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Recasting K-nets[1]

REFERENCE: Michael Buchler, "Reconsidering Klumpenhouwer Networks," *Music Theory Online* 13.2

ABSTRACT: In this article I propose a new notation for hyper-transpositions and hyper-inversions, in which the usual subscripts are divided by 2. This new notation allows us to recast the hyper operations as ordinary transpositions and inversions operating on equivalence classes in 24-tone equal temperament. This suggests a response to Buchler and Losada, who both criticize the conceptual foundations of standard K-net analysis.

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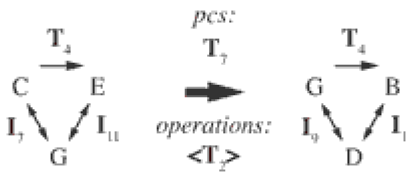
I. The Problem

[1] In a recent article, Michael Buchler observes that K-nets such as those in Figure 1, which I will notate as {C, E} + {G} and {G, B} + {D}, are related by $\langle \mathbf{T}_2 \rangle$ ("hyper-$\mathbf{T}_2$").[2], [3] He asks, in effect, "what's $\mathbf{T}_2$-like about the relation between C major and G major triads?"

Figure 1.

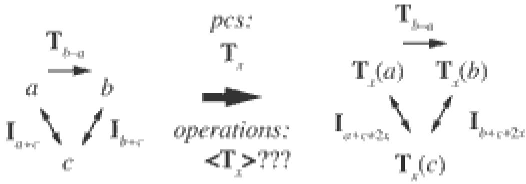


(click to enlarge)

Figure 2.

[2] This question is worth taking seriously. Although

hyper-transposition is different from ordinary transposition, being a function over functions rather than a function over pitch classes, comparisons between these two types of "transposition" are intrinsic to the practice of K-net analysis. And although the primary analytical use of K-nets is to relate sets belonging to different set classes, the technology applies equally well to chords such as C major and G major. Buchler has uncovered an example that seems to demonstrate that there is only a tenuous analogy between the two sorts of transposition. It will not do simply to reiterate that they are different. For Buchler's challenge is, *given* that they disagree so dramatically about such a simple case, what's the musical value of comparing them?



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[3] It is possible, however, that a simple change in notation

might help meet his objection. For suppose we used the label <$\mathbf{T}_7$> to refer to the hyper-transposition linking {C, E} + {G} to {G, B} + {D}. In that case, the force of Buchler's worry would be significantly ameliorated, since there is *obviously* something $\mathbf{T}_7$-like about the relationship. We might therefore ask whether it is possible to label the members of the hyper-TI group <TI>, such that, if $\mathbf{T}_x$ or $\mathbf{I}_y$ transforms the pitch classes in one K-net K into those of another K′, with arrows being updated accordingly, <$\mathbf{T}_x$> or <$\mathbf{I}_y$> transforms the arrow-labels in K into the arrow-labels in K′? (See Figure 2.) In other words, can we label the elements of the hyper-TI group in a way that is consistent with the TI group?

II. The Solution

[4] Yes. Simply divide the current hyper-T and hyper-I labels by 2. The only complication is that division is not uniquely defined in circular pitch-class space.

[5] As shown in Figure 1, {C, E} + {G} relates to {G, B} + {D} by <$\mathbf{T}_2$> in ordinary K-net notation. I propose instead that we label this hyper-transposition <$\mathbf{T}_1$ *or* $\mathbf{T}_7$>.

This is because 1 + 1 = 7 + 7 = 2 (mod 12). In other words, “2 divided by 2” can be either 1 or 7 in pitch-class space. Note that the label <**T**$_1$ *or* **T**$_7$> does *not* refer to a “dual transposition”: it does not mean that one part of the K-net moves by 1 semitone while the other moves by 7 semitones. Instead, it says that the two parts of the K-net move by a total distance that is equal to both 1 + 1 and 7 + 7 (mod 12). Intuitively, the parts move by an *average* distance of 1 or 7 (mod 12): thus they might move by 1 and 1, 7 and 7, 0 and 2, 6 and 8, and so on.

