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$\overline{\mathrm{NC}}$-p-GROUPS SATISFYING THE NORMALIZER CONDITION

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Abstract

Let G be a perfect locally nilpotent p-group in which every proper subgroup is nilpotent-by-Chernikov. It is shown that in $G/Z(G)$ every proper subgroup is a Chernikov extension of a nilpotent subgroup of finite exponent (Theorem 1). This result is then used to give a characterization of G if, in addition, it satisfies the normalizer condition (Theorem 2).

1. Introduction

A group G is called **locally graded** if every nontrivial finitely generated subgroup of G has a proper subgroup of finite index. B. Bruno in [5] called a locally graded group G a **minimal non-(nilpotent-by-finite) group ($\overline{\mathbf{NF}}$-group)** if every subgroup of G, other than G itself, is nilpotent-by-finite(NF-group). Later Otal and Penã in [14] introduced the class of $\overline{\mathbf{NC}}$**-groups** by substituting "Chernikov" for "finite" above Examples of $\overline{\mathrm{NF}}$-groups are the Heineken-Mohamed type groups given in [9], [10], [6], [7] and [11].

It seems that not much is known yet about perfect $\overline{\mathrm{NF}}$-p-groups and $\overline{\mathrm{NC}}$-p-groups (which are necessarily perfect by Theorem A of [3]). These groups are also studied in [1], [3] and [4] and it is not yet known whether or not they exist. If they do exist, we do not know whether or not they satisfy the normalizer condition.

Here two results on $\overline{\mathrm{NC}}$-p-groups satisfying the normalizer condition are obtained. The first result is a partial improvement of Theorem A(i) of [3] (p always denotes a prime number).

Theorem 1. *Let G be an $\overline{NC}$-p-group satisfying the normalizer condition. Then every proper subgroup of $G/Z(G)$ is a Chernikov extension of a nilpotent subgroup of finite exponent and every normal nilpotent subgroup of $G/Z(G)$ has finite exponent. Furthermore $G/Z(G)$ has a proper subgroup $E/Z(G)$ of finite exponent such that $G = E^G$.*

As an application of this theorem the following is obtained.

Theorem 2. *Let G be an $\overline{NC}$-p-group satisfying the normalizer condition. Suppose that for any two HM-subgroups S and T of $G/Z(G)$ the following holds. For every $a \in S'$*

$$[a, S, T] \leq [a, S].$$

Then either G has an epimorphic image which is an $\overline{NF}$-group or it has a proper normal subgroup K such that

$$G/K = \bigcup_{i=1}^{\infty} (T_i/K)'$$

where for each $i \geq 1, T_i/K$ is a normal HM-subgroup of G/K such that

$$(T_i/K)' \leq T_{i+1}/K)'.$$

A study of $\overline{NC}$-p-groups satisfying the normalizer condition began in [3]. One is tempted to think that such a group might have an epimorphic image which is an $\overline{NF}$-group but this is not yet known. In Section 3 of [3] an HM*-group was defined as a generalization of an Heineken-Mohammed group and some properties of it were given there. A locally nilpotent p-group $X \neq 1$ is called an **HM*-group** if X' is nilpotent and

$$X/X' \cong C_{p^\infty} \times \cdots \times C_{p^\infty} \quad (n \, factors \, for \, some \, n \geq 1).$$

If $n = 1$, $X' \neq 1$ and every subgroup of X is subnormal, then X is called an **HM-group**. In this case it follows from (1) Lemma of [12] that every proper subgroup of X is nilpotent (See the third sections of [3] and this paper for elementary properties of HM*-groups).

2. Proof of Theorem 1

We begin by stating without proof a slightly more general form of Lemma 2.1 of [3]. Indeed an easy inspection shows that in the statement of that lemma "$a^m \in Z(H)$" may be replaced by "$a^m \in C_G(H)$". So the following holds.

