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Without a Safety (k)-Net

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[1] When analyzing a passage of music with K-nets, the transformational operation from one K-net to the next is just as important as the construction of each individual K-net. To ensure that different K-nets are related, various authors have proposed that either an interval (Lewin 1990 and Klumpenhouwer 1991) or an inversional relationship (Stoecker 2002) from one K-net to the next must remain invariant. And with all the isographic tools that are available—strong, positive, negative, and axial isography—virtually every trichordal K-net can be isographically related to any other trichordal K-net. This high degree of isographic relatedness is a concern for Michael Buchler. In his critique of K-nets and K-net analyses, Buchler points out that

"[s]ince the most promiscuous trichord classes include many of the most common and familiar melodic and harmonic structures found in a wide range of repertoire, trichordal isography generally comes easily to those who seek it. When the standard for pcset relatedness is this low, analysts ought to exercise particular diligence and discretion in making a strong case for the uniqueness and musicality of their readings."(1)

[2] Though Buchler warns us about the promiscuous nature of trichordal isography, there are times when adjacent, trichordal collections do not share an interval or an inversional relationship. As a result, the K-nets that model these sonorities cannot be isographically related, creating a gap in our transformational pathway. These isographic holes are the focus of my article. Trichordal K-nets that cannot be conventionally and axially isographic are extremely rare in the literature. It is unusual

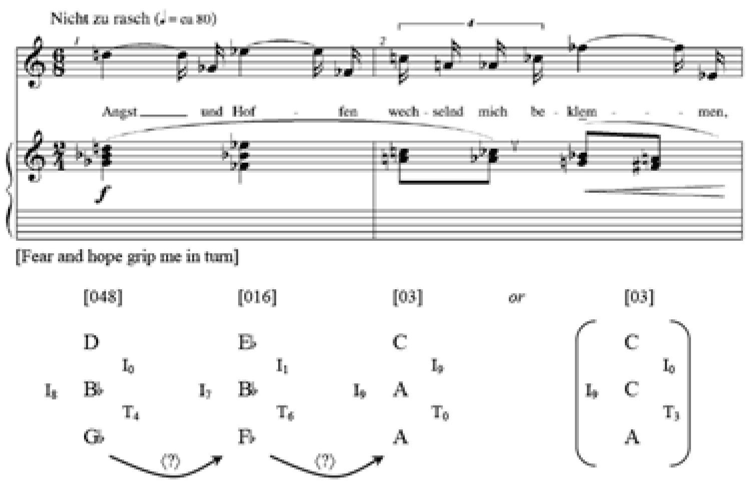
to find two collections that do not share an interval class or an inversional relationship.(2) To better understand how these analytical gaps occur, I will focus on the voice leading between K-nets that cannot be isographically related.

[3] In his first published study of K-nets, David Lewin analyzes a few brief passages from “Eine blasse Wäscherin,” the fourth song from Arnold Schoenberg’s *Pierrot Lunaire*, op. 21.(3) The *Pierrot* passages that Lewin analyzes are good ones for illustrating the strengths of a K-net analysis. Lewin effectively demonstrates how to construct coherent, transformational pathways and introduces K-net

Example 1. Schoenberg, “Eine blasse Wäscherin,” *Pierrot Lunaire*, op. 21, mm. 1–4

(click to enlarge)

Example 2. Schoenberg, “Angst und Hoffen,” *Das Buch der hängenden Gärten*, op. 15, mm. 1–2



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recursion. But if we analyze the entire song with a K-net lens, we will find that the isographic tools that Lewin introduced cannot relate a few K-nets.

[4] Example 1 includes the score for the opening instrumental phrase of “Eine blasse Wäscherin.” Most of the trichords in this passage contain a dyad of interval-class 3, and the K-nets that model these collections could be configured in such a way that T_3 and T_9 arrows are drawn. From an isographic

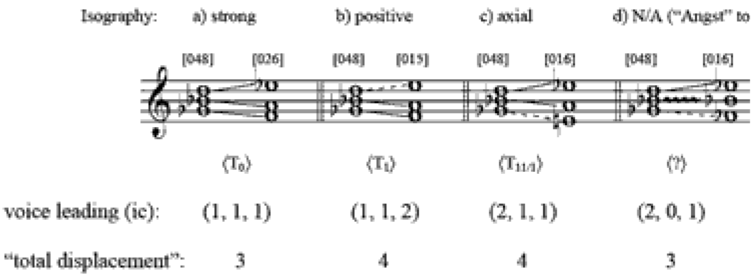
perspective, virtually every trichordal K-net in this passage is strong, positively, or negatively isographic to its adjacent K-net. In m. 3, however, the K-net that interprets the [036] diminished trichord cannot be isographically related to the K-net that interprets the [048] augmented trichord. Since these two collections do not share an interval class, the K-nets cannot be related by strong, positive, or negative isography. And since these particular diminished and augmented trichords do not share an inversional relationship, they cannot be related by axial isography. As a result, the K-nets that interpret these two collections cannot, in any way, be isographically related.