[6] Now what’s **T**$_1$ or **T**$_7$-ish about the relation between C major and G major? The **T**$_7$ relationship is obvious. What about **T**$_1$? Well, {C, E} + {G} is strongly isographic (<**T**$_0$>) to {G♭, B♭} + {D♭} and the G♭ major chord has to be transposed up one semitone to become G major. This is illustrated in Figure 3. The hyper-transpositional labels thus reflect actual transpositions: C major is related to G major by <**T**$_1$ *or* **T**$_7$> because C major is strongly isographic to chords that are **T**$_1$- and **T**$_7$-related to G major (G♭ major and C major respectively). For the sake of clarity, I’m going to drop the clumsy <**T**$_1$ *or* **T**$_7$> notation and use whichever of the pair is most

Figure 3.

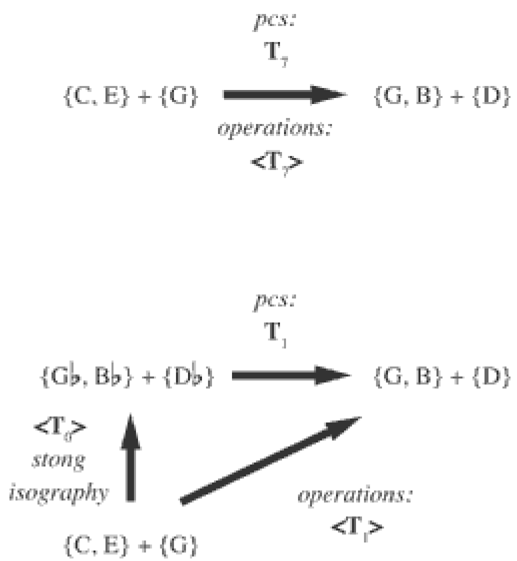

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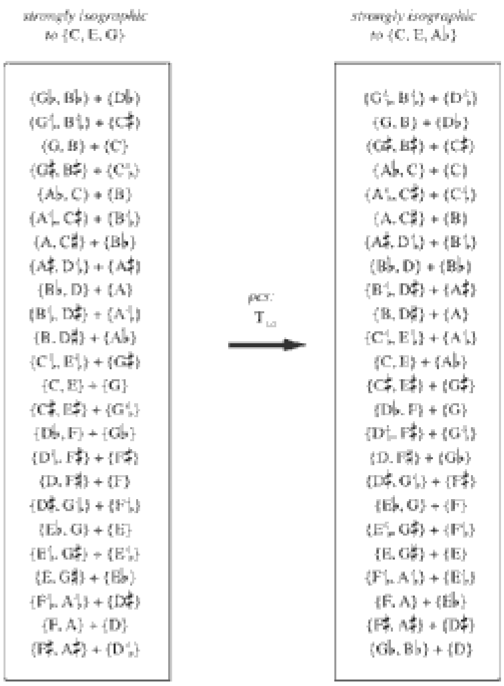
Figure 4.

appropriate to the context.[(4)]

[7] Q: In the new system, what hyper-transposition relates {C, E} + {G} and {C, E} + {A♭}? A: <**T**$_{1/2}$> ("hyper-T one-half"). What could that mean? Here's an informal way to think about it: in 24-tone equal-temperament, {C, E} + {G} and {C𝄳, E𝄳} + {G𝄲} (C and E quarter-tone flat, G quarter-tone sharp) are strongly isographic, and hence related by <**T**$_0$>. Transposing the second chord up by half a semitone yields {C, E} + {A♭}. Again, in the new system, the hyper-T labels reflect *actual transpositional relationships*: if *A* is related to *B* by <**T**$_x$> then there is a chord strongly isographic to *A* that can be transposed by *x* semitones to get chord *B*. Note however that this chord *need not live in 12-tone equal temperament*—in fact for half-integer hyper-T, it will live in 24-tone equal-temperament. Happily, once we pay attention to these quarter-tone chords, then



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"positively isographic to *A*" simply means "transpositionally related to a chord that is strongly isographic to *A*." Likewise, "negatively isographic to *A*" means "inversionally related to a chord that is strongly isographic to *A*."