Lemma 2.1. *Let H be a normal subgroup and a be an element of a group G such that [H, a] $\leq$ Z(H). Let m, n, be positive integers such that $a^m \in C_G(H)$ and $< a >$ has subnormal index n in $H < a >$. Then*

$$H^{m^n} \leq C_G(a).$$

Proof. Same as the proof of Lemma 2.1 of [3]. □

Lemma 2.2. *Let E be a subgroup of finite exponent of an $\overline{NC}$-p-group G. If $E^G < G$ then E^G is nilpotent and has finite exponent.*

Proof. Let $L = E^G$; then L is normal in G. Assume that $L < G$. Then L is an NC-subgroup of G so it has a normal nilpotent subgroup K such that L/K is Chernikov. Thus by Lemma 4.7(i) of [8] L has a G invariant nilpotent subgroup U such that L/U is Chernikov. Then $L/U \leq Z(G/U)$ by Theorem A of [3] and Theorem 3.29.2 of [15]. But now EU/U must be finite which implies that L/U is finite and so L is nilpotent by (1) Lemma of [12] since finite subgroups of G are subnormal by Theorem A(i) of [3]. Moreover L/L' has finite exponent. Therefore it follows from the Corollary on p. 55 of [15] that L his finite exponent. □

Lemma 2.3. *Let G be an $\overline{NC}$-p-group satisfying the normalizer condition. Suppose that every proper subgroup of G is a Chernikov extension of a nilpotent subgroup of finite exponent. Then G has a subgroup E of finite exponent such that $E^G = G$.*

Proof. Assume that $E^G < G$ for every subgroup E of finite exponent of G. Let X be any proper subgroup of G. We claim that X is subnormal in G.

First we show that $X^G < G$. Without loss of generality X has infinite exponent. By hypothesis X has a normal nilpotent subgroup Y of finite exponent such that X/Y is Chernikov. Thus by Lemma 3.3 of [3] X has a unique maximal HM*-subgroup U such that X/U has finite exponent. Now since X/UY is both Chernikov and has finite exponent it must be finite, so $X = F(UY)$ for some finite subgroup F of X. Clearly then $X^G = (UFY)^G = U^G(FY)^G$. But $(FY)^G < G$ by assumption since FY has finite exponent and U is normal in G by Lemma 3.2 [3]. Consequently it follows that $X^G < G$ since X^G is an NC-group.

Now Since U is normal in G and $FY \leq (FY)^G$ which is nilpotent by Lemma 2.2, it follows that X is subnormal in G. Thus it follows that every subgroup of G is subnormal in G. But then G is solvable by (7) Satz of [13] which is a contradiction. □

Proof of Theorem 1. By Theorem A of [3] G is perfect which implies that $Z(G/Z(G)) = 1$. Therefore without loss of generality we may suppose that $Z(G) = 1$. Furthermore by Theorem A(i) of [3] G has a normal nilpotent subgroup N such that in G/N every proper subgroup is a Chernikov extension of a nilpotent subgroup of finite exponent and for every normal nilpotent subgroup M of G MN/N has finite exponent. First we need to show that N has finite exponent. For this it suffices to show that $Z(N)$ has finite exponent by Theorem 2.23 of [15] since N is nilpotent.

Let $Z = Z(N)$. By Lemma 2.3 G/N has a subgroup E/N of finite exponent such that $E^G = G$. Evidently E is nilpotent since it is an NC-subgroup of G. Let c be the nilpotency class of E and m be the exponent of E/N. Let $x \in E$. Then $[Z, x] \leq Z$ and $x^m \in N \leq C_G(Z)$. So letting $H = Z$ in Lemma 2.1 gives that

$$Z^{m^c} \leq C_G(x)$$

But since x is any element of E and Z is normal in G if follows from this that

$$E^G \leq C_G(Z^{m^c})$$

which implies $Z^{m^c} = 1$ since $E^G = G$. Clearly it follows from this that N has finite exponent. In particular now every proper normal subgroup of G has finite exponent.

Now let K be any proper subgroup of G and let $L = KN$. By the choice of $N, L/N$ has a normal nilpotent subgroup U/N of finite exponent such that L/U is Chernikov. Also U has finite exponent since N does and hence U is nilpotent since it is an NC-subgroup of G. Therefore $K \cap U$ is a normal nilpotent subgroup of K of finite exponent such that $K/K \cap U$ is Chernikov. Consequently it follows that every proper subgroup of G is a Chernikov extension of a nilpotent subgroup of finite exponent. Therefore G has a subgroup E of finite exponent such that $E^G = G$ by Lemma 2.3. This completes the proof of the theorem.

