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FUZZY IDEALS IN GAMMA NEAR-RINGS

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Abstract

The aim of this paper is to introduce the notion of fuzzy left (resp. right) ideals of Γ-near-rings, and to study the related properties.

1. Introduction

Γ-near-rings were defined by Bh. Satyanarayana [18], and the ideal theory in Γ-near-rings was studied by Bh. Satyanarayana [18] and G. L. Booth [1, 2]. Fuzzy ideals of rings were introduced by W. Liu [13], and it has been studied by several authors [4, 10, 11, 19]. The notion of fuzzy ideals and its properties were applied to various areas: semigroups [6, 12, 14], BCK-algebras [9, 16], and semirings [8]. In this paper we consider the fuzzification of left (resp. right) ideals of Γ-near-rings, and investigate the related properties.

2. Preliminaries

We first recall some basic concepts for the sake of completeness. Recall from [15, p.3] that a non-empty set R with two binary operations "+"(addition) and "·" (multiplication) is called a *near-ring* if it satisfies the following axioms:

(i) $(R, +)$ is a group,

(ii) $(R, \cdot)$ is a semigroup,

(iii) $(x + y) \cdot z = x \cdot z + y \cdot z$ for all $x, y, z \in R$.

Precisely speaking, it is a right near-ring because it satisfies the right distributive law. We will use the word “near-ring” to mean “right near-ring”. We denote xy instead of $x \cdot y$.

A Γ*-near-ring* ([18]) is a triple $(M, +, \Gamma)$ where

(i) $(M, +)$ is a group,

(ii) Γ is a nonempty set of binary operators on M such that for each $\alpha \in \Gamma$, $(M, +, \alpha)$ is a near-ring,

(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

A subset A of a Γ-near-ring M is called a *left (*resp. *right) ideal* of M if

(i) $(A, +)$ is a normal divisor of $(M, +)$,

(ii) $u\alpha(x + v) - u\alpha v \in A$ (resp. $x\alpha u \in A$) for all $x \in A$, $\alpha \in \Gamma$ and $u, v \in M$.

We now review some fuzzy logic concepts. A fuzzy set in a set M is a function $\mu : M \to [0, 1]$. We shall use the notation μ_t, called a *level subset* of μ, for $\{x \in M | \mu(x) \geq t\}$ where $t \in [0, 1]$. If μ is a fuzzy set in M and f is a function defined on M, then the fuzzy set ν in $f(M)$ defined by

$$\nu(y) = \sup_{x \in f^{-1}(y)} \mu(x)$$

for all $y \in f(M)$ is called the *image* of μ under f. Similarly if ν is a fuzzy set in $f(M)$, then the fuzzy set $\mu = \nu \circ f$ in M (that is, the fuzzy set defined by $\mu(x) = \nu(f(x))$ for all $x \in M$) is called the *preimage* of ν under f (see [17]). We say that a fuzzy set μ in M has the *sup property* ([17]) if, for any subset T of M, there exists $t_0 \in T$ such that

$$\mu(t_0) = \sup_{t \in T} \mu(t).$$

3. Fuzzy ideals of Γ-near-rings

Definition 3.1 *A fuzzy set μ in a Γ-near-ring M is called a fuzzy left (*resp. *right) ideal of M if*

(i) *μ is a fuzzy normal divisor with respect to the addition,*

(ii) *$\mu(u\alpha(x+v)-u\alpha v)\geq\mu(x)$ (resp. $\mu(x\alpha u)\geq\mu(x)$) for all $x,u,v\in M$ and $\alpha\in\Gamma$.*

The condition (i) of Definition 3.1 means that μ satisfies:

(i) $\mu(x-y)\geq\min\{\mu(x),\mu(y)\}$,

(ii) $\mu(y+x-y)\geq\mu(x)$,

for all $x,y\in M$.

Note that if μ is a fuzzy left (resp. right) ideal of a Γ-near-ring M, then $\mu(0)\geq\mu(x)$ for all $x\in M$, where 0 is the zero element of M.

Theorem 3.2 *Let M be a Γ-near-ring and μ be a fuzzy left (resp. right) ideal of M. Then the set*

$$M_\mu:=\{x\in M|\mu(x)=\mu(0)\}$$

is a left (resp. right) ideal of M.

