Anomalous parity asymmetry of WMAP power spectrum data at low multpoles: is it cosmological or systematics?

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We have investigated the odd-parity preference of the WMAP 7 year power spectrum. Our investigation shows parity asymmetry of the WMAP data ($2 \leq l \leq 22$) is anomalous at 4-in-1000 level. We also find it likely that low quadrupole power is part of this parity asymmetry rather than an isolated anomaly. We have investigated non-cosmological causes for the odd-parity preference, but have not found a definite non-cosmological origin. WMAP7 data possesses most anomalous odd-parity preference, while they have more accurate calibration and less foreground contamination than earlier data [1–5]. Besides that, the anomaly is associated with the WMAP power spectrum data, in which most efforts have been exerted to minimize systematics. Therefore, we find it unlikely that calibration or foregrounds are the source of the anomaly. We have also considered primordial origin for the parity asymmetry. However, we find primordial origin requires violation of translational invariance on large scales.

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I. INTRODUCTION

For the past years, there have been great successes in measurement of Cosmic Microwave Background (CMB) anisotropy by ground and satellite observations [1, 3–11]. Very recently, the seven year data of the Wilkinson Microwave Anisotropy Probe (WMAP) [4, 5, 12] is released, and the recent ground-based CMB observations such as the ACBAR [7, 8] and QUaD [9–11, 13] provide information complementary to the WMAP data. For the ongoing observation, Planck surveyor [14] has been successfully launched and is measuring CMB temperature and polarization anisotropy with very fine angular resolution. Using the recent and future CMB data, we may test cosmological hypotheses and impose significant constraints on cosmological models [15–17]. For the past years, WMAP data have gone through scrutiny, and various anomalies have been reported [18–37]. Among a few anomalies, anomalous mirror-parity asymmetry of WMAP data had been reported [38]. In our previous work [39], we have investigated point-parity, using WMAP 5 year power spectrum. We have reported anomalous odd-parity preferences at multipoles ($2 \leq l \leq 18$). In this paper, we present the investigation on WMAP 7 year power spectrum data and report anomalous odd-parity preference at ($2 \leq l \leq 22$). We have also investigated origins for the anomaly, but do not find any definite origin associated with WMAP systematics.

The outline of this paper is as follows. In Section II, we discuss the basic properties of CMB anisotropy. In Section III, we investigate the point-parity anomaly of the WMAP data. In Section IV and V, we discuss non-cosmological and cosmological origin for the anomaly. In Section VII, we summarize our investigation and discuss prospects.

II. CMB ANISOTROPY

The temperature anisotropy $T(\theta, \phi)$ over a whole-sky is conveniently decomposed in terms of spherical harmonics $Y_{lm}(\theta, \phi)$ as follows:

$$T(\hat{\mathbf{n}}) = \sum_{lm} a_{lm}\, Y_{lm}(\hat{\mathbf{n}}), \tag{1}$$

where a_{lm} is a decomposition coefficient, and $\hat{\mathbf{n}}$ is a sky direction. Decomposition coefficients are related to primordial perturbation as follows:

$$a_{lm} \;=\; 4\pi(-\imath)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3}\Phi(\mathbf{k})\, g_l(k)\, Y^*_{lm}(\hat{\mathbf{k}}), \tag{2}$$

where $\Phi(\mathbf{k})$ is primordial perturbation in Fourier space, and $g_l(k)$ is a radiation transfer function. For a Gaussian seed fluctuation model, decomposition coefficients satisfy the following statistical properties:

$$\langle a_{lm} \rangle \;=\; 0, \tag{3}$$
$$\langle a^*_{lm} a_{l'm'} \rangle \;=\; C_l\, \delta_{ll'} \delta_{mm'}, \tag{4}$$

where $\langle \ldots \rangle$ denotes the average over the ensemble of universes. Given a standard cosmological model, we expect Sach-Wolf plateau for CMB power spectrum at low multipoles [15]:

$$l(l+1)C_l \sim \text{const.} \tag{5}$$

III. PARITY ASYMMETRY

Spherical harmonics behave under parity inversion as follows [40]: $Y_{lm}(\hat{\mathbf{n}}) = (-1)^l\, Y_{lm}(-\hat{\mathbf{n}})$. Therefore, power