Proof of Theorem 2. First we need to give some properties of HM*-groups. Some of the following results are contained in [3] but they are included here for convenience.

Lemma 3.1. *Let X be an HM*-group for a prime p. Then the following hold.*

(i) $X' = [X, X']$

(ii) There does not exist any proper normal subgroup N of X satisfying $X = NX'$. In particular X/N cannot have finite exponent.

(iii) If X satisfies the normalizer condition then X' is not properly supplemented in X.

Proof. (i) Let $\overline{X} = X/[X, X']$. Then $\overline{X}' \leq Z(\overline{X})$ and so $\overline{X}$ is nilpotent and $\overline{X}/\overline{X}'$ is radicable abelian which implies that $\overline{X}$ is abelian and hence $X' \leq [X, X']$, by Theorem 9.23 of [15].

(ii) Assume that $X = NX'$ for some proper normal subgroup N of X. Then

$$\begin{aligned} X' &= [X', NX'] = [X', N]X'' \\ &= [X', N] \\ &\leq N \end{aligned}$$

and so $X = N$ by (i) and by Lemma 2.22 of [15] since X' is nilpotent which is a contradiction. Also if X/N has finite exponent then as X/NX' is radicable abelian and has finite exponent it follows that $X = NX'$ and so $X = N$ by the first part of (ii).

(iii) Assume that $X = CX'$ for some $C < X$. Let $D = C \cap X'$ and put $\overline{X} = X/DX''$. Then $\overline{X} = \overline{CX}$ and $\overline{C}$ is radicable abelian and Chernikov. Let $\overline{Y} = N_{\overline{X}}(\overline{C})$.

Then $\overline{Y} = \overline{C}(\overline{Y} \cap \overline{X}')$ which implies that $\overline{Y}$ is nilpotent since $\overline{C}$ and $\overline{Y} \cap \overline{X}'$ are normal nilpotent subgroup of $\overline{Y}$. In particular $\overline{C} \leq Z(\overline{Y})$ by Lemma 3.13 of [15]. Assume that $\overline{Y} < \overline{X}$ and put $\overline{V} = N_{\overline{X}}(\overline{Y})$. Then $\overline{Y} < \overline{V}$ by hypothesis. But since $\overline{V} = \overline{C}(\overline{V} \cap \overline{X})$ it follows as in the first case that $\overline{C} \leq Z(\overline{V})$ and so $\overline{V} \leq \overline{Y}$ which is a contradiction. Consequently it follows that $\overline{Y} = \overline{X}$ and so $\overline{C} \leq Z(\overline{X})$. But now

$$\begin{aligned} \overline{X}' = [\overline{X}', \overline{X}] &= [\overline{X}', \overline{C}\overline{X}'] \\ &= [\overline{X}', \overline{X}'] \\ &= \overline{X}'' \end{aligned}$$

which is possible only if $\overline{X}' = 1$ and so $X' \leq DX'' \leq D \leq C$ since X' is nilpotent. This is a contradiction since $C < X$. □

Lemma 3.2. *Let X be an NC-p-group and T be an HM*-supgroup of X. If N is a normal subgroup of X such that X/N is Chernikov then $T' \leq N$.*

Proof. Let $D = T' \cap N$ and put $\dot{T} = T/D$. Then $\overline{T}$ is an HM*-group. But $\overline{T}'$ is Chernikov and nilpotent so the Corollary to Theorem 3.29.2 of [15] gives that $\overline{T}/C_{\overline{T}}-(\overline{T}')$ is finite which implies that $\overline{T}' = 1$ and $\overline{T}' = D \leq N$ by (ii) and (i) of Lemma 3.1. □

Lemma 3.3. *Let X be an HM*-group for a prime p such that X' has finite exponent. Then X is a product of normal HM*-subgroups Y such that $Y/Y' \cong C_{p^\infty}$*

