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AUGMENTED GRADED RINGS

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Abstract

In this paper we study the augmented graded rings and give the ralationship between these rings and stronger properties of graded rings.

0.Introduction.

Let G be a multiplicative group with identity e and R be an associative G-graded ring with unity 1. Then R_e (the identity component of R) is a subring of R and $1 \in R_e$ (See [5]). One of the most important problems in graded ring theory is to study the link between a certain property for R or (R, G) and R_e.

Now, one can think about grading R_e by a group G. This types of rings appear naturally when we deal with the group ring of G over a G-graded ring R.

In this paper, we discuss the G-graded rings in which the identity component is itself a G-graded ring satisfying some related conditions with the graduation of G. We call these rings augmented G-graded rings. Also, we give the relationship between these rings and the stronger properties of G-graded rings given in [8]. This simple effort will help for studying those rings in details and that will be a good tool to solve many problems in Graded Ring Theory.

1.Preliminaries.

In this section we give some basic definitions and facts of graded rings. For more details one can look in [8].

Definition 1.1. *Let G be a group with identity e. Then a ring R is G-graded if there exist additive subgroups R_g of R such that $R = \bigoplus_{g \in G} R_g$ and $R_g R_h \subseteq R_{gh}$ for all $g, h \in G$. We denote for this graduation by (R, G) and R_e will be the identity component of R. The elements of R_g are called homogeneous of dimension g. For $x \in R$, we write x_g for the component of x in R_g, so that x can be written uniquely as $\sum_{g \in G} x_g$.*

We denote supp $(R,G)=\{g\in G : R_g\neq 0\}$. *Note that supp* (R,G) *need not be a subgroup of* G.

Definition 1.2. *For a* G*-graded ring* R *we say :*

1) (R,G) *is strong if* $R_g R_h = R_{gh}$ *for all* $g,h\in G$. *But this definition is equivalent to* $1\in R_g R_{g^{-1}}$ *for all* $g\in G$ *(See proposition 1.6 of [3]).*

2) (R,G) *is first strong if* $1\in R_g R_{g^{-1}}$ *for all* $g\in$ *supp* (R,G).

3) (R,G) *is second strong if* $R_g R_h = R_{gh}$ *for all* $g,h\in$ *supp* (R,G) *and supp* (R,G) *is a monoid in* G.

4) (R,G) *is faithful if for any* $a_g\in R_{g^{-0}}, a_g R_h\neq 0$ *and* $R_h a_g\neq 0$ *for all* $h\in G$.

5) (R,G) *is nondegenerate if for any* $a_g\in R_g - 0, a_g R_{g^{-1}}\neq 0$ *and* $R_{g^{-1}} a_g\neq 0$.

The nondegenerate and faithful properties are motivated by the work of Cohen and Rowen in [1]. The strongly graded rings have been studied by E. C.Dade in [2], where they were called clifford systems. In [8] we studied the other stronger properties and gave relationship between all of them.

Lemma 1.3 *([5]).* R_e *is a subring of* R *and* $1\in R_e$.

2.Augmented Graded Rings.

In this section we study the augmented graded rings and then give the relationship between these rings and some stronger properties G-graded rings.

Definition 2.1 *A ring* R *is said to be an augmented* G*-graded ring if it statisfies the following conditions:*

1) $R=\bigoplus_{g\in G} R_g$ *where* R_g *is an additive subgroup of* R *and* $R_g R_h\subseteq R_{gh}$ *for all* $g,h\in G$. *(*R *is a* G*-graded ring).*

2) *If* R_e *is the identity component of the graduation given in 1 then* $R_e=\bigoplus_{g\in G} R_{e-g}$ *where* R_{e-g} *is an additive subgroup of* R_e *and* $R_{e-g}R_{e-h}\subseteq R_{e-gh}$ *for all* $g,h\in G$. *(*R_e *is a* G*-graded ring).*

3) *For each* $g\in G$, *there exists* $r_g\in R_g$ *such that* $R_g=\bigoplus_{h\in G} R_{e-h}\, r_g$ *we assume* $r_e=1$.

4) If $g, h \in G$ and $r_g\,, r_h$ are both non-zero, then $r_g\, r_h = r_{gh}$ and $(x\, r_g)\,(y\, r_h) = xy\, r_{gh}$ for all $x,\ y \in R_e$.