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asymmetry between even and odd multipoles may be thought as power asymmetry between even and odd parity map, because a map consisting of only even(odd) multipoles possesses even(odd) parity. Hereafter, we will denote it as 'parity asymmetry'. In Fig. 1, we show the

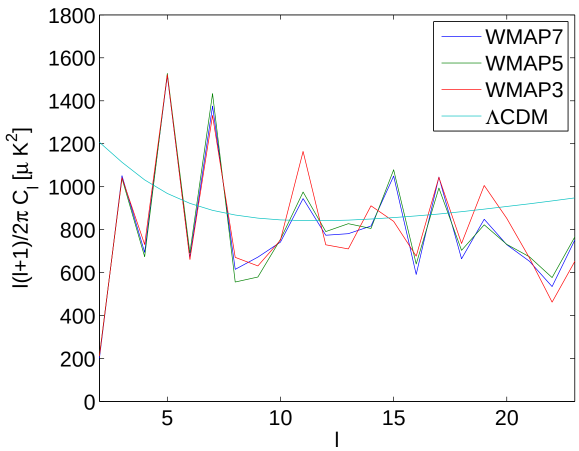

FIG. 1: CMB power spectrum: ΛCDM model (cyan), WMAP 7 year data (blue), WMAP 5 year data (green) and WMAP 3 year data (red)

WMAP 7 year, 5 year, 3 year data and the theoretical power spectrum of the WMAP concordance model [3, 41, 42].

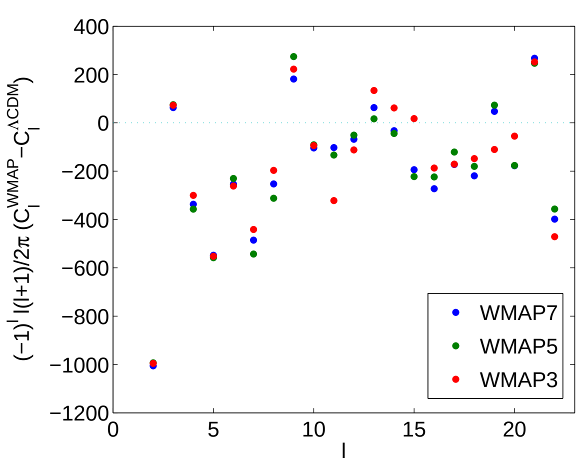

FIG. 2: $(-1)^l\times$ difference between WMAP power spectrum data and ΛCDM model

In Fig. 2, we show $(-1)^l l(l+1)/2\pi\,(C_l^{\rm WMAP}-C_l^{\Lambda\rm CDM})$ at low multipoles. As shown in Fig. 2, most of them posses negative values, which indicates there exist power deficit (excess) in comparison to the ΛCDM model at most of even (odd) multipoles. In the case of WMAP7 or WMAP5 data, there is only 5 points of positive values among 22 data points. A order-of-magnitude estimation shows that such events require the odd of $22!/(5!\,17!2^{22})\approx 0.006$. However, power spectrum is estimated from cut-sky to avoid Galactic foreground contamination. Therefore, statistical fluctuation in estimated C_l is correlated among multipoles, and larger than whole-sky estimation. In order to assess odd of the parity asymmetry more properly, we have produced 10^4 simulated CMB maps (HEALPix Nside=8) of Gaussian ΛCDM model. We have degraded the WMAP processing mask (Nside=16) to Nside=8, and set pixels to zero, if any of their daughter pixels is zero. After applying the mask, we have estimated power spectrum from cut-sky maps by a pixel-based Maximum-Likelihood method. We have neglected instrument noise, since noise is subdominant on multipoles of interest (e.g. S/N ~ 100 for C_l at $l=30$) [5]. Noting Eq. 5, we consider the following quantities:

$$P^+ = \sum (l+1-2\left\lfloor\frac{l+1}{2}\right\rfloor)\, l(l+1)/2\pi\, C_l \qquad (6)$$

$$P^- = \sum (l-2\left\lfloor\frac{l}{2}\right\rfloor)\, l(l+1)/2\pi\, C_l \qquad (7)$$

where $\lfloor\cdots\rfloor$ denotes the greatest integer smaller than or equal to the argument. Using the WMAP power spectrum data and simulations respectively, we have computed the ratio P^+/P^- for various multipole ranges $2\le l\le l_{\rm max}$, where $l_{\rm max}$ is between 3 and 23. By comparing P^+/P^- of the WMAP data with simulation, we have estimated p-value, where p-value denotes fractions of simulations as low as P^+/P^- of the WMAP data. In Fig. I, we show p-value of WMAP7, WMAP5 and

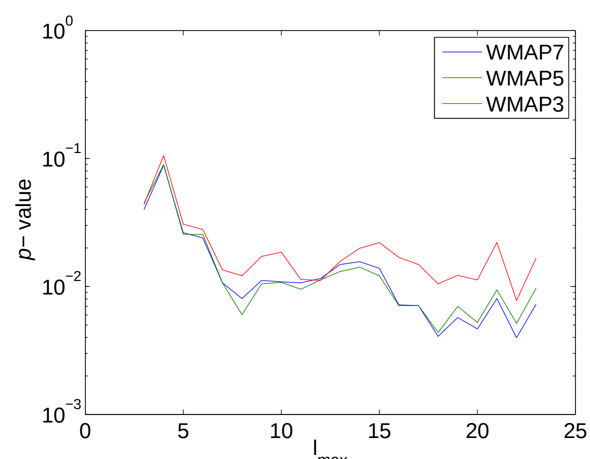

FIG. 3: Probability of getting P^+/P^- as low as WMAP data for multipole range $2\le l\le l_{\rm max}$.

WMAP3 respectively for various $l_{\rm max}$. As shown in Fig. I, the parity asymmetry of WMAP7 data at multipoles $(2\le l\le 22)$ is most anomalous, where p-value is 0.004. In Table I, we summarize the P^+/P^- and p-values of WMAP7, WMAP5 and WMAP3 at $2\le l\le 22$ As shown in Fig. 3 and Table I, odd-parity preference of WMAP7 is most anomalous, while WMAP7 data have more accurate calibration and less foreground contamination than the

TABLE I: the parity asymmetry of WMAP data ($2 \leq l \leq 22$)

data	P^+/P^-	p-value
WMAP7	0.7076	0.004
WMAP5	0.7174	0.0051
WMAP3	0.7426	0.0078

earlier data [1–5]. Therefore, we find it unlikely that calibration or foregrounds are the source of the anomaly. It should be also noted that the anomaly is associated with the WMAP power spectrum data, in which most efforts have been exerted to minimize systematics. For multipole range ($2 \leq l \leq 22$), we find $P^+/P^- \approx 1.1$ is most likely for simulations. In Fig. 4, we show P^+/P^- values of WMAP data and cumulative distribution of P^+/P^- for 10^4 simulated maps.