Proof. There exists an $n \geq 1$ such that $X/X' \cong C_{p^\infty} \times \cdots \times C_{p^\infty}$ (n copies). Therefore X has normal subgroups $V_1, ..., V_n$ each containing X' such that

$$X = V_1 \cdots V_n \text{ and } V_i/X' \cong C_{p^\infty}$$

for all $i \geq 1$. By Lemma 3.3 of [3] each V_i has a unique maximal normal HM*-subgroup Y_i such that $Y_i/Y_i' \cong C_{p^\infty}$ and V_i/Y_i has finite exponent. Clearly then $Y = Y_1 \cdots Y_n$ is a normal subgroup of X such that X/Y has finite exponent whence $X = Y$ by Lemma 3.1 (ii) which was to be shown. □

Lemma 3.4. *Let G be an $\overline{NC}$-p-group such that every proper subgroup of it is a Chernikov extension of a nilpotent subgroup of finite exponent. Suppose that G has no epimorphic image which is an $\overline{NF}$-group. Then G is generated by its HM*-subgroups.*

Proof. Since no epimorhic image of G is an $\overline{\text{NF}}$-group it has a nonnilpotent proper subgroup X. In particular X has infinite exponent by hypothesis. So by Lemma 3.3 of [3]

X contains a normal HM*-subgroup Y such that X/Y has finite exponent. Consequently G contains HM*-subgroups.

Let W be the set of all the HM*-subgroups of G and define $L = < X : X \in W >$. Evidently L is normal in G. Assume if possible that $L \neq G$. Then G/L is an $\overline{\text{NC}}$-group which is not an $\overline{\text{NF}}$-group by hypothesis. So as above G/L contains an HM*-subgroup Y/L. Then Y contains a unique maximal HM*-subgroup T such that Y/T has finite exponent. But since $T \leq L, Y/L$ must have finite exponent which is impossible and so $L = G$ as claimed.

The first part of the following is known (see p. 64, Part II of [15]). □

Lemma 3.5. *Let G be a group and $a \in G$ such that a^G is abelian. If $[a, G'] = 1$ then*

$$[a, x, y] = [a, y, x], \text{ for all } x, y \in G.$$

If G is locally nilpotent then the converse is also true.

Proof. Let $x, y \in G$. Then the following hold.

$$[a, xy] = [a, y][a, x][a, x, y] \tag{1}$$

$$[a, yx] = [a, x][a, y][a, y, x] \tag{2}$$

$$[a, (yx)^{-1}(xy)] = [a, xy][a, yx]^{-[x,y]} \tag{3}$$

Now if $[a, G'] = 1$ then (3) gives that [a, xy] = [a, yx] and then (1) and (2) give that $[a, x, y] = [a, y, x]$. Conversely if the given condition holds then (1) and (2) give that $[a, xy] = [a, yx]$ and then (3) gives that $[a, [x, y]] = [a, xy, [x, y]]^{-1} = [a, [x, y], xy]^{-1}$ which implies that $[a, [x, y]] = 1$ since $< [a, [x, y]], xy >$ is nilpotent. □

Proof of Theorem 2. Let G be an $\overline{\text{NC}}$-p-group satisfying the hypothesis but not the conclusion of the theorem. Then $G = G'$ and so in particular every finite subgroup of G is subnormal in G by Theorem A of [3]. Also $Z(G/Z(G)) = 1$ so without loss of generality we may suppose that $Z(G) = 1$. Thus now it follows from Theorem 1 that every proper subgroup of G is a Chernikov extension of a nilpotent subgroup of finite exponent and every normal nilpotent subgroup of G has finite exponent. Assume that no proper epimorphic image of G is an $\overline{\text{NF}}$-group.

First we show that G contains HM-subgroups. By our assumption G contains a proper subgroup X such that X is an NC-group but not an NF-group. So by Lemma 3.3, X contains a unique maximal HM*-subgroup V such that $V = Y_1 Y_2 \cdots Y_t$, where for each $1 \leq i \leq t, Y_i$ is a normal HM*-subgroup of V such that $Y_i/Y_i' \cong C_{p^\infty}$. If $Y_i' = 1$ for some $i \geq 1$, then $Y_i \cong C_{p^\infty}$ and hence $Y_i \cong Z(G) = 1$ by hypothesis and by Lemma 3.2 of [3], which is a contradiction. Therefore each Y_i is a (normal) HM-subgroup of G, that is $Y_j' \neq 1$ and $Y_i/Y_i' \cong C_{p^\infty}$ and every proper subgroup of Y_i is subnormal (See Lemma 3.1 (iii)).