[8] The new notation suggests a response to those, like Buchler, who worry that hyper-T and hyper-I are too abstract. Every K-net defines an equivalence class of strongly isographic chords in 24-tone equal-temperament, as shown in Figure 4. The statement that two (12-tone equal-tempered) K-nets are related by $<\mathbf{T}_x>$ or $<\mathbf{I}_y>$ is logically equivalent to the statement that these equivalence classes relate by $\mathbf{T}_x$ or $\mathbf{I}_y$ (Figure 4). Thus we need not think of "hyper-transposition" and "hyper-inversion" as being extremely abstruse "functions over functions": instead, we can understand them as ordinary transpositions and inversions of equivalence classes.(5) In principle, there is nothing

problematic about using transposition and inversion to relate equivalence classes; we do this every time we talk about transpositionally related pitch-class sets. The only question is whether equivalence classes of strongly isographic chords are analytically or perceptually meaningful—an issue we will return to shortly.

[9] I know that many music theorists dislike quarter tones. So here's an argument for my new notation that relies on 12-tone equal temperament exclusively: in twelve-tone equal-temperament, *no* chord that is strongly isographic to {C, E} + {G} is transpositionally related to {C, E} + {A♭}. Hence we need to identify the hyper-T operation that takes {C, E} + {G} to {C, E} + {A♭} using a label *not* used by any equal-tempered transposition. (Remember, we want to ensure consistency between labels for the TI group and the <TI> group.) Furthermore, we want to

label this hyper-transposition with a number x such that $\langle \mathbf{T}_x \rangle + \langle \mathbf{T}_x \rangle = \langle \mathbf{T}_{x+x} \rangle = \langle \mathbf{T}_1 \rangle$. (This is because doing $\langle \mathbf{T}_x \rangle$ twice produces what we have decided to call $\langle \mathbf{T}_1 \rangle$.) This strongly suggests taking x to be 1/2.[6]

[10] Since there are several ways to notate ordinary inversions, there are several ways to notate hyper-inversions.

Alternative 1. If you label inversions with index numbers, you should divide the current hyper-I labels by 2. In this new system, $\langle \mathbf{T}_x \rangle \mathbf{I}_y = \mathbf{I}_{2x+y}$ and $\langle \mathbf{I}_x \rangle \mathbf{I}_y = \mathbf{I}_{2x-y}$. Some hyper-inversions will be labeled with half-integers, just like hyper-transpositions.

Alternative 2. If you use Lewin-type labels, such as $\mathbf{I}_{E}^{E\flat}$, then $\langle \mathbf{T}_x \rangle \mathbf{I}_{b}^{a}$ will transpose *a* and *b* by an average of *x* semitones. Thus $\langle \mathbf{T}_{1/2} \rangle \mathbf{I}_{E}^{E\flat} = \mathbf{I}_{E}^{E}$, since here we transpose E$\flat$ by 1 and E by 0, for an average transposition of (1 + 0)/2 = 1/2. In this system, some hyper-inversions will be labeled with quarter tones.

Alternative 3. Perhaps the best solution is to divide *ordinary* inversion labels by 2, labeling inversions by their fixed points, or $\frac{\textit{index numbers}}{2}$. In this system, $\mathbf{I}_{E}^{E\flat}$ is represented by $\mathbf{I}_{3.5}$. (Informally, this is an inversion that maps pitch class 3.5, or E$\overset{4}{\flat}$, to itself.) This allows us to keep the traditional rule $\langle \mathbf{T}_x \rangle \mathbf{I}_y = \mathbf{I}_{x+y}$. However, in this system, some hyper-inversions will have *quarter*-integer labels, such as $\langle \mathbf{I}_{1/4} \rangle$.