We need condition 3 of the definition to give a connection between the graduation of R and the graduation of R_e. Also, condition 4 is necessary for proving some of the following useful remarks.

Remarks 2.2.

1) Condition 3 of the definition implies $R_h = R_e r_h$ for all $h \in G$.

   To see this, let $x \in R_h$. Then $x = s_{e-g_1} r_h + ... + s_{e-g_n}\, r_h$ where $s_{e-g_i} \in R_{e-g_i}$, and hence $x = (s_{e-g_1} + ... + s_{e-g_n})\, r_h \in R_e\, r_h$. Conversely, let $s\, r_h \in R_e\, r_h$. Then $s\, r_h = (s_{e-g_1} r_h + ... + s_{e-g_n}\, r_h) \in R_h$ where $s_{e-g_i} \in R_{e-g_i}$.

2) Condition 4 of the definition is equivalent to :

   $(x_{e-h_1}\, r_g)\,(y_{e-h_2}\, r_h) = x_{e-h_1}\, y_{e-h_2}\, r_{gh}$ for all $x_{e-h_1} \in R_{e-h_1}, y_{e-h_2} \in R_{e-h_2}$ and non-zero $r_g\,, r_h$. Clearly condition 4 implies this condition. Let $x, y \in R_e$. For simplicity we assume $x = x_{e-h_1} + x_{e-h_2}$ and $y = y_{e-g_1} + y_{e-g_2}$. Then $(x\, r_g)\,(y\, r_h) = x_{e-h_1}\, y_{e-g_1}\, r_{gh} + x_{e-h_1}\, y_{e-g_1}\, r_{gh} + x_{e-h_2}\, y_{e-g_1}\, r_{gh} + x_{e-h_2}\, y_{e-g_1}\, r_{gh} = (xy)\, r_{gh}$.

3) R_g is a G-graded R_e-module with the usual multiplication on R and with the graduation $R_{g-h} = R_{e-h}\, r_g$ for all $h \in G$.

   (i) R_{g-h} is an additive subgroup of R_g.

   (ii) $R_{e-e}\, R_{g-h} = R_{e-e}\, R_{e-h}\, r_g \subseteq R_{e-h}\, r_g\,, i.e., R_{g-h} \in R_{e-e}$ -mod.

   (iii) $R_{e-h_1}\, R_{g-h_2} = (R_{e-h_1}\, R_{e-h_2}) r_g \subseteq R_{e-h_1 h_2} \quad r_g = R_{g-h_1 h_2}$.
   Clearly $R_g = \bigoplus_{h \in G} R_{g-h}$.

4) $R_{g-h}\, R_{g'-h'} \subseteq R_{gg'-hh'}$ for all $g, g', h, h' \in G . R_{g-h} R_{g'-h'} = R_{e-h}\, r_g R_{e-h'} r_{g'}$. If $r_g = 0$ or $r_{g'} = o$ then done. Suppose $r_g\,, r_{g'}$ are both non zero. By condition 4 of the definition we have $R_{g-h}\, R_{g'-h'} = R_{e-h}\, R_{e-h'} r_{gg'} \subseteq R_{e-hh'} \quad r_{gg'} = R_{gg'-hh'}$.

5) If $r_g\,, r_{g^{-1}}$ are both non-zero then $r_g\, R_e = R_e\, r_g$. Let $x \in R_e$. Then $r_g\, x = r_g x.1 = (r_g x) r_e = (r_g\, x)\,(r_{g^{-1}}\, r_g) = (1.r_g)\,(x\, r_{g^{-1}})\ (1.r_g) = x\, r_g\, r_{g^{-1}} r_g = x\, r_g$.

In the following example , we see that if R is an associative ring with unity 1 and G is a group with identity e then R can be regarded as an augmented G-graded ring.