Let W be the set of all the HM-subgroups of G. Then $W \neq \emptyset$ by the above paragraph. For each $T \in W$ let

$$W(T) = \{X \in W : T' \leq X'\}.$$

First suppose that $W(T)$ generates G for all $T \in W$. Choose $T_1 \in W$. Assume if possible that $T_1' = X'$ for all $X \in W(T_1)$. Put $\overline{G} = G/T_1'$. Then $\overline{G} =< \overline{X} : X \in W(T_1) >$. But since X is normal in G and $\overline{X}$ is radicable abelian and Chernikov, it follows that $\overline{X} \leq Z(\overline{G})$ for all $X \in W(T_1)$ and hence $\overline{G}$ is abelian which implies that $\overline{G} = 1$ and then $G' = T_1'$ which is a contradiction. Therefore there exists a $T_2 \in W(T_1)$ such that $T_1' < T_2'$. Assume that for some $n > 1$ we have obtained $T_1, ..., T_n$ such that

$$T_1' < \cdots < T_n'$$

but T_n is not contained in $T_1 \cdots T_{n-1}$. Now the set

$$\{X \in W : T_n' \leq X' \text{ but } X \text{ is not contained in } T_1 \cdots T_n\}$$

generates G since $W(T_n)$ does. So as above this set contains an element T_{n+1} such that $T_n' < T_{n+1}'$. Thus continuing in this way we obtain an infinite sequence

$$T_1, ..., T_n, ...$$

of elements of W such that

$$T_1' < T_2' < \cdots$$

and T_{n+1} is not contained in $T_1 \cdots T_n$ for all $n \geq 1$.

Let $L =< T_n : n \geq 1 >$. First we show that $L = G$. Assume if possible that $L \neq G$. Then L is an NC-subgroup of G so by Lemma 3.3 of [3] it contains a unique maximal HM*-subgroup R. Clearly then $T_n \leq R$ for all $n \geq 1$ and so $L = R$. Thus $L/L' \cong C_{p^\infty} \times \cdots \times C_{p^\infty}$ (m factors) and also L' has finite exponent by Lemma 3.2 since $Z(G) = 1$. Therefore we can find an $r \geq m$ such that $L = L'(T_1 \cdots T_r)$ by definition of the T_n and then $L = T_1 \cdots T_r$ by Lemma 3.1(ii). This is impossible since T_{r+1} is not contained in $T_1 \cdots T_r$. Consequently it follows that $L = G$.

Next let $H =< T_n' : n \geq 1 >$. Then it can be shown as above that $G = H$, since $L = G$ and $T_nH/H \leq Z(G/H)$ for all $n \geq 1$. Thus we have shown that

$$G = \bigcup_{n=1}^{\infty} T'_n$$

and such that $T'_n < T'_{n+1}$ for all $n \geq 1$. This contradicts our assumption. Therefore there exists an $S \in W$ such that $W(S)$ cannot generate G. In this case the set $W'(S) = W \setminus W(S)$ must generate G by Lemma 3.4 in which case also the set of all X' such that $X \in W'(S)$ generates G as before. In particular then there exists a $T \in W'(S)$ such that $[S', T'] \neq 1$ since $Z(G) = 1$. Also S' is not contained in T' by the choice of T.

First suppose that S' is elementary abelian. Since $S' = [S', S]$ is not contained in T' there exists an $a \in S'$ such that $[a, S]$ is not contained in T'. In the same way there exists a $b \in [a, S]$ such that $[b, S]$ is not contained in T'. First suppose that

$$[b, S, T] < [a, S, T]$$

Let K be the largest subgroup of S' such that $[K, S] \leq [b, S, T]$. Put $R = ST$ and $\overline{R} = R/K$. Then by Lemma 3.2 of [3] $C_{\overline{S}'}(\overline{S}) = 1$. Furthermore it is easy to see that

$$1 \neq [\overline{a}, \overline{S}, \overline{T}] \leq \overline{T}', [\overline{b}, \overline{S}, \overline{T}] = 1$$

and

$$[\overline{a}, \overline{S}, \overline{T}, \overline{T}] = [\overline{a}, \overline{S}, \overline{T}] \neq 1.$$