Proof. Let μ be a fuzzy left ideal and let $x,y\in M_\mu$. Then

$$\mu(x-y)\geq\min\{\mu(x),\mu(y)\}=\mu(0),$$

and so $\mu(x-y)=\mu(0)$ or $x-y\in M_\mu$. For every $y\in M$ and $x\in M_\mu$, we have $\mu(y+x-y)\geq\mu(x)=\mu(0)$. Hence $y+x-y\in M_\mu$, which shows that M_μ is a normal divisor of M with respect to the addition. Let $x\in M_\mu$, $\alpha\in\Gamma$ and $u,v\in M$. Then

$$\mu(u\alpha(x+v)-u\alpha v)\geq\mu(x)=\mu(0),$$

and hence $\mu(u\alpha(x+v)-u\alpha v)=\mu(0)$, i.e., $u\alpha(x+v)-u\alpha v\in M_\mu$. Therefore M_μ is a left ideal of M. Similarly we have the desired result for the right case. □

Theorem 3.3 *Let A be a non-empty subset of a Γ-near-ring M and μ_A be a fuzzy set in M defined by*

$$\mu_A(x) := \begin{cases} s, & \text{if } x \in A, \\ t, & \text{otherwise,} \end{cases}$$

for all $x \in M$ and $s, t \in [0,1]$ with $s > t$. Then μ_A is a fuzzy left (resp. right) ideal of if and only M if A is a left (resp. right) ideal of M. Moreover $M_{\mu_A} = A$.

Proof. Let μ_A be a fuzzy left (resp. right) ideal of M and let $x, y \in A$. Then

$$\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\} = s,$$

and so $\mu_A(x-y) = s$. This implies $x - y \in A$. For any $y \in M$ and $x \in A$, we have $\mu_A(y+x-y) \geq \mu_A(x) = s$ and so $y+x-y \in A$. Now let $x \in A$, $\alpha \in \Gamma$ and $u, v \in M$. Then $\mu_A(u\alpha(x+v) - u\alpha v) \geq \mu_A(x) = s$ (resp. $\mu_A(x\alpha u) \geq \mu_A(x) = s$), and hence $\mu_A(u\alpha(x+v) - u\alpha v) = s$ (resp. $\mu_A(x\alpha u) = s$). Thus $u\alpha(x+v) - u\alpha v \in A$ (resp. $x\alpha u \in A$). This shows that A is a left (resp. right) ideal of M. Conversely assume that A is a left (resp. right) ideal of M. Let $x, y \in M$. If at least one of x and y does not belong to A, then $\mu_A(x-y) \geq t = \min\{\mu_A(x), \mu_A(y)\}$. If $x, y \in A$, then $x - y \in A$ and so $\mu_A(x-y) = s = \min\{\mu_A(x), \mu_A(y)\}$. If $x \in A$, then $y + x - y \in A$ and hence $\mu_A(y+x-y) = s = \mu_A(x)$. Clearly $\mu_A(y+x-y) \geq t = \mu_A(x)$ for all $x \notin A$ and $y \in M$. This shows that μ_A is a fuzzy normal divisor with respect to the addition. Now let $x, u, v \in M$ and $\alpha \in \Gamma$. If $x \in A$, then $u\alpha(x+v) - u\alpha v \in A$ (resp. $x\alpha u \in A$) and thus $\mu_A(u\alpha(x+v) - u\alpha v) = s = \mu_A(x)$ (resp. $\mu_A(x\alpha u) = s = \mu_A(x)$). If $x \notin A$, then clearly $\mu_A(u\alpha(x+v) - u\alpha v) \geq t = \mu_A(x)$ (resp. $\mu_A(x\alpha u) \geq t = \mu_A(x)$). Hence μ_A is a fuzzy left (resp. right) ideal of M. Moreover

$$\begin{aligned} M_{\mu_A} &= \{x \in M | \mu_A(x) = \mu_A(0)\} \\ &= \{x \in M | \mu_A(x) = s\} \\ &= \{x \in M | x \in A\} \\ &= A. \end{aligned}$$

□

Corollary 3.4 *Let M be a Γ-near-ring and χ_A be the characteristic function of a subset $A \subset M$. Then χ_A is a fuzzy left (resp. right) ideal if and only if A is a left (resp. right) ideal.*

Theorem 3.5 *Let μ be a fuzzy set in a Γ-near-ring M. Then μ is a fuzzy left (resp. right) ideal of M if and only if each level subset μ_t, $t \in \mathrm{Im}(\mu)$, of μ is a left (resp. right) ideal of M.*

We then call μ_t a *level left* (resp. *right*) *ideal* of μ.