[5] An even more remarkable passage to explore non-isographic K-nets occurs in Schoenberg's song "Angst und Hoffen" from his *Buch der hängenden Gärten*, op. 15,

no. 7, given in Example 2.[4] The first three collections of the piano accompaniment are members of three different set classes: [048], [016], and [03] respectively. The augmented triad accompanies the word “Angst” while set-class [016] accompanies the word “Hoffen.” The A4/C5 dyad on the downbeat of the second measure accompanies the word “wechselnd.” Throughout the song, the “Angst” [048] and “Hoffen” [016] sonorities alternate with one another, reflecting the anxious longing for love by the narrator. What makes the opening accompanimental passage so remarkable from a K-net perspective is that these three adjacent collections do not share an interval or an inversional relationship, two requirements to ensure K-net relatedness.(5)

[6] Example 3 presents models that review voice-leading properties for three different types of

Example 3. Voice-leading scenarios from Schoenberg’s “Angst” trichord



isographies—the example does not include negative isography—and the last progression is Schoenberg's "Angst" trichord to the "Hoffen" trichord. When trichords are strongly isographic (Ex. 3a) the registral, voice-leading lines feature three moves, all by the same interval class. When trichords are positively (Ex. 3b) or axially isographic (Ex. 3c) the registral, voice-leading

(click to enlarge)

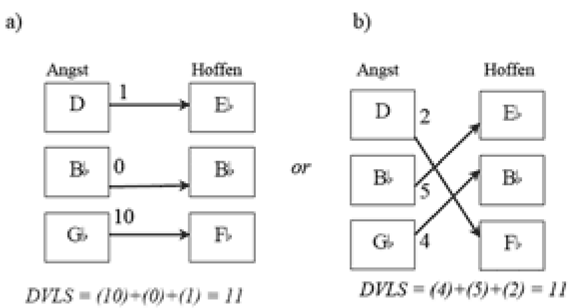
lines will always feature two different interval-class moves. When trichordal K-nets *cannot* be isographically related, the registral lines will always feature three different voice-leading moves.(6) In Example 3d, the progression from the "Angst" trichord to the "Hoffen" trichord features three different registral voice-leading moves.

[7] An effective way to measure the voice leading of non-isographic K-nets is to use Joseph Straus's "total displacement," which is the sum in absolute value of the three

voice-leading moves from one trichord to the next.(7) In Example 3d, the registral voice leading from the "Angst" to the "Hoffen" trichord features three different interval classes (2, 0, and 1), and these three voice-leading intervals sum to a total displacement of 3. A total displacement of three is significant since this is the smoothest possible voice leading for trichordal K-nets that cannot be conventionally and axially isographic. Since non-isographic K-nets will always feature three *different* voice-leading moves, a total displacement of 0 (0+0+0), 1 (0+0+1), and 2 ((0+1+1) or (0+0+2)) is not possible.(8)

[8] Another way to measure all the possible voice-leading lines between non-isographic K-nets is to use Richard Cohn's ***directed voice-leading sum*** (DVLS), which is the sum of the ordered pitch-class intervals from one trichord to the next.(9) As shown in Figure 1a, three different voice-leading lines are drawn from "Angst" to "Hoffen": 10, 0, and 1. These three voice-leading intervals sum to 11. If different voice-leading lines are chosen, as shown in Figure 1b, the DVLS number will be the same.(10) Because Cohn takes into account the direction of the pitch-class intervals, a consistent DVLS number will occur, regardless of the voice-leading mappings drawn.(11)

Figure 1. Cohn's Directed Voice-Leading Sum (DVLS)
applied to "Angst" and "Hoffen"



(click to enlarge)

Figure 2. The [016] presets that cannot be isographically related—strong, positive, negative, and axial—to {G♭,B♭,D}, the "Angst" sonority

[048] "Angst"
{G♭,B♭,D} 6

X [016]	SUM(X)	DVLS
{C,C♯,F♯}	7	1
{F♯,G,C}	1	7
{D,E♭,A♭}	1	7
{A♭,A,D}	7	1
{E,F,B♭}	7	1
{B♭,B,E}	1	7
{C,F,F♯}	11	5
{F♯,B,C}	5	11
{D,G,A♭}	5	11
{A♭,C♯,D}	11	5
{E,A,B♭}	11	5
{B♭,E♭,E}	5	11

(click to enlarge)