Example 2.3. Let R be any ring and G be any group. Then R is an augmented G-graded ring with :

$$R_g = \begin{cases} R & \text{if } g = e \\ O & \text{if } g \neq e \end{cases}, \quad R_{e-g} = \begin{cases} R & \text{if } g = e \\ O & \text{if } g \neq e \end{cases},$$

$$\text{and } r_g = \begin{cases} 1 & \text{if } g = e \\ O & \text{if } g \neq e \end{cases}.$$

1. Clearly R_g is an additive subgroup of R, $R_g R_h \subseteq R_{gh}$ for all $g, h \in G$ and $R = \bigoplus_{g \in G} R_g$.
2. R_{e-g} is an additive subgroup of R_e, $R_{e-g} R_{e-h} \subseteq R_{e-gh}$ and $R_e = \bigoplus_{g \in G} R_{e-g}$.
3. $R_g = \bigoplus_{h \in G} R_{e-h}\, r_g$.
4. If r_g, r_h are both non-zero then $g = h = e$, and hence $r_g r_h = 1 = r_e = r_{gh}$. Also, $(x r_g)(y r_h) = xy\, r_{gh}$ for all $x, y \in R_e$. We call this graduation the trivial augmented graduation of R by G.

Now , we will try to connect the augmented graded rings with the following facts which we proved in [8].

Proposition 2.4 ([8]).

1. *(R,G) strong $\Longrightarrow$ (R,G) first strong $\Longrightarrow$ (R,G) second strong.*
2. *(R,G) strong $\Longrightarrow$ (R,G) first strong $\Longrightarrow$ (R,G) nondegenerate.*
3. *(R,G) strong $\Longrightarrow$ (R,G) faithful $\Longrightarrow$ (R,G) nondegenerate.*
4. *(R,G) first strong $\Longleftrightarrow$ (R,G) is second strong and nondegenerate.*
5. *(R,G) strong $\Longleftrightarrow$ (R,G) is second strong and faithful.*
6. *(R,G) second strong and supp $(\mathbf{R},G) \leq G \Longrightarrow$ (R,G) first strong.*
7. *(R,G) second strong and G finite group $\Longrightarrow$ (R,G) first strong.*

Remark 2.5. If $\mid G \mid \geq 2$ then the trivial augmented G-graded rings are not faithful. In fact, supp (R, G) = supp $(R_e, G) = \{e\}$. Therefore, if R is an augmented G-graded ring then (R, G) is not necessarily faithful.

Proposition 2.6. *Let (R, G) be augmented G-graded ring and supp $(R, G) = G$. Then R is a strongly G-graded ring.*

Proof. : Suppose (R, G) is augmented G-graded ring. Let $g \in G$. Since supp $(R, G) = G$, R_g and $R_{g^{-1}}$ are non-zero and hence there exist non-zero elements $r_g \in R_g$ and $r_{g^{-1}} \in R_{g^{-1}}$. By condition 4, $1 = r_e = r_g r_{g^{-1}} \in R_g R_{g^{-1}}$, *i.e.*, $1 \in R_g R_{g^{-1}}$ and hence R is a strongly G-graded ring.

From this proposition we notice that the augmented graded rings are very useful in case supp $(R, G) = G$. In the following example, $R = K[x]$ is an augmented $\mathbb{Z}$-graded ring with supp $(R, \mathbb{Z}) = \mathbb{N} \cup \{o\}$ and supp $(R_o, \mathbb{Z}) = \{o\}$. But in Example 2.9 , we use the same ring with different graduations of R and R_o such that supp $(R, \mathbb{Z}) = \{o\}$ and supp $(R_o, \mathbb{Z}) = \mathbb{N} \cup \{o\}$. □

Example 2.7. Let $R = K[x]$. Then R is an augmented $\mathbb{Z}$-graded ring with :

$$R_j = \begin{cases} K & \text{if } j = O \\ Kx^j & \text{if } j > O \\ O & \text{if } j < O \end{cases}, \quad R_{o-i} = \begin{cases} K & \text{if } i = O \\ O & \text{if } i \neq O \end{cases},$$

$$\text{and } r_j = \begin{cases} 1 & \text{if } j = O \\ x^j & \text{if } j > O \\ O & \text{if } j < O \end{cases}.$$

1. $R = \bigoplus_{j \in \mathbb{Z}} R_j$ (This is the usual graduation of $K[x]$ by $\mathbb{Z}$), and supp $(R, \mathbb{Z}) = \mathbb{N} \cup \{o\}$.
2. $R_o = \bigoplus_{i \in \mathbb{Z}} R_{o-i}$ (This is the trivial graduation of K by $\mathbb{Z}$), and supp $(R_o, \mathbb{Z}) = \{o\}$.
3. Clearly $R_j = \bigoplus_{i \in \mathbb{Z}} R_{o-i}\, r_j$.
4. If r_i, r_j are non-zero then $i, j \geq o$ and hence $r_i r_j = r_{i+j}$. Since R is commutative ring we have $(x\, r_i)(y\, r_j) = (xy)\, r_{i+j}$.