Proof. Let μ be a fuzzy left (resp. right) ideal of M and let $t \in \mathrm{Im}(\mu)$. For any $x, y \in \mu_t$, we have

$$\mu(x - y) \geq \min\{\mu(x), \mu(y)\} \geq t$$

and so $x - y \in \mu_t$. Let $y \in M$ and $x \in \mu_t$. Then $\mu(y + x - y) \geq \mu(x) \geq t$; whence $y + x - y \in \mu_t$. Now let $x \in \mu_t$, $\alpha \in \Gamma$ and $u, v \in M$. Then

$$\mu(u\alpha(x + v) - u\alpha v) \geq \mu(x) \geq t$$

(resp. $\mu(x\alpha u) \geq \mu(x) \geq t$), which implies that $u\alpha(x + v) - u\alpha v \in \mu_t$ (resp. $x\alpha u \in \mu_t$). Hence μ_t is a left (resp. right) ideal of M. Conversely assume that μ_t is a left (resp. right) ideal of M for every $t \in \mathrm{Im}(\mu)$. If $\mu(x_0 - y_0) < \min\{\mu(x_0), \mu(y_0)\}$ for some $x_0, y_0 \in M$, then by taking

$$t_0 = \frac{1}{2}(\mu(x_0 - y_0) + \min\{\mu(x_0), \mu(y_0)\})$$

we have $\mu(x_0 - y_0) < t_0$, $\mu(x_0) > t_0$ and $\mu(y_0) > t_0$. Hence $x_0 - y_0 \notin \mu_{t_0}$, $x_0 \in \mu_{t_0}$ and $y_0 \in \mu_{t_0}$. This is a contradiction, and so $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in M$. Assume that $\mu(y_0 + x_0 - y_0) < \mu(x_0)$ for some $x_0, y_0 \in M$. Putting

$$s_0 = \frac{1}{2}(\mu(y_0 + x_0 - y_0) + \mu(x_0));$$

then $\mu(y_0 + x_0 - y_0) < s_0 < \mu(x_0)$. It follows that $x_0 \in \mu_{s_0}$ and $y_0 + x_0 - y_0 \notin \mu_{s_0}$ which is impossible. Hence $\mu(y + x - y) \geq \mu(x)$ for all $x, y \in M$. If the condition (ii) of Definition 3.1 is not true, then for a fixed $\alpha \in \Gamma$ there exist $x, u, v \in M$ such that $\mu(u\alpha(x+v) - u\alpha v) < \mu(x)$ (resp. $\mu(x\alpha u) < \mu(x)$). Let $p_0 = \frac{1}{2}(\mu(u\alpha(x+v) - u\alpha v) + \mu(x))$

(resp. $q_0 = \frac{1}{2}(\mu(x\alpha u)+\mu(x))$. Then $u\alpha(x+v)-u\alpha v \notin \mu_{p_0}$ and $x \in \mu_{p_0}$ (resp. $x\alpha u \notin \mu_{q_0}$ and $x \in \mu_{q_0}$). This is a contradiction, and we are done. □

Theorem 3.6 *Let A be a left (resp. right) ideal of a Γ-near-ring M. Then for any $t \in (0,1]$ there exists a fuzzy left (resp. right) ideal μ of M such that $\mu_t = A$.*

Proof. Let $\mu : M \to [0,1]$ be a fuzzy set defined by

$$\mu(x) = \begin{cases} t, & \text{if } x \in A, \\ 0, & \text{otherwise,} \end{cases}$$

for all $x \in M$, where $t \in (0,1]$. Then clearly $\mu_t = A$. It is easy to prove that

$$\mu(x-y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in M.$$

Assume that $\mu(y+x-y) < \mu(x)$ for some $x, y \in M$. Since μ is two-valued, i.e., $|Im(\mu)| = 2$, $\mu(y+x-y) = 0$ and $\mu(x) = t$ and hence $y+x-y \notin A$ and $x \in A$. This contradicts the fact that $(A,+)$ is a normal divisor of $(M,+)$. Hence $\mu(y+x-y) \geq \mu(x)$ for all $x, y \in M$. Now assume that $\mu(u\alpha(x+v)-u\alpha v) < \mu(x)$ (resp. $\mu(x\alpha u) < \mu(x)$) for some $x, u, v \in M$ and $\alpha \in \Gamma$. Since $|Im(\mu)| = 2$, we have $\mu(u\alpha(x+v)-u\alpha v) = 0$ and $\mu(x) = t$ (resp. $\mu(x\alpha u) = 0$ and $\mu(x) = t$); whence $u\alpha(x+v)-u\alpha v \notin A$ and $x \in A$ (resp. $x\alpha u \notin A$ and $x \in A$). This is impossible because A is a left (resp. right) ideal of M, which proves the theorem. □