[11] Table 1 compares these alternatives. Note the consistency, in Alternative 3, between the way the TI operations operate on pitches, collections, and operations: in each case, **T** adds *x* to something (a pitch or an inversion subscript), while **I** subtracts something from 2*x* (a pitch or an inversion subscript). In standard notation, there is an inconsistency between the way TI operate on collections and operations, while in Alternative 1, there is an inconsistency between the way TI operate on pitches and on

collections and operations. My goal here is to ensure consistency between collections and operations.[7]

Table 1.

	Pitches	Collections	Operations
Standard Notation	$\mathbf{T}_x(p) = x + p$	$\mathbf{T}_x(p, \mathbf{I}_y(p)) = (p + x, \mathbf{I}_{2x+y}(p + x))$	$<\mathbf{T}_x>\mathbf{I}_y = \mathbf{I}_{(x+y)}$
	$\mathbf{I}_x(p) = x - p$	$\mathbf{I}_x(p, \mathbf{I}_y(p)) = (x - p, \mathbf{I}_{2x-y}(x - p))$	$<\mathbf{I}_x>\mathbf{I}_y = \mathbf{I}_{(x-y)}$
Alternative 1	$\mathbf{T}_x(p) = x + p$	$\mathbf{T}_x(p, \mathbf{I}_y(p)) = (p + x, \mathbf{I}_{2x+y}(p + x))$	$<\mathbf{T}_x>\mathbf{I}_y = \mathbf{I}_{(2x+y)}$
(index numbers)	$\mathbf{I}_x(p) = x - p$	$\mathbf{I}_x(p, \mathbf{I}_y(p)) = (x - p, \mathbf{I}_{2x-y}(x - p))$	$<\mathbf{I}_x>\mathbf{I}_y = \mathbf{I}_{(2x-y)}$
Alternative 2	$\mathbf{T}_x(p) = x + p$	$\mathbf{T}_x(p, \mathbf{I}_b^a(p)) = (p + x, \mathbf{I}_{b+x}^{a+x}(p + x))$	$<\mathbf{T}_x>\mathbf{I}_b^a = \mathbf{I}_{b+x}^{a+x}$
(Lewin labels)	$\mathbf{I}_b^a(p) = a + b - p$	$\mathbf{I}_b^a(p, \mathbf{I}_d^c(p)) = (a + b - p, \mathbf{I}_{a+b-d}^{a+b-c}(a + b - p))$	$<\mathbf{I}_b^a>\mathbf{I}_d^c = \mathbf{I}_{a+b-d}^{a+b-c}$
Alternative 3	$\mathbf{T}_x(p) = x + p$	$\mathbf{T}_x(p, \mathbf{I}_y(p)) = (p + x, \mathbf{I}_{x+y}(p + x))$	$<\mathbf{T}_x>\mathbf{I}_y = \mathbf{I}_{(x+y)}$
(inversional centers)	$\mathbf{I}_x(p) = 2x - p$	$\mathbf{I}_x(p, \mathbf{I}_y(p)) = (2x - p, \mathbf{I}_{2x-y}(2x - p))$	$<\mathbf{I}_x>\mathbf{I}_y = \mathbf{I}_{(2x-y)}$

(click to enlarge)

III. Problems for recursivity

[12] What happens to existing K-net analyses if we adopt my proposal? Well, ordinary T labels stay the same while hyper-T labels are divided by two. This means that in the new notation, existing "recursive" K-net analyses—purporting to demonstrate isographies between K-nets and "hyper K-nets"[8]—will no longer seem to be recursive. It follows that their apparent "recursivity" depends on a particular (and possibly ad hoc) way of labelling hyper-operations. I therefore agree with Buchler that we should be concerned about the significance of these analyses.

