Observational Constraints on Transverse Gravity: a Generalization of Unimodular Gravity.

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Abstract. We explore the hypothesis that the set of symmetries enjoyed by the theory that describes gravity is not the full group of diffeomorphisms (*Diff(M)*), as in General Relativity, but a maximal subgroup of it (*TransverseDiff(M)*), with its elements having a jacobian equal to unity; at the infinitesimal level, the parameter describing the coordinate change $x^\mu \to x^\mu + \xi^\mu(x)$ is transverse, i.e., $\partial_\mu \xi^\mu = 0$. Incidentally, this is the smaller symmetry one needs to propagate consistently a graviton, which is a great theoretical motivation for considering these theories. Also, the determinant of the metric, g, behaves as a "transverse scalar", so that these theories can be seen as a generalization of the better-known unimodular gravity. We present our results on the observational constraints on transverse gravity, in close relation with the claim of equivalence with general scalar-tensor theory. We also comment on the structure of the divergences of the quantum theory to the one-loop order.

1. Introduction & Formalism

The gravitational theory of General Relativity presents as a remarkable feature the property of (active) diffeomorphism gauge invariance. The Einstein-Hilbert action

$$S_{EH} = -\frac{1}{2\kappa}\int d^4x\sqrt{g}R \tag{1}$$

is symmetric[1] under the "Diff" transformation $g_{\mu\nu}(x) \to g_{\mu\nu}(x) + \nabla_\mu \xi_\nu(x) + \nabla_\nu \xi_\mu(x)$, which represents the active version of the infinitesimal coordinate transformation $x^\mu \to x^\mu + \xi^\mu(x)$.

When attempting to quantize this theory in a perturbative expansion, one can take the ansatz $g_{\mu\nu} = \widetilde{g}_{\mu\nu} + h_{\mu\nu}$, where $\widetilde{g}_{\mu\nu}$ is a classical background that fulfills the classical equations of motion (e.o.m), and $h_{\mu\nu}$ is the quantum perturbation. In this way, one gets

$$S = -\frac{1}{2\kappa}\int d^4x[const + o(h^2_{\mu\nu}) + o(h^3_{\mu\nu}) + ...] \tag{2}$$

where the quadratic part in $h_{\mu\nu}$ constitutes the free-theory. In the case of a flat space, with a Minkowski classical background, the free-theory is given by the Fierz-Pauli Lagrangian:

$$\mathcal{L}_{FP} = \mathcal{L}^I + \mathcal{L}^{II} + a\mathcal{L}^{III} + b\,\mathcal{L}^{IV} \quad with\, b = \left(1 - 2a + 3a^2\right)/2 \tag{3}$$

[1] Unlike internal gauge transformations which do not involve the coordinates (e.g., U(1) of electromagnetism), this gauge invariance is not attained at the level of the Lagrangian, but only at the level of the action.

where $\mathcal{L}^I \equiv \frac{1}{4}\partial_\mu h^{\nu\rho}\partial^\mu h_{\nu\rho}$, $\mathcal{L}^{II} = -\frac{1}{2}\partial_\mu h^{\mu\rho}\partial_\nu h^\nu_\rho$, $\mathcal{L}^{III} = \frac{1}{2}\partial^\mu h\partial^\rho h_{\mu\rho}$, $\mathcal{L}^{IV} = -\frac{1}{4}\partial_\mu h\partial^\mu h$. Note that we have allowed for field redefinitions $h_{\mu\nu} \to \phi h_{\mu\nu}$ with respect to the usual presentation of the Fierz-Pauli Lagrangian, when $a = b = 1$. The corresponding action is symmetric under the "linear Diff" transformation

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu \tag{4}$$

and represents the action for a free massless spin-two particle (graviton). Here, gauge invariance is essential to eliminate the unphysical propagating modes, much like in electromagnetism, where the bad-behaved longitudinal mode is made a gauge artifact through the symmetry $A_\mu \to A_\mu + \partial_\mu\alpha$.

The authors of [1] undertook the task of finding out the most general conditions for a consistent description of the free massless spin-two particle. This involved considerations regarding the little group ISO(2) of the masless spin-two particle and the necessity of triviality of the generators of translations. Their conclusion is that, actually, one only needs the somehow smaller gauge symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu, \quad with\ \partial_\mu\xi^\mu = 0, \tag{5}$$

which we will denote as the "Linear TDiff" transformation, to indicate the transverse condition on the vector parameter. Hence, one can think of alternative theories of gravity to General Relativity, based on a slightly different symmetry principle, which however contain the graviton in the linear regime as well. From this observation, there are basically two ways to proceed forward.

The first route was already explored in the aforementioned seminal paper [1]. One should note that the transversality condition in (5) is directly affecting the transformation rule of the trace of $h_{\mu\nu}$: $h^\mu_\mu \to h^\mu_\mu + \partial_\mu\xi^\mu$, such that this quantity is an invariant. Therefore, one can decide to restrict the value of the trace in the theory to have a fixed value, tipically $h^\mu_\mu = 0$. In the full non-linear regime, this naturally translates into the condition $g \equiv -det(g_{\mu\nu}) = 1$. This approach has been named *unimodular gravity.*

In this work we are concerned with the second way to proceed. One can ask himself about the most general theory involving an unconstrained second-rank tensor $h_{\mu\nu}$ and which is compatible with the TDiff symmetry. Since we will be using less symmetry than the standard Diff case, we expect to get a *more general* action also in the full non-linear regime:

$$S_{TDiff} = S_{TDiff}[g_{\mu\nu}, \psi] = S_{Diff}[g_{\mu\nu}, \psi] + S'[g_{\mu\nu}(x), \psi]. \tag{6}$$

We wondered whether this fact could be useful in addressing such problems as the dark energy of dark matter puzzles.