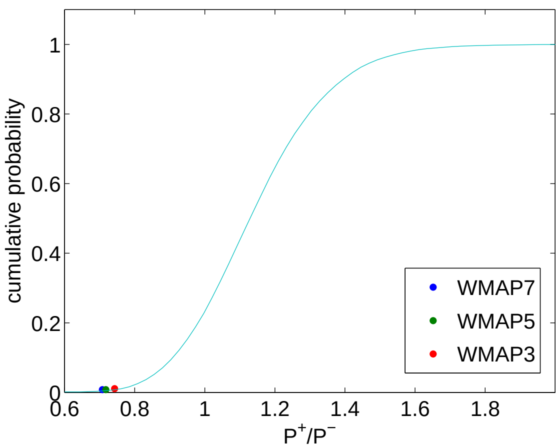

FIG. 4: Parity asymmetry at multipoles ($2 \leq l \leq 22$): cumulative distribution of P^+/P^- for 10^4 simulated maps (red), P^+/P^- of WMAP7 (blue), WMAP5 (green) and WMAP3 (red)

We have also compared P^+/P^- of the WMAP7 with whole-sky simulation (i.e. no mask), and found only 0.0013 of simulations has P^+/P^- as low as the WMAP7. The difference from the cut-sky result is attributed to the increased statistical fluctuation in cut-sky C_l estimation. By using whole-sky simulations, we have also investigated p-value for $l_{\max} \gg 23$, but have not found p-value as low as that of $l_{\max} = 22$.

IV. NON-COSMOLOGICAL ORIGINS

It is known that 1/f noise, when coupled with WMAP scanning pattern, may result in less accurate measurement at certain low multipoles [41, 43, 44]. Therefore, one may attribute the parity asymmetry of the WMAP data to 1/f noise. However, WMAP7 power spectrum data, whose signal-to-noise ratio is higher than earlier data, possess most anomalous odd-parity preference [1–5]. It should be also noted that the signal-to-noise ratio of WMAP temperature data is quite high at low multipoles (e.g S/N∼ 100 for $l = 30$) [2, 41, 44]. Therefore, we find that instrument noise, including 1/f noise is unlikely to be the source of the anomaly.

To reduce Galactic foreground contamination, the WMAP team have subtracted diffuse foregrounds by template-fitting, and masked the regions that cannot be cleaned reliably. The WMAP team used the difference between K and Ka band maps, dust emission "Model 8" and Hα map for templates [1, 45–48]. In Fig. 5, we

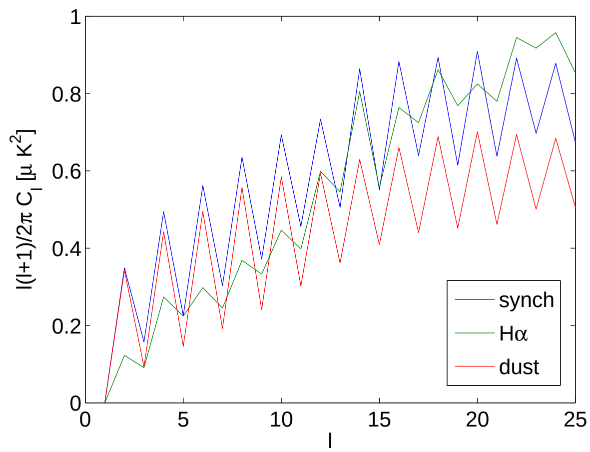

FIG. 5: the power spectra of the templates (synchrotron, Hα, dust): plotted with arbitrary normalization.

show the power spectrum of templates. As shown in Fig. 5, templates show strong even parity preference, which is opposite to that of the WMAP power spectrum data. Therefore, one may consider the possibility that over-subtraction by templates might lead to the odd-parity preference in WMAP data. However, we find it unlikely for the following reasons. Spherical harmonic coefficients of a foreground-reduced map by template-fitting is given by:

$$a^{\rm obs}_{lm} = a^{\rm cmb}_{lm} + a^{\rm fg}_{lm} - b\, a^{\rm tpl}_{lm}, \quad (8)$$

where $a^{\rm obs}_{lm}$, $a^{\rm fg}_{lm}$ and $b\, a^{\rm tpl}_{lm}$ correspond to a foreground-cleaned map, a residual foreground and a template with a fitting coefficient b. For simplicity, we consider only a single foreground component and a single template, but the conclusion is equally valid for multi-component foreground. Since there should be no correlation between foregrounds and CMB, the observed power spectrum is given by:

$$C^{\rm obs}_l \approx C^{\rm cmb}_l + \langle \left| a^{\rm fg}_{lm} - b\, a^{\rm tpl}_{lm} \right|^2 \rangle. \quad (9)$$