[13] One can save these analyses by asserting that they require only a *group isomorphism* between the hyper-TI group and the ordinary TI group. My rejoinder is threefold:

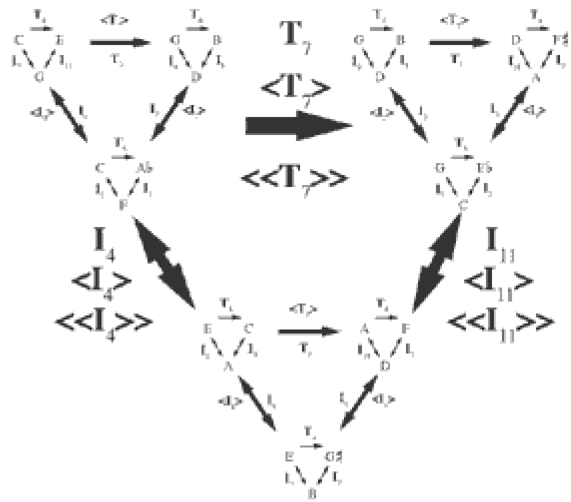
1. Group isomorphism is actually a weak relationship.[9]
2. The use of labels like $\langle T_1 \rangle$ and T_1 misleads people by making the isomorphism look stronger than it actually is: if all we care about is group isomorphism, then we can associate $\langle T_1 \rangle$ with either T_1, T_5, T_7, or T_{11}. From the standpoint of group theory, there's no principled reason for choosing among these.

3. In the new notation, we can have recursive K-net analyses that are underwritten by something stronger than mere group-theoretical isomorphism. My $\langle T_x \rangle$ really can be described as transpositions: they transpose equivalence classes of strongly isographic chords by *x* semitones. (Similar points apply to $\langle I_x \rangle$, which invert these equivalence classes.) Consequently, there is a canonical way to associate each ordinary transposition T_x to a particular hyper-transposition $\langle T_x \rangle$—something that group theory alone does not provide. Furthermore, in the new system, there is a clear analogy between the techniques of K-net analysis and those of standard set theory—the main difference being that K-net analysis manipulates equivalence classes of strongly isographic chords, rather than equivalence classes of transpositionally or inversionally related chords.

All of this, in my view, argues strongly in favor of the proposal.

IV. Going up the hierarchy

Figure 5.



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[14] Q. Can we do this again at the next level? That is, can we label hyper-hyper-transpositions $\langle\langle \mathbf{T}_x \rangle\rangle$ consistently with our labeling of the transpositions and hyper-transpositions?

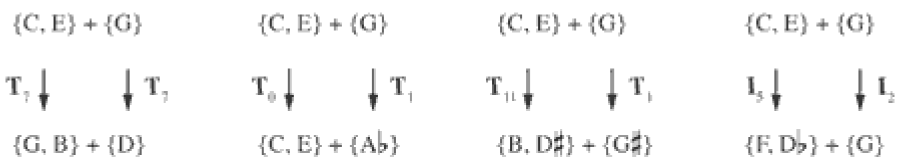
[15] A. Yes, as Figure 5 demonstrates. There are no new difficulties here—and no need for anything other than half-integer labels. (In particular, there is no cascade to ever-smaller fractions, as the hyper-operations get more hyper.) I'll leave the details up to the reader. The resulting system

has the nice feature that if you take a big hyper-hyper-hyper -...-hyper-network of pitch classes, transpose all the pitch classes by *x* semitones, update all your arrow-labels accordingly, doing the same for your hyper-arrows, your hyper-hyper-arrows, etc., your two networks are related by $\mathbf{T}_x$, $<\mathbf{T}_x>$, $<<\mathbf{T}_x>>$, and so on. (Similar points apply to inversion and hyper-hyper-...-hyper-inversional labels.) In other words, the ordinary transpositional and inversion labels propagate up the system. This is not the case in standard notation, a fact that can lead to philosophical puzzles.[10]

V. Hyper-transposition and dual transpositions

[16] Buchler perceptively remarks that K-net analyses try to pack too much information into a single number. He suggests that we use O'Donnell's "dual transpositions" and "dual inversions" rather than hyper-transposition and hyper-inversion, identifying the particular TI operations that apply to each of the K-net's two parts.[11] This is illustrated in Figure 6. I think this is a reasonable suggestion. However, it should be noted that there are two independent questions here. First, should we describe the relation between K-nets using one number or two? And second, should we use "dual transpositions" or hyper-transpositions?