Remark 2.8. If R is an augmented G-graded ring then (R, G) is not necessarily nondegenerate. In the previous example choose $g = 1$ and $a_g = x$. Then $a_g \in R_g - o$, $R_{g^{-1}} = R_{-1} = o$ and hence $a_g R_{g^{-1}} = o$, *i.e.*, (R, G) is degenerate.

Example 2.9. Let $R = K[x]$ and $G = \mathbb{Z}$. Then R is augmented G-graded ring with :

$$R_j = \begin{cases} R & \text{if } \; j = O \\ \\ O & \text{if } \; j \neq O \end{cases} , \; R_{o-j} = \begin{cases} K & \text{if } \; j = O \\ Kx^j & \text{if } \; j > O \\ O & \text{if } \; j < O \end{cases} ,$$

$$\text{and } r_j = \begin{cases} 1 & \text{if } \; j = O \\ O & \text{if } \; j \neq O \end{cases} .$$

1. $R = \bigoplus_{j \in \mathbb{Z}} R_j$, and supp $(R, \mathbb{Z}) = \{o\}$.

2. $R_o = \bigoplus_{i \in \mathbb{Z}} R_{o-i}$, and supp $(R_o, \mathbb{Z}) = \mathbb{N} \cup \{o\}$. Conditions 3 and 4 are obvious.

Now , we give two essential examples of augmented G-graded rings. In fact, we have built the concept augmented G-graded ring by taking the common properties between those examples.

Example 2.10. Let R be a G-graded ring and let $R[G]$ be the group ring of R over G. Then R[G] is an augmented G-graded ring with : $R[G]_g = R.g \, , R[G]_{e-g} = R_g.e$ and $r_g = 1.g$ for all $g \in G$.

1. $R[G]_g$ is an additive subgroup of $R[G]$, $R[G]_g \, R[G]_h = (R.g)\,(R.h) = R\,.gh = R[G]_{gh}$ and $R[G] = \bigoplus_{g \in G} R[G]_g$. Also, $\text{supp}(R[G]\,, G) = G$.

2. $R[G]_{e-g} = R_g.e$ is an additive subgroup of $R[G]_e \, , R[G]_{e-g} R[G]_{e-h} = R_g \,.eR_h \,.e \subsetneq R_{gh} \,.e = R[G]_{e-gh}$ and $R[G]_e = \bigoplus_{g \in G} R[G]_{e-g}$. Also, $\text{supp}(R[G]_e, G)$ is not necessarily G.

3. $R[G]_g = R.g = \bigoplus_{h \in G} R_h.g = \bigoplus_{h \in G} (R_h \,.e)\,(1.g) = \bigoplus_{h \in G} R[G]_{e-h} \, r_g$.

4. $r_g \, r_h = (1.g)\,(1.h) = 1.gh = r_{gh}$. Let $x, y \in R$ and $x.e, y.e \in R[G]_e$. Then $(x.e \, r_g) = (y.e \, r_h) = (x.e)\,(1.g)\,(y.e)\,(1.h) = (x.g)\,(y.h) = xy \, gh = (x.e)\,(y.e) r_{gh}$.

Example 2.11. Let R be a G-graded ring. Let $\overline{R[G]}$ be the left free R-module with basis G.

Let $\lambda_\sigma \in R_\sigma$ and $\lambda\sigma' \in R_{\sigma'}$ then for the elements $\lambda_\sigma \tau, \lambda_{\sigma'} \, \tau'$ define $(\lambda_\sigma \tau)\,(\lambda_{\sigma'} \tau') = (\lambda_\sigma \, \lambda_{\sigma'}) \; \sigma'^{-1} \tau \, \sigma' \tau'$. With this product $\overline{R[G]}$ is an associative ring with unity $1.e$ (See [4]).