As shown Eq. 9, the second term on the right hand side propagates the parity preference of templates, as far as templates are good tracer of foregrounds (i.e. $a^{\rm fg}_{lm}/a^{\rm tpl}_{lm} \approx$ const). Therefore, the opposite parity preference cannot be produced, whether oversubtracted or undersubtracted. At lowest multipoles, cross term $\sum_m \mathrm{Re}[a_{lm}^{\rm cmb}(a_{lm}^{\rm fg} - b\, a_{lm}^{\rm tpl})^*]$ may not be negligible [49], making Eq. 9 a bad approximation. However, more rigorous investigation by [50] shows that, as long as the source of contamination is statistically independent of signal, the presence of contamination makes power deficit less likely. For these reasons, we find contamination associated with foregrounds is unlikely to be the cause of the anomaly.

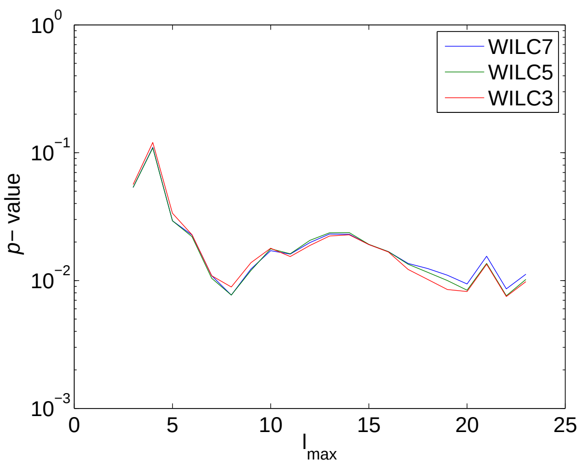

FIG. 6: Probability of getting P^+/P^- as low as ILC maps for multipole range $2 \leq l \leq l_{\rm max}$

TABLE II: the parity asymmetry of WMAP ILC maps ($2 \leq l \leq 22$)

data	P^+/P^-	p-value
WILC7	0.7726	0.0086
WILC5	0.7673	0.0076
WILC3	0.7662	0.0075

The WMAP team have masked the region that cannot be reliably cleaned by template fitting, and estimated CMB power spectrum from sky data outside the mask [3, 5, 41, 48]. One may attribute incomplete sky coverage to the parity asymmetry. To see whether the parity asymmetry is the artificial anomaly produced by incomplete sky coverage, we have investigated the WMAP team's Internal Linear Combination map (ILC) map. It is believed to provide a reliable estimate of CMB signal over whole-sky on angular scales larger than 10° [41, 48]. In Fig. 7, we show the odd and even parity map, which are derived from the 7 year ILC map ($2 \leq l \leq 22$). It is worth to note the wider temperature range of odd-parity map in comparison with the even-parity map. We have compared P^+/P^- of the WMAP team's ILC map with whole-sky simulations. In Fig. 6, we show p-value for WILC7 (7 year), WILC5 (5 year) and WILC3 (3 year) respectively. In Table II, we summarize P^+/P^- and p-values at multipoles ($2 \leq l \leq 22$). As shown in Fig. 6 and