Figure 6.



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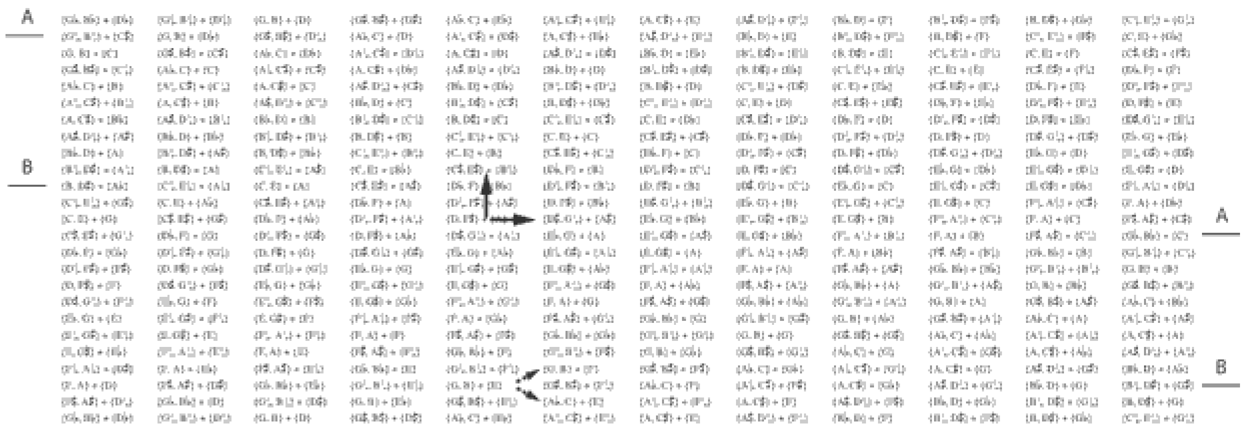
[17] Buchler labels the relation between K-nets {C, E} + {G} and {C, E} + {A♭} using the dual transposition $\mathbf{T}_0/\mathbf{T}_1$, indicating that the major third stays fixed while the singleton moves by one semitone. But we can also label this relation **<**$\mathbf{X}_{1/2}/\mathbf{T}_{1/2}$>. (Here “X” stands for “eXpand.”) The subscript of <$\mathbf{X}_x$> is calculated by taking half of the *difference* between the two components of Buchler’s dual transposition, while <$\mathbf{T}_x$> is equal to their average: mathematically, the dual transposition $\mathbf{T}_a/\mathbf{T}_b$ is equivalent to the pair of transformations <$\mathbf{X}_{(b-a)/2}/\mathbf{T}_{(a+b)/2}$>. Thus, since {C, E} + {G} and {C, E} + {A♭} relate by $\mathbf{T}_0/\mathbf{T}_1$, they also relate by <$\mathbf{X}_{(1-0)/2}/\mathbf{T}_{(1+0)/2}$>= <$\mathbf{X}_{1/2}/\mathbf{T}_{1/2}$>.