Now $\overline{R[G]}$ is an augmented G-graded ring with :
$\overline{R[G]}_g = \bigoplus_{\sigma\in G} R_{g\sigma^{-1}}\sigma$, $\overline{R[G]}_{e-g} = R_g g^{-1}$ and $r_g = 1.g$ for all $g \in G$.

1. $\overline{R[G]}_g$ is an additive subgroup of $\overline{R[G]}$, and $\overline{R[G]}_g \overline{R[G]}_h \subseteq \overline{R[G]}_{gh}$ because $R_{g\sigma^{-1}}\sigma R_{h\tau^{-1}}\tau \subseteq R_{g\sigma^{-1}h\tau^{-1}}\tau h^{-1}\sigma h \subseteq \overline{R[G]}_{gh}$ for all $\sigma, \tau \in G$. Also, $\overline{R[G]} = \bigoplus_{g\in G} \overline{R[G]}_g$.
2. $\overline{R[G]}_{e-g} = R_g g^{-1}$ is an additive subgroup of $R[G]_e$, $\overline{R[G]}_{e-g}\overline{R[g]}_{e-h} = (R_g g^{-1})(R_h h^{-1}) \subseteq R_{gh} h^{-1} g^{-1} = \overline{R[G]}_{e-gh}$ and $\overline{R[g]}_e = \bigoplus_{g\in G} \overline{R[G]}_{e-g}$.
3. $\overline{R[G]}_h = \bigoplus_{\sigma\in G} R_{h\sigma^{-1}}\sigma$. But $R_{h\sigma^{-1}}\sigma = (R_{h\sigma^{-1}}\sigma h^{-1})(1.h) = (R_{h\sigma^{-1}}\sigma h^{-1}) r_h$, *i.e.*, $\overline{R[G]}_h = \bigoplus_{g\in G} \overline{R[G]}_{e-g}\, r_h$ for all $h \in G$.
4. Let $\lambda_\sigma \sigma^{-1}, \lambda_\tau \tau^{-1} \in R[G]_e$ and $g, h \in G$. Then $(\lambda_\sigma\sigma^{-1}) r_h (\lambda_\tau \tau^{-1}) r_g = (\lambda_\sigma \sigma^{-1})(1.h)(\lambda_\tau\tau^{-1})(1.g) = (\lambda_\sigma\sigma^{-1} h)(\lambda_\tau\tau^{-1} g) = \lambda_\sigma \lambda_\tau (\tau^{-1}\sigma^{-1} hg)$. On the other hand , $(\lambda_\sigma\sigma^{-1})(\lambda_\tau\tau^{-1}) r_{hg} = (\lambda_\sigma \lambda_\tau \tau^{-1}\sigma^{-1})(1.hg) = \lambda_\sigma\lambda_\tau(\tau^{-1}\sigma^{-1} hg)$. Clearly ,$r_h r_g = (1.h)(1.g) = 1.hg = r_{gh}$.

Remark 2.12. If R is an augmented G-graded ring , then $R \in R_e{-}gr$ with the usual product in R and with $R_{(g)} = \bigoplus_{h\in G} R_{h-g}$.

1. $R_{(g)}$ is an additive subgroup of R and $R_{(g)} \in R_{e-e}$-mod.
2. $R_{e-g}R_{(h)} \subseteq R_{(gh)}$ for all $g, h \in G$ because $R_{e-g}R_{\sigma-h} \subseteq R_{\sigma-gh} \subseteq R_{(gh)}$ for all $\sigma \in G$. Clearly $R = \bigoplus_{g\in G} R_{(g)}$.

In Example 2.10 , $R[G]_{(g)} = \bigoplus_{h\in G} R[G]_{h-g} = \bigoplus_{h\in G} R_g.h$.
In Example 2.11 , $\overline{R[G]}_{(g)} = \bigoplus_{h\in G} \overline{R[G]}_{h-g} = \bigoplus_{h\in G} R_g g^{-1} h$.