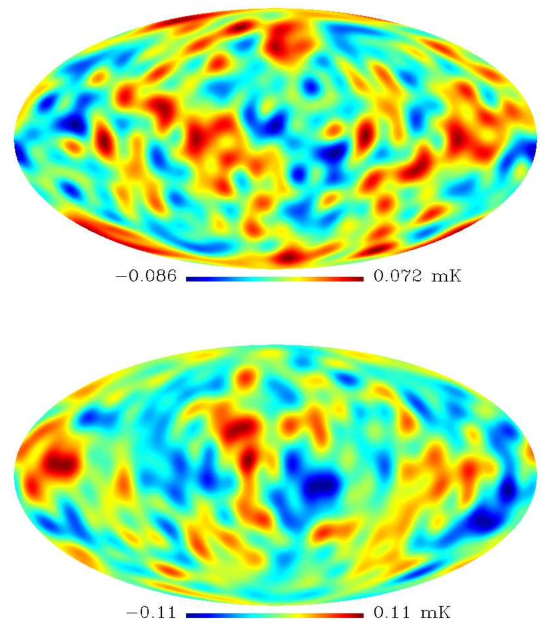

FIG. 7: Internal Linear Combination map ($2 \leq l \leq 22$): even parity map (top) and odd parity map (bottom)

Table II, we find there exist anomalous odd-parity preference of whole-sky CMB maps as well. Besides that, we have estimated p-values of WMAP power spectrum by comparing it with cut-sky simulations. Therefore, we find it unlikely that odd-parity preference is not produced by cut-sky.

V. COSMOLOGICAL ORIGIN

In this section, we take the WMAP power spectrum at face values. (i.e. truly cosmic origin), and consider what might produce such an anomaly. So far, various topological models including multi-connected Universe and Bianchi VII model have been proposed to explain cold spot or low quadrupole power [51, 52]. However, those topological models do not produce the parity asymmetry over a range of multipoles, though some of them predict low quadrupole power. Therefore, the parity asymmetry is not easily explained in terms of the topological models proposed so far. We may consider the parity asymmetry is associated with primordial perturbation $\Phi(\mathbf{k})$. To see what kind of primordial origin leads to such parity asymmetry, let us get back to Eq. 2, which is equivalently given:

$$
\begin{aligned}
a_{lm} &= \frac{(-\imath)^l}{2\pi^2}\int_0^\infty dk \int_0^\pi d\theta_{\mathbf{k}}\sin\theta_{\mathbf{k}}\int_0^{2\pi} d\phi_{\mathbf{k}}\,\Phi(\mathbf{k})\,g_l(k)\,Y^*_{lm}(\hat{\mathbf{k}}),\\
&= \frac{(-\imath)^l}{2\pi^2}\int_0^\infty dk \int_0^\pi d\theta_{\mathbf{k}}\sin\theta_{\mathbf{k}}\int_0^{\pi} d\phi_{\mathbf{k}}\,g_l(k)\\
&\times\;\left(\Phi(\mathbf{k})\,Y^*_{lm}(\hat{\mathbf{k}})+\Phi(-\mathbf{k})\,Y^*_{lm}(-\hat{\mathbf{k}})\right),\\
&= \frac{(-\imath)^l}{2\pi^2}\int_0^\infty dk \int_0^\pi d\theta_{\mathbf{k}}\sin\theta_{\mathbf{k}}\int_0^{\pi} d\phi_{\mathbf{k}}\,g_l(k)Y^*_{lm}(\hat{\mathbf{k}})\\
&\times\;\left(\Phi(\mathbf{k})+(-1)^l\Phi^*(\mathbf{k})\right), \qquad (10)
\end{aligned}
$$

where we used the reality condition $\Phi(-\mathbf{k}) = \Phi^*(\mathbf{k})$ and $Y_{lm}(\hat{-\mathbf{n}}) = (-1)^l\,Y_{lm}(\hat{\mathbf{n}})$. Using Eq. 10, it is trivial to show, for the odd number multipoles $l = 2n-1$,