Theorem 3.7 *If μ is a fuzzy left (resp. right) ideal of a Γ-near-ring M, then*

$$\mu(x) = \sup\{t \in [0,1] | x \in \mu_t\}, \forall x \in M.$$

Proof. Let $s := \sup\{t \in [0,1] | x \in \mu_t\}$ and let $\varepsilon > 0$ be given. Then $s - \varepsilon < t$ for some $t \in [0,1]$ such that $x \in \mu_t$, and so $s - \varepsilon < \mu(x)$. Since ε is arbitrary, it follows that $s \leq \mu(x)$. Now let $\mu(x) = u$. Then $x \in \mu_u$ and so $u \in \{t \in [0,1] | x \in \mu_t\}$. Hence $\mu(x) = u \leq \sup\{t \in [0,1] | x \in \mu_t\} = s$. Therefore $\mu(x) = s$, as desired. □

We now consider the converse of Theorem 3.7. Let Λ be a non-empty subset of $[0,1]$. Without loss of generality, we may use Λ as an index set in the following:

Theorem 3.8 *Let $\{A_t | t \in \Lambda\}$ be a collection of left (resp. right) ideals of a Γ-near-ring M such that*

(i) $M = \underset{t\in\Lambda}{\cup} A_t$,

(ii) $s > t$ if and only if $A_s \subset A_t$ for all $s, t \in \Lambda$.

Define a fuzzy set μ in M by

$$\mu(x) = \sup\{t \in \Lambda | x \in A_t\}, \forall x \in M.$$

Then μ is a fuzzy left (resp. right) ideal of M.

Proof. Using Theorem 3.5, it is sufficient to show that μ_p $(\neq \emptyset)$ is a left (resp. right) ideal of M for every $p \in [0, 1]$. We consider the following two cases:

$$(1)\ p = \sup\{t \in \Lambda | t < p\} \text{ and } (2)\ p \neq \sup\{t \in \Lambda | t < p\}.$$

Case (1) implies that

$$\begin{aligned} x \in \mu_p &\Leftrightarrow x \in A_t \text{ for all } t < p \\ &\Leftrightarrow x \in \underset{t<p}{\cap} A_t, \end{aligned}$$

whence $\mu_p = \underset{t<p}{\cap} A_t$, which is a left (resp. right) ideal of M. For the case (2), there exists $\varepsilon > 0$ such that $(p-\varepsilon, p) \cap \Lambda = \emptyset$. We claim that $\mu_p = \underset{t\geq p}{\cup} A_t$. If $x \in \underset{t\geq p}{\cup} A_t$, then $x \in A_t$ for some $t \geq p$. It follows that $\mu(x) \geq t \geq p$ so that $x \in \mu_p$. Conversely if $x \notin \underset{t\geq p}{\cup} A_t$, then $x \notin A_t$ for all $t \geq p$, which implies that $x \notin A_t$ for all $t > p - \varepsilon$, that is, if $x \in A_t$ then $t \leq p - \varepsilon$. Thus $\mu(x) \leq p - \varepsilon$ and so $x \notin \mu_p$. Consequently $\mu_p = \underset{t\geq p}{\cup} A_t$ which is a left (resp. right) ideal of M. This completes the proof. □

Definition 3.9 ([2]) *Let M and N be Γ-near-rings. A map $\theta : M \to N$ is called a Γ-near-ring homomorphism if $\theta(x + y) = \theta(x) + \theta(y)$ and $\theta(x\alpha y) = \theta(x)\alpha\theta(y)$ for all $x, y \in M$ and $\alpha \in \Gamma$.*

Theorem 3.10 *A Γ-near-ring homomorphic preimage of a fuzzy left (resp. right) ideal is a fuzzy left (resp. right) ideal.*