[18] What do these numbers mean? Figure 7 presents a two-dimensional graph listing all the K-nets, in 24-tone equal-temperament, that are positively isographic to {C, E} + {G}, and that belong to set classes found in 12-tone equal temperament. Strongly isographic chords lie on the same vertical line. Transpositionally related chords lie on the same horizontal line. The graph should be interpreted as a 2-torus, with its right edge glued to its left, and the top edge glued to the bottom. (As in early video-games such as *Pac-Man* or *Asteroids*, one can move off of an edge to reappear on the opposite side of the figure.[(12)]) What I am calling the <$\mathbf{X}_x$> transform moves the K-net 2x vertical steps on this figure, while hyper-transposition <$\mathbf{T}_y$> moves 2y steps horizontally. The first transformation represents motion within the equivalence class of strongly isographic chords, and describes how the structure of the set class is altered: under <$\mathbf{X}_x$> each part of the K-net moves by x semitones in contrary motion, possibly changing the set class in the process.[(13)] The second transformation represents motion from one equivalence class to another, moving all the pitch-classes in the K-net upward by y semitones. As can be seen from the figure, dual transpositions represent an alternative way of describing relationships on this

two-dimensional surface: the dual transposition $\mathbf{T}_a/\mathbf{T}_b$ moves a chord *a* positions diagonally southeast, and *b* positions diagonally northeast. The two notational systems are fundamentally equivalent, and are related by what physicists would call a "coordinate transformation."(14)

Figure 7.



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[19] Let's now return to the issue of K-nets' abstractness. Buchler is right to observe that standard K-net analyses use one number where it is possible to use two. In fact, we can restate his observation more precisely: K-net analyses use only the *x*-coordinate to refer to relationships between objects situated on a two-dimensional surface. One possible response is to adopt "dual transposition" labels. A slightly more conservative alternative, from the standpoint of traditional K-net theory, is to use the <$\mathbf{X}_x/\mathbf{T}_y$> labels described here. These simply extend the techniques of K-net analysis by adding the missing coordinate, representing motion within equivalence classes of strongly isographic chords.

[20] To my mind, the deepest question raised by Buchler's article is this: what coordinate system should we use when navigating Figure 7? I share his feeling that "dual transformations" are somewhat more general than hyper-transpositions and inversions. (In particular, I think we should be reluctant to use standard K-net technology in cases where the musical surface does not clearly project exact contrary

motion.) Furthermore, I am sympathetic with Buchler's complaint that K-net analysis throws away too much information. I see no reason why we should have developed an analytical tradition that pays attention to only *one* of the two dimensions of Figure 7. Buchler's worry, which I share, is that our analytical practices may derive not from deep conceptual reflection, or from underlying musical necessity, but simply from the force of institutional habit: we disregard the second coordinate because we have always done so, perhaps without even noticing that it could be incorporated into our K-net analyses.

VI. Philosophical issues

[21] The proposal in this paper is at bottom notational, a suggestion that we use new words to describe familiar relationships. It might therefore seem that I do not go to the musical heart of the matter. My response is that notation is not at all trivial, but is rather something that shapes thought. The importance of good notation is well understood by physicists and mathematicians, and is an issue that deserves more music-theoretical scrutiny—especially since some of our basic notational conventions are conceptually quite confusing.

[22] Among these is the practice of referring to inversions by index number. Ultimately, the notational question I have been pursuing is this: what should we call the function $F(\mathbf{I}_x) = \mathbf{T}_1(\mathbf{I}_x)\mathbf{T}_{-1} = \mathbf{I}_{x+2}$? My suggestion is that we should call it <$\mathbf{T}_1$> , or 揾 yper-$\mathbf{T}_1$,?because it shifts the axis of inversional symmetry up by semitone: C is mapped to C under $\mathbf{I}_0$, and C♯ is mapped to C♯ under $\mathbf{T}_1(\mathbf{I}_0)\mathbf{T}_{-1} = \mathbf{I}_2$. This fact is obscured by index numbers, which represent the one-semitone shift of the axis of inversional symmetry by a *two*-unit change to the inversion 扭 subscript. Standard notation might therefore mislead the unwary theorist into thinking that, when the inversional axis of symmetry shifts up by one semitone, something else—the index number, whatever that is—has shifted by *two*.