$$
a_{lm} = \qquad (11)
$$
$$
-\frac{(-\imath)^{l+1}}{\pi^2}\int_0^\infty dk \int_0^\pi d\theta_{\mathbf{k}}\sin\theta_{\mathbf{k}}\int_0^{\pi} d\phi_{\mathbf{k}}\,g_l(k)Y^*_{lm}(\hat{\mathbf{k}})\,\mathrm{Im}[\Phi(\mathbf{k})],
$$

and, for even number multipoles $l = 2n$,

$$
a_{lm} = \qquad (12)
$$
$$
\frac{(-\imath)^{l}}{\pi^2}\int_0^\infty dk \int_0^\pi d\theta_{\mathbf{k}}\sin\theta_{\mathbf{k}}\int_0^{\pi} d\phi_{\mathbf{k}}\,g_l(k)Y^*_{lm}(\hat{\mathbf{k}})\,\mathrm{Re}[\Phi(\mathbf{k})].
$$

Therefore, explaining the parity asymmetry in terms of primordial origin requires $|\mathrm{Re}[\Phi(\mathbf{k})]|$ to be suppressed in comparison with $|\mathrm{Im}[\Phi(\mathbf{k})]|$ for $k \lesssim 22/\eta_0$, where η_0 is the present conformal time. However, in a new coordinate of an origin shifted by $\delta\mathbf{x}$, the primordial perturbation is given by $\Phi(\mathbf{k})\exp[\imath\mathbf{k}\cdot\delta\mathbf{x}]$. Therefore, the condition $|\mathrm{Re}[\Phi(\mathbf{k})]| \ll |\mathrm{Im}[\Phi(\mathbf{k})]|$ for $k \lesssim 22/\eta_0$ imposed at our vantage point breaks a translational invariance on the scales of $\sim 2\pi/(22\,\eta_0) \sim 3/(10\,\eta_0)$. It is rather intriguing from the viewpoint of cosmological homogeneity, though it is not in direct conflict with observable Universe.

VI. ASSOCIATION WITH OTHER ANOMALIES

It has been known that CMB quadrupole power of WMAP data is unusually low, compared with the theoretical value [22]. Therefore, one may attribute low P^+/P^- of the WMAP data simply to low quadrupole power. However, as shown in Fig. I, the anomalously low P^+/P^- (i.e. low p-value) persists over extended range of multipoles. Therefore, we find it likely that low quadrupole power is not an isolated anomaly, but part of the parity asymmetry on multipoles ($2 \le l \le 22$). It was also shown that hemispherical power asymmetry is much more anomalous on multipoles ($2 \le l \le 19$) than multipoles ($20 \le l \le 40$) [33]. It is also worth to note that the associated multipoles have some relevance to the characteristic scale of the WMAP cold spot ($\sim 10°$) [18–20]. Given all these circumstantial evidences, we find it likely that there exists an underlying common origin for these anomalies, whether cosmological origin or WMAP systematics.

VII. DISCUSSION

The parity asymmetry under point reflection as well as mirror reflection was noted [38], but point-parity was not given enough attention, since the statistical significance was not high. Using a slightly different estimator for point-parity asymmetry, we have computed the parity asymmetry of newly released WMAP 7 year data and compared it with cut-sky simulation. Our investigation shows that odd-parity preference of the WMAP7 power spectrum data ($2 \le l \le 22$) is anomalous at 4-in-1000 level. We have considered non-cosmological origins for the anomaly, but find they are unlikely to be the cause. Besides that, WMAP7 data, which have more accurate calibration and less foreground contamination than earlier data, possesses most anomalous odd-parity preference [1–5]. Therefore, we find it unlikely that calibration or foregrounds are the source of the anomaly. We have also considered what kind of primordial anomaly produce such parity asymmetry in CMB signal. However, we find explanation in terms of primordial origin requires violation of a translational invariance on large scales. Unfortunately, we are unable to resolve a definite non-cosmological or cosmological origin for the anomaly. However, with future data from the Planck surveyor, we may be able to resolve the origin of odd-parity preference in WMAP data.

VIII. ACKNOWLEDGMENTS

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