Definition 2.13 ([10]) . *Let R be a ring with two graduations (R, G) and (R, H). Then (R, G) is almost aquivalent to (R, H) if there exists a ring isomorphism $f : R \longrightarrow R$ such that for each $h \in H$ there exists $g \in G$ with $f(R_g) = R_h$.*

In [8,10] we proved that first strong, second strong and nondegenerate are preserved between almost equivalent graduations , but the faithfulness and strong properties are not. We finish this paper by giving similar result on the augmented graded rings.

Proposition 2.14 . *Let R be an augmented G-graded ring such that $supp(R,G) = supp(R_{e_G},G)$ and $R_{g_1}R_{g_2} \neq o$ for all $g_1, g_2 \in supp(R,G)$. Suppose (R,G) is almost equivalent to (R,H). Then R is an augmented H-graded ring with the same graduation of R by H and with a graduation on R_{e_H} defined as follows :*

$$R_{e_H} - h = \begin{cases} f(R_{e_G} - g) & \text{if } h \in \text{supp(R,H)} \quad \text{where } f(R_g) = R_h \\ O & \text{if } h \notin \text{supp(R,H)} \end{cases}$$

$$\text{Let } r_h = \begin{cases} f(r_g) & \text{if } h \in \text{supp (R,G) and} f(R_g) = R_h \\ O & \text{otherwise} \end{cases}$$

Proof. First , we show that R_{e_H} is an H-graded ring. Since f is an isomorphism , R_{e_H-h} is an additive subgroup of R_{e_H}. Let $h_1, h_2 \in \text{supp}(R,H)$. Then $R_{e_H-h_1}R_{e_H-h_2} = f(R_{e_G-g_1})f(R_{e_G-g_2})$ where $R_{h_1} = f(R_{g_1})$ and $R_{h_2} = f(R_{g_2})$.

Since $g_1, g_2 \in$ supp $(R,G), R_{g_1}R_{g_2} \neq O$ and hence $O \neq f(R_{g_1}R_{g_2}) \subseteq R_{h_1h_2}$. So , $f(R_{g_1g_2}) = R_{h_1h_2}$ (See Lemma 2.2 of [10]). Therefore , $R_{e_H-h_1}R_{e_H-h_2} \subseteq f(R_{e_G-g_1g_2}) = R_{e_H-h_1h_2}$. Clearly , $R_{e_H} = f(R_{e_G}) = \bigoplus_{g\in G} f(R_{e_G-g}) = \bigoplus_{h\in H} R_{e_H-h}$, *i.e.*, R_{e_H} is an H-graded ring.

Let $h \in supp(R,H)$. Then $R_h = f(R_g)$ for some $g \in$ supp (R,G). But $f(R_g) = \bigoplus_{\sigma\in G} f(R_{e_G-\sigma}\, r_g) = \bigoplus_{\sigma\in G} f(R_{e_G-\sigma)f(r_g)} = \bigoplus_{\tau\in H} R_{e_H-\tau}\, r_h$.

Suppose $h_1, h_2 \in$ supp (R,H). Then $r_{h_1} = f(r_{g_1})$ and $r_{h_2} = f(r_{g_2})$ where $f(R_{g_1}) = R_{h_1}$ and $f(R_{g_2}) = R_{h_2}$. Now , $r_{h_1}r_{h_2} = f(r_{g_1}r_{g_2}) = f(r_{g_1g_2}) \neq O$. But $f(R_{g_1g_2}) = R_{h_1h_2}$ implies $f(r_{g_1g_2}) = r_{h_1h_2}$, *i.e.*, $r_{h_1}r_{h_2} = r_{h_1h_2}$.

Finally, let $x, y \in R_{e_H}$. Then $(x\,r_{h_1})(y\,r_{h_2}) = f(x')f(r_{g_1})f(y')f(r_{g_2})$ where $f(x') = x$ and $f(y') = y$. Hence $f(x')f(r_{g_1})f(y')f(r_{g_2}) = f(x'\,r_{g_1}\,y'\,r_{g_2}) = f(x'y'\,r_{g_1g_2})$ $= f(x')f(y')f(r_{g_1g_2}) = xy\,r_{h_1h_2}$. □

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Genişletilmiş Katmanlı Halkalar

Özet

Bu çalışmada genişletilmiş katmanlı halkalar incelenmiş ve kuvvetli katmanlı halkalarla olan ilişkileri ortaya konmuştur.

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