[23] Familiar K-net terminology inherits this problem, using the label <$\mathbf{T}_2$> to refer to the operation $F(\mathbf{I}_x) = \mathbf{T}_1(\mathbf{I}_x)\mathbf{T}_{-1} = \mathbf{I}_{x+2}$. This may be a case of bad notation leading to confused thought. For though it can *feel* like we are transposing by two when we

increase the index numbers by two, and though thinking in this way may help us calculate, this is not at all what is happening musically: transposition by *x* changes index numbers by 2*x*, as both Figure 2 and Table 1 demonstrate. I worry that David Lewin may have been mislead by this simple but pernicious feature of our ordinary notation when he invented the now-standard labels for hyper-transpositions and hyper-inversions.

[24] However, it may be that Lewin was motivated not by conceptual confusion, but by the desire to identify $\mathbf{T}_1$, a *generator* of the T group, with $<\mathbf{T}_1>$, a generator of the hyper-T group. Presumably, this desire was in turn motivated by Lewin's goal of exploiting the group isomorphism between the <TI> hyper-operations and the TI operations. But is it so clear that this particular group isomorphism is, musically speaking, the most important one? My notation emphasizes a *different* isomorphism: that between the quotient group TI/$\mathbf{T}_6$ (the TI group for tritone-symmetrical objects, such as equivalence classes of strongly isographic chords) and a particular subgroup of <TI>—those that relate K-net arrows in networks whose pitch classes are related by ordinary transposition and inversion. (In Lewin's notation, these are the hyper-operations with even-numbered subscripts; in my notation, they have integer subscripts.) This relationship is more than a mere group isomorphism: any action of TI/$\mathbf{T}_6$ on K-net nodes induces a corresponding action of the <TI> subgroup on arrows; conversely, any action of the <TI> subgroup on arrows can be realized by networks whose PCs are related by a corresponding action of TI/$\mathbf{T}_6$ on nodes. In many circumstances, the two perspectives provide *alternate descriptions* of the same musical process.

[25] The interesting point is that Lewin chose to emphasize a relatively weak relationship (group isomorphism) at the expense of this stronger relationship—and that almost all subsequent users of K-nets have followed him. In doing so, they have asked us to overlook the fact that $<\mathbf{T}_1>$ (in my notation) *actually transposes something by one semitone* and focus instead on the very abstract fact that my $<\mathbf{T}_1>$ generates only half of the <T> operations. But it should be understood that this approach is rooted in a discrete, group-theoretical perspective. I would argue that it's

not necessary—and perhaps not even productive—to think about K-nets in this way. In fact, there's a very beautiful (and in my view much more natural) geometrical interpretation of K-nets and their significance.[15]

[26] What rests on the choice between standard notation and my own? First, convenience and conceptual clarity: the system I advocate is (I claim) simpler and more logical, once you get used to it. Second, generalizability: K-nets define "wedge" voice leadings in which the two parts of the K-net move in exact contrary motion; the system I propose can be extended to "generalized K-nets" whose parts move along arbitrary voice leadings—and not simply those involving exact contrary motion. These arbitrary voice leadings define generalized analogues to "strong isography" and give rise to equivalence classes related by generalized analogues to the <TI> operations. (Unfortunately, describing this generalization further is beyond the scope of the current paper.[16]) Third, our understanding of the relationship between <$\mathbf{T}_x$> and $\mathbf{T}_x$. Under my proposal they're extremely close, both transposing something by x—chords in one case, equivalence classes of strongly isographic chords in the other.

[27] How should we understand Buchler's criticisms in light of my proposed notational reforms? As I have indicated, I believe that some of his complaints can be ameliorated by a simple change of notation. There is indeed a close relationship between hyper-transposition and ordinary transposition, though it is obscured by standard K-net terminology. At the same time, even in my new notational system, it is clear that some of Buchler's criticisms remain intact. K-nets are woven with a very coarse mesh, allowing a lot of useful musical information to wriggle free. (Indeed, relative to Figure 7, K-nets are not nets at all, but rather threads—using one dimension where two are needed!) I applaud Michael for having the courage to point this out, and for challenging us to think about how the practice of K-net analysis might be improved.