The concrete answer in the linear regime was found in [2]. In general, one gets an extra scalar-like mode propagating as well as the two tensor modes corresponding to the massless graviton — the only exception is when the TDiff symmetry is enhanced with an additional Weyl conformal symmetry, in which case the unimodular approach is recovered. Specifically:

$$\mathcal{L}_{L-TDiff} = \mathcal{L}^I + \mathcal{L}^{II} + a\mathcal{L}^{III} + b\ \mathcal{L}^{IV} \quad with\, a,\, b\ arbitrary \tag{7}$$

where the form of the different pieces in the Lagrangian is given below (3). It should be stressed that this TDiff Langrangian, as well as the following ones, are not covariant with respect to general coordinate transformations[2], so that one has to formulate them in a specific set of coordinates. This is the reason why we are not talking of a *proper* additional scalar mode.

In the non-linear regime, the symmetry (5) *naturally* generalizes[3] to

$$g_{\mu\nu} \to g_{\mu\nu} + \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu, \quad with\ \partial_\mu\xi^\mu = 0 \tag{8}$$

[2] In passing, one could render the theory covariant by introducing extra objects in it.

[3] This is, however, not a unique answer. The other most "sensible" generalization, where ξ^μ is restricted with $\nabla_\mu\xi^\mu = 0$ (and $\sqrt{g} \to \sqrt{g}$) is doomed since one cannot accommodate a kinetic term for g: $\frac{1}{2}f_k(g)g^{\mu\nu}\partial_\mu g\partial_\nu g$.

$$\implies \sqrt{g} \to \sqrt{g} + \xi^\mu \partial_\mu \sqrt{g} \tag{9}$$

Hence, the determinant of the metric acts as a scalar under this restricted transformation (it is not, however, a scalar under coordinate transformations). What this entails is that one can have arbitrary powers of g appearing everywhere in the action of the full TDiff theory:

$$\begin{aligned} S_{TDiff} \;=\; & -\frac{1}{2\kappa^2}\int_{\mathcal{M}} d^4x\sqrt{g}\left(f(g)R + 2f_\lambda(g)\Lambda + \frac{1}{2}f_k(g)g^{\mu\nu}\partial_\mu g\partial_\nu g\right) \\ & + \int_{\mathcal{M}} d^4x\sqrt{g}\mathcal{L}_{Matter}[\psi, g_{\mu\nu};\, g], \end{aligned} \tag{10}$$

formulated in a specific set of coordinates.

2. Results

The first question that was examined was the ultraviolet behaviour of these theories. Less symmetry translates in this case into more divergences, as compared with the template of General Relativity. Specifically, a calculation employing the background field method was undertaken in order to extract the 1-loop order divergences [3]. We made use of a correspondence between TDiff theories and scalar-tensor theories in the so-called "unitary gauge", which helped simplify the computations. To 1-loop order, General Relativity is UV (on-shell) finite [4]. These theories are not, due to extra terms arising from the additional g-mode; in fact, the result is very similar to a scalar-tensor theory with the scalar replaced by g. The only exceptions are two already-mentioned limiting cases: General Relativity and the presence of an additional conformal Weyl symmetry (unimodular case).

Then, the observational signatures of these kind of theories were inspected [5]. A general remark is that an almost exact correspondence can be found between these theories and scalar-tensor theories where the g is promoted to a true scalar field ϕ, and a lagrangian multiplier is added to the action to force the constraint $\phi = g$ (the precise form of this correspondence remains to be stablished [in preparation]). In practice, we can thus apply all existing experimental bounds on scalar-tensor gravity to constrain these theories. One should note that, in general, the presence of g "alone" in the matter action will translate into a *non-metric coupling* of the scalar ϕ, when viewed in terms of the scalar-tensor language; hence, we will get wild violations of the (Weak) Equivalence Principle. A mechanism in the spirit of the metric postulate [6], or the one usually advocated in string theory [7] will be generically necessary to render the theory compatible with observations.

3. Conclusions

- TDiff theories are well motivated alternative theories of gravity from a quantum perspective.
- To the 1-loop level, these theories present UV divergences — while General Relativity is UV finite —, with only one significant exception that corresponds, arguably, to Unimodular Gravity.
- The experimental bounds on scalar-tensor theories of gravity are generically applicable to these theories. A mechanism to avoid "excessive" violations of the (Weak) Equivalence Principle is thus necessary to render the theory compatible with experiments.

[1] J. J. van der Bij, H. van Dam and Y. J. Ng, "The Exchange Of Massless Spin Two Particles," Physica **116A**, 307 (1982).

[2] E. Alvarez, D. Blas, J. Garriga and E. Verdaguer, "Transverse Fierz-Pauli symmetry," Nucl. Phys. B **756** (2006) 148 [arXiv:hep-th/0606019].

[3] E. Alvarez, A. F. Faedo and J. J. Lopez-Villarejo, "Ultraviolet behavior of transverse gravity," JHEP **0810** (2008) 023 [arXiv:0807.1293 [hep-th]].

[4] G. 't Hooft and M. J. G. Veltman, "One loop divergencies in the theory of gravitation," Annales Poincare Phys. Theor. A **20** (1974) 69.

[5] E. Alvarez, A. F. Faedo and J. J. Lopez-Villarejo, "Observational constraints on transverse gravity," JCAP **0907** (2009) 002 [arXiv:0904.3298 [hep-th]].

[6] C. M. Will, "The confrontation between general relativity and experiment," Living Rev. Rel. **9**, 3 (2005) [arXiv:gr-qc/0510072].

[7] T. Damour, "Questioning the equivalence principle," arXiv:gr-qc/0109063.