Proof. Let $\theta : M \to N$ be a Γ-near-ring homomorphism, ν a fuzzy left (resp. right) ideal of N and μ the preimage of ν under θ. Then

$$\begin{aligned}\mu(x-y) &= \nu(\theta(x-y)) \\ &= \nu(\theta(x)-\theta(y)) \\ &\geq \min\{\nu(\theta(x)), \nu(\theta(y)) \\ &= \min\{\mu(x), \mu(y),\end{aligned}$$

$$\begin{aligned}\mu(y+x-y) &= \nu(\theta(y+x-y)) \\ &= \nu(\theta(y)+\theta(x)-\theta(y)) \\ &\geq \nu(\theta(x)) \\ &= \mu(x),\end{aligned}$$

and

$$\begin{aligned}\mu(u\alpha(x+v)-u\alpha v) &= \nu(\theta(u\alpha(x+v)-u\alpha v)) \\ &= \nu(\theta(u)\alpha(\theta(x)+\theta(v))-\theta(u)\alpha\theta(v)) \\ &\geq \nu(\theta(x)) \\ &= \mu(x)\end{aligned}$$

(resp. $\mu(x\alpha u) = \nu(\theta(x\alpha u)) = \nu(\theta(x)\alpha\theta(u)) \geq \nu(\theta(x)) = \mu(x)$) for all $x, y, u, v \in M$ and $\alpha \in \Gamma$. Hence μ is a fuzzy left (resp. right) ideal of M. □

Let $\theta : M \to N$ be a Γ-near-ring homomorphism. Assume that μ is a fuzzy left ideal of M with the sup property and let ν be the image of μ under θ. Given $\theta(x), \theta(y) \in \theta(M)$, let $x_0 \in \theta^{-1}(\theta(x))$, $y_0 \in \theta^{-1}(\theta(y))$, $u_0 \in \theta^{-1}(\theta(u))$ and $v_0 \in \theta^{-1}(\theta(v))$ be such that $\mu(x_0) = \sup_{z\in\theta^{-1}(\theta(x))} \mu(z)$, $\mu(y_0) = \sup_{z\in\theta^{-1}(\theta(y))} \mu(z)$, $\mu(u_0) = \sup_{z\in\theta^{-1}(\theta(u))} \mu(z)$ and $\mu(v_0) = \sup_{z\in\theta^{-1}(\theta(v))} \mu(z)$, respectively. Then

$$\begin{aligned}\nu(\theta(x)-\theta(y)) &= \sup_{z\in\theta^{-1}(\theta(x)-\theta(y))}\mu(z)\\ &\geq \mu(x_0-y_0)\\ &\geq \min\{\mu(x_0),\mu(y_0)\\ &= \min\{\sup_{z\in\theta^{-1}(\theta(x))}\mu(z),\sup_{z\in\theta^{-1}(\theta(y))}\mu(z)\}\\ &= \min\{\nu(\theta(x)),\nu(\theta(y)),\end{aligned}$$

$$\begin{aligned}\nu(\theta(y)+\theta(x)-\theta(y)) &= \sup_{z\in\theta^{-1}(\theta(y)+\theta(x)-\theta(y))}\mu(z)\\ &\geq \mu(y_0+x_0-y_0)\\ &\geq \mu(x_0)\\ &= \sup_{z\in\theta^{-1}(\theta(x))}\mu(z)\\ &= \nu(\theta(x)),\end{aligned}$$

and for any $\alpha\in\Gamma$,

$$\begin{aligned}&\nu(\theta(u)\alpha(\theta(x)+\theta(v))-\theta(u)\alpha\theta(v))\\ =\ &\sup_{z\in\theta^{-1}(\theta(u)\alpha(\theta(x)+\theta(v))-\theta(u)\alpha\theta(v))}\mu(z)\\ \geq\ &\mu(u_0\alpha(x_0+v_0)-u_0\alpha v_0)\\ \geq\ &\mu(x_0)\\ =\ &\sup_{z\in\theta^{-1}(\theta(x))}\mu(z)\\ =\ &\nu(\theta(x)).\end{aligned}$$

This proves that ν is a fuzzy left ideal of N. Similarly if μ is a fuzzy right ideal of M with the sup property, then the image ν of μ under θ is a fuzzy right ideal of N. Hence we have the following result.

Theorem 3.11 *A Γ-near-ring homomorphic image of a fuzzy left (resp. right) ideal possessing the sup property is a fuzzy left (resp. right) ideal.*

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