentially a geometric property of spacetime, which is determined by the energy-momentum contained in that spacetime, not only at the classical level but also at the quantum level. In other words, gravity is as fundamental as the quantum.

On the basis of the equivalence principle, there are some common steps to "derive" the Einstein field equations, the concrete relation between the geometry of spacetime and the energy-momentum contained in that spacetime, in terms of Riemann geometry and tensor analysis as well as the conservation of energy and momentum etc. For example, it can be shown that there is only one symmetric second-rank tensor that will satisfy the following conditions: (1) Constructed solely from the spacetime metric and its derivatives; (2) Linear in the second derivatives; (3) The four-divergence of which is vanishes identically (this condition guarantees the conservation of energy and momentum); (4) Is zero when spacetime is flat (i.e. without cosmological constant). These conditions will yield a tensor capturing the dynamics of the curvature of

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[2] Similarly, we will also have $\Delta t \geq \frac{\hbar}{2\Delta E} + \frac{T_U{}^2 \Delta E}{2\hbar}$ for time uncertainty. In some sense, our analysis here can be regarded as a reverse application of the generalized uncertainty principle [5].

spacetime, which is proportional to the stress-energy density, and we can then obtain the Einstein field equations[3]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \tag{5}$$

where $R_{\mu\nu}$ the Ricci curvature tensor, R the scalar curvature, $g_{\mu\nu}$ the metric tensor, κ is the Einstein gravitational constant, and $T_{\mu\nu}$ the stress-energy tensor.

The left thing is to determine the value of the Einstein gravitational constant κ. It is usually derived by requiring that the weak and slow limit of the Einstein field equations must recover Newton's theory of gravitation. In this way, the gravitational constant is determined by experience as a matter of fact. If the above argument is right, the Einstein gravitational constant can also be determined in theory in terms of the discreteness of spacetime. Consider an energy eigenstate limited in a region with radius *R*. The spacetime outside the region can be described by the Schwarzschild metric by solving the Einstein field equations:

$$ds^2 = (1 - \frac{r_S}{r})^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin\theta^2 d\phi^2 - (1 - \frac{r_S}{r}) c^2 dt^2 \tag{6}$$

where $r_S = \frac{\kappa E}{4\pi}$ is the Schwarzschild radius. By assuming that the metric tensor inside the region *R* is in the same order as that on the boundary, the proper size of the region is

$$L \approx 2\int_0^R (1 - \frac{r_S}{R})^{-1/2} dr \approx 2R + \frac{\kappa E}{4\pi} \tag{7}$$

Therefore, the change of the proper size of the region due to the containing energy E is

$$\Delta L \approx \frac{\kappa E}{4\pi} \tag{8}$$

By comparing with Eq. (4) we find $\kappa = 2\pi \frac{L_U T_U}{\hbar}$ in Einstein's field equations. It can be seen that this formula itself also suggests that the discreteness of spacetime results in the existence of gravity. In continuous spacetime where $T_U = 0$ and $L_U = 0$, we have $\kappa = 0$, and thus gravity does not exist.

The above argument for the fundamental existence of gravity may have an important implication for quantum gravity. As we know, there exists a fundamental conflict between the superposition principle of QM and the general covariance principle of GR[4] [6-7]; QM requires a presupposed fixed spacetime structure to define quantum state and its evolution, but the spacetime structure is dynamical and determined by the state according to GR. The conflict indicates that at least one of these basic principles must be compromised in order to combine into a coherent theory of quantum gravity. But there has been a hot

[3] Another route to deriving the Einstein field equations is through an action principle using a gravitational Lagrangian.

[4] This conflict between QM and GR can be regarded as a different form of the problem of time in quantum gravity. It is widely acknowledged that QM and GR contain drastically different concepts of time (and spacetime), and thus they are incompatible in nature. In QM, time is an external (absolute) element (e.g. the role of absolute time is played by the external Minkowski spacetime in quantum field theory). In contrast, spacetime is a dynamical object in GR. This then leads to the notorious problem of time in quantum gravity [8-9].

debate on which one should yield to the other. The problem is actually two-fold. On the one hand, QM has been plagued by the measurement problem, and thus it is still unknown whether its superposition principle is universally valid, especially for macroscopic objects. On the other hand, it is not clear whether the gravity described by GR is emergent or not either. The existing heuristic "derivation" based on Newton's theory cannot determine whether gravity as a geometric property of spacetime described by GR is fundamental.

If gravity is really emergent, for example, GR is treated as an effective field theory, then the dynamical relation between the geometry of spacetime and the energy-momentum contained in that spacetime, as described by Einstein's field equations, will be not fundamental. As a consequence, different from the superposition principle of QM, the general covariance principle of GR will be not a basic principle, and thus no conflict will exist between quantum and gravity and we may directly extend the quantum field theory to include gravity (e.g. in string theory). In fact, the general covariance principle of GR has been compromised here because it is not fundamental. Note that, besides the string theory, there are also some interesting suggestions that gravity may be emergent, e.g. Sakharov's induced gravity [10-11], Jacobson's gravitational thermodynamics [12] and Verlinde's latest conjecture of gravity as entropic force [13]. On the other hand, if gravity is not emergent but fundamental as we have argued above, then quantum and gravity may be combined in a way different from the string theory. Now that the general covariance principle of GR is universally valid, the superposition principle of QM probably needs to be compromised when considering the fundamental conflict between them [6, 14].

To sum up, we show that the existence of a minimum size of spacetime or the discreteness of spacetime implies that gravity as a geometric property of spacetime described by GR is fundamental. In particular, the dynamical relationship between matter and spacetime holds true not only for macroscopic objects, but also for microscopic particles. The argument may provide a deeper basis for the Einstein equivalence principle. Moreover, the Einstein gravitational constant in GR can also be determined in terms of the discreteness of spacetime. It is suggested that the fundamental existence of gravity as argued above may have further implication for a complete theory of quantum gravity.

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