[9] If the original "Hoffen" trichord is transposed, different DVLS numbers emerge. Figure 2 shows that the "Angst" trichord cannot be conventionally or axially isographic with twelve of the twenty-four members from the [016] "Hoffen" family.(12) The figure also illustrates that four different DVLS numbers are possible when the "Angst" trichord is followed by one of the non-isographic, [016] pitch-class sets: 7 and its complement 5, 1 and its complement 11.(13)

[10] One of the advantages of using Cohn's DVLS is that it helps catalog the different types of voice leadings for non-isographic K-nets, which I have included in Figure 3.(14) Each DVLS number is paired with its complement and listed along the top. Beneath each DVLS category I have included all set-class pairs that cannot be conventionally or axially isographic. For example, DVLS 0 contains only two set-class pairs that cannot be isographic: [015][036] and [024][036]. What the figure does not tell us is which [015] is not isographic with which particular [036]. Note that each non-isographic progression listed in the figure has at least one symmetrical trichord; these sonorities contain a limited number of different interval classes, which increases the chances that it will not be isographic with its adjacent sonorities. In addition to the symmetrical set classes, another manifestation of symmetry occurs in Figure 3: the trichordal progressions that are listed in DVLS 0 also appear in DVLS 6; the set-class pairs listed in 1/11 also appear in 5/7, and so on.(15)

Figure 3. Set-class pairs that are not isographic organized into DLVS categories

DVLS	0	1/11	2/10	3/9	4/8	5/7	6
	[015][036]	[013][048]	[014][027]	[012][036]	[014][027]	[013][048]	[015][036]
	[024][036]	**[016][048]**[6]	[024][016]	[012][048]	[024][016]	**[016][048]**[3/5]	[024][036]
		[024][016]	[037][012]	[024][036]	[037][012]	[024][016]	
		[025][048][4]		[027][036]		**[025][048]**[2]	
				[027][048][2]			
				[036][048][1/7]			

(1) Erik Satie, *Vexations*, first variation
(2) Schoenberg, "Mondestrunken," *Pierrot Lunaire*, op. 21, mm. 29–30
(3) Schoenberg, "Colombine," *Pierrot Lunaire*, op. 21, mm. 9–11, right-hand piano accompaniment only
(4) Schoenberg, "Valse de Chopin," *Pierrot Lunaire*, op. 21, m. 29, right-hand piano accompaniment only
(5) Schoenberg, "Valse de Chopin," *Pierrot Lunaire*, op. 21, mm. 21–22, right hand piano accompaniment only
(6) Schoenberg, "Angst und Hoffen," *Das Buch der hängenden Gärten*, op. 15, mm. 1–2
(7) Schoenberg, "Eine blasse Wäscherin," *Pierrot Lunaire*, op. 21, mm. 1–4

(click to enlarge)

[11] The set-class pairs highlighted in bold (Fig. 3) represent examples I found in the literature. Of the twenty-four possible pairs of non-isographic K-nets listed in the figure, only six are represented here. And in every case, an augmented triad plays a role in establishing the non-isographic relationship. In fact, the chance that an augmented triad plays the spoiler is quite high. Recall that in Figure 2, the [048] "Angst" trichord cannot be isographic with twelve different pitch-class sets from the [016] family.[16] In addition to its intervallic redundancy, all four members of the [048] family feature only even I_n labels. This is a significant property since all the other trichordal set classes feature two odd I_n labels and only a single even I_n label.[17] So when it comes to network isography, [048] functions as an isographic renegade.

[12] Though the augmented triad is responsible for most of the transformational gaps, trichordal K-nets that are not conventionally or axially isographic are quite rare. If we were to analyze the rest of the trichords in Schoenberg's "Eine blasse Wäscherin," network isography will be easily achieved. It should come as no surprise that in all the analyses that use trichordal K-nets, the analyst has presented coherent, transformational pathways since K-net relatedness is so common. Still, there is a definite limit to the abundant isographic relations among trichordal K-nets. But what do these isographic holes tell us about the music it models? As Figure 3 illustrates, all non-isographic progressions include at least one trichord that is symmetrically

organized. In addition, non-isographic K-nets will always feature three different voice-leading moves, creating a maximally diverse pitch-class counterpoint. Recall that in Schoenberg's "Angst und Hoffen," the opening [048] to [016] progression features a variety of different voice-leading lines, and it is not difficult to imagine Schoenberg choosing two trichords that do not share any similarities; for me, Schoenberg's diverse voice-leading lines appropriately accompany the words fear and hope of the text. So when it finally happens that K-nets cannot be isographically connected, the transformational paths are momentarily thwarted and the voice leading from one trichord to the next is maximally diverse. Rather than search for cues elsewhere to bridge these transformational gaps, I wish to highlight and celebrate those extraordinarily unique moments when our transformational safety (k)-net has been taken away.