The above equalities follow from Lemma 3.2 of [3]. Moreover $[\overline{a}, \overline{S}, \overline{T}] \leq [\overline{a}, \overline{S}]$ by hypothesis. Consequently $[\overline{a}, \overline{S}, \overline{T}]$ and $[\overline{b}, \overline{S}]$ are $\overline{S}$-invariant normal subgroups of $[\overline{a}, \overline{S}]$ neither of which can contain the other, since only $[\overline{b}, \overline{S}]$ is centralized by $\overline{T}$ and only $[\overline{a}, \overline{S}, \overline{T}]$ is contained in $\overline{T}'$. Now let $N = <\overline{a}>^{\overline{S}} = <\overline{a}> [\overline{a}, \overline{S}]$. Since S' is elementary abelian, $\overline{S}/\overline{S}' \cong C_{p^\infty}$ acts on N by conjugation and so induces an endomorphism ring on N which is a quotient of $Z_p[C_{p^\infty}]$. Then every $\overline{S}$-invariant subgroup of N corresponds uniquely to an ideal of this ring (see p. 438 of [10]). But since the ideals of this ring are linearly ordered by Lemma 3 of [9], one of $[\overline{a}, \overline{S}, \overline{T}]$ and $[\overline{b}, \overline{S}]$ must be contained in the other, which is a contradiction. Consequently if follows that

$$[b, S, T] = [a, S, T]$$

whenever $b \in [a, S]$ and $[b, S]$ is not contained in T'.

Assume that $[a, S, T] \neq 1$ for some $a \in S' \setminus T'$ such that $[a, S]$ is not contained in T'. Since $[a, S] = [a, S, S]$ there exists an $s \in S$ such that $[a, S, s]$ is not contained in T'. Furthermore as $[a, S, T] < s >$ is nilpotent, it is easy to see that

$$[a, S, T, s] < [a, S, T]$$

Moreover

$$[a, S, T, s] = [a, s, T, S]$$

by Lemma 3.5. But now letting $b = [a, s]$ gives

$$[b, S, T] < [a, S, T]$$

and $[b, S]$ is not contained in T' which is a contradiction. Therefore $[a, S, T] = 1$ for all $a \in S'$ for which $[a, S]$ is not contained in T'.

Let L be the largest subgroup of S' for which $[L, S] \leq T'$. Then $L \neq S'$ since S' is not contained in T'. Let $a \in S' \setminus L$. Then $[a, S, T] = 1$ as was shown above. Let $y \in L$. Then $ay \in S' \setminus L$ and so we get

$$\begin{aligned} 1 = [ay, S, T] &= [a, S, T][y, S, T] \\ &= [y, S, T] \end{aligned}$$

Since y is any element of L it follows that

$$1 = [S', S, T] = [S', T]$$

which is a contradiction.

Now we consider the general case. Let $U = S''(S')^p$ and put $Z/U = Z(S/U)$. Let $\overline{G} = G/Z$. Obviously then $\overline{S}'$ is elementary abelian and nontrivial by Lemma 2.22 of [15]. Also $Z(\overline{S}) = 1$. Now it can be shown as in the first case that the set of all $\overline{X}$ for $X \in W$ such that $\overline{S}'$ is not contained in $\overline{X}'$ can generate $\overline{G}$. Then also the set of all $\overline{T}'$ not containing $\overline{S}'$ can generate $\overline{G}$. Therefore we can find a $T \in W$ such that $\overline{S}'$ is not contained in $\overline{T}'$ and $[\overline{S}', \overline{T}'] \neq 1$. Clearly now we get another contradiction as in the first case which completes the proof of the theorem.

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Normalleyen Şartını Sağlayan $\overline{NC}$-p-Grupları

Özet

G bir mükemmel lokal nilpotent p-grubu ve G'nin her öz altgrubu nilpotent bir grubun Chernikov genişlemesi olsun. $G/Z(G)$'nin her öz altgrubunun sonlu üslü bir nilpotent grubun Chernikov genişlemesi olduğu gösterilmiştir (Theorem 1). Daha sonra bu sonuç kullanılarak G'nin, normalleyen şartını sağlaması halinde, bir karakterizasyonu verilmiştir (Theorem 2).

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