A Study Of Direct Two-Proton Transfer Reactions Using Optical Potential Model

Abdel Khaliq Abdel Rahman AL-FARRA
Physics Department, Faculty Of Science, AL-Azhar University
Gaza-PALESTINE
e-mail: a_a_farra@hotmail.com

Abstract

^{12}C, ^{26}Mg(^{3}He,n) reactions have been studied at high bombardment energy with evaluation of the differential cross sections to the ground and the first excited states evaluated in the framework of the exact finite-range Born approximation (DWBA) calculations using the optical model. With the constraint that bound-state interactions have a Thomas-Fermi spin orbit form, different formations of the bound-state wave functions are explored. The calculated angular distributions are found to be in a good agreement with the experimental data. The present DWBA predictions are normalized to give the best overall fits to the data. The extracted spectroscopic factors are reasonable.

Key Words: A Study of Direct Two-Proton Transfer Reactions Using Optical Potential Model

1.   Introduction

The theory of direct transfer reactions introduce a powerful tool with which one can obtain information about the structure of nuclei. Therefore, the differential cross sections of $\left(^3He, n\right)$ reactions to the ground and the first several excited-states in ^{28}Si nuclei, with excitation energies up to 11 MeV, have been calculated [1,2] at the low bombardment energies of 5.7 MeV and 13 MeV using the shell-model two-nucleon spectroscopic amplitudes. The calculations, which follow from [2], describe the ground state angular distribution reasonably well and provide an important contribution to understanding the compound-nucleus reaction process. The $^{26}Mg\left(^3He, n\right)$ reaction has been studied at higher bombardment energies using coupled-channel calculations [3] and the $\left(^3He, n\right)$ reaction on ^{12}C has been studied using DWBA calculations [4,5]. Calculations of the $^{12}C\left(^3He, n\right)^{14}O$ reaction [6] at higher energies show that the low-lying states in residual excitation were satisfactorily reproduced in a unified way by the same coupled-channel method [3] via a two-step process. Recently, the $\left(^3He, n\right)$ reaction leading to different excited states in residual nuclei have been reasonably well reproduced via a microscopic DWBA analysis [7,8].

In the present work, the differential cross sections of $^{12}C\left(^3He, n\right)^{14}O$ and $^{26}Mg\left(^3He, n\right)^{28}Si$ reactions leading to the low-lying states in the final nuclei at 45.5 MeV incident energy have been calculated in terms of DWBA calculations. In section 2, the theoretical expressions of the reaction cross sections are introduced. Numerical calculations and results are given in Section 3. Section 4 is left for discussion and conclusions.

2.   Reaction Cross-Section

In the present work, the differential cross sections for the Stripping reaction A(a, b)B in which a and b represent the light particles products has been evaluated in the framework of the DWBA calculations via

the following relationship for the differential cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{K_a}{K_b} \frac{(2I_B + 1)(2I_b + 1)}{(2I_A + 1)(2I_a + 1)} \sum |T_{fi}|^2. \tag{1}$$

Here the μ's and K's are the reduced masses and asymptotic wave numbers, respectively, and I_i is the total angular momentum of the ith particle. The details of the DWBA calculations are introduced explicitly in [9,10] via direct single step process. The post-formulation DWBA transition amplitude has the form

$$T_{fi} = \left\langle \chi_f^{(-)} \middle| V_{bA} + V_{cA} - \tilde{V}_{aA} \middle| \chi_i^{(+)} \right\rangle, \tag{2}$$

where $\chi_i^{(+)}$ and $\chi_f^{(-)}$ are the distorted wave functions in the initial and final channels, respectively, and V_{ij} is the interaction potential between the particles i and j, while $\tilde{V}$ is the optical potential generating the distorted waves. As is done in such calculations, we use the differential cross sections without considering a spin-orbit coupling term [11,12]. Generally, the present spectroscopic factors are extracted from the relation

$$\left(\frac{d\sigma(\theta)}{d\omega}\right)_{\exp} = N \frac{C^2 S_{ij}}{2J + 1} \left(\frac{d\sigma(\theta)}{d\omega}\right)_{theory}, \tag{3}$$

where N is the normalization factor for the reaction [13], S_{ij} is the spectroscopic factors, and C^2 stands for the isospin Clebsch-Gorden coefficient.

3. Numerical Calculations and Results

In the present section, we examine the differential cross sections of the $\left(^3He, n\right)$ reaction leading to the two lowest states of residual nuclei ^{16}O and ^{28}Si as calculated via the exact finite-range DWBA methodology.

We assume the nucleus-nucleus interaction has the form

$$\begin{aligned} V(r) &= -Vf(r, r_0, a_o) - i[W_V - 4a_I W_D \frac{d}{dr}]f(r, r_i, a_i) \\ &+ \left(\frac{\hbar}{m_\pi c}\right)^2 V_{s.o.} \frac{1}{r} \frac{d}{dr} f(r, r_{s.o.}, a_{s.o.}) \\ &+ V^C(r), \end{aligned} \tag{4}$$

where

$$f(r) = \left\{1 + \exp[(r - r_o A^{\frac{1}{3}})/a]\right\}^{-1}. \tag{5}$$

Here, $V^C(r)$ is the Coulomb potential for a uniformly charged sphere of radius $R_c = r_c A^{1/3}$ fm. The optical potential parameters used in the present calculations have been taken from the literature [14-16]. Thusly, we have tested some various combinations of ^3He-particle and distributions neutron parameters. The parameters of set II shown in Table 1 are found to give the best fit to the angular distributions. In the first set of calculations, the Bayman and Kallio form factor [17] was used with bound state parameters of $r_o = 1.25$ fm, $a = 0.65$ fm and $\lambda = 25$, together with the same normalization factor as used in previous work [3]. In another set of calculations, the wave function by Cohen and Kurath [18] was used for the positive-parity states and those obtained by the Millener-Kurath interaction [19] for the negative-parity states. In these calculations, a Woods-Saxon type potential with $r_o = 1.25$ fm, $a = 0.65$ fm and spin-orbit strength $V_{LS} = 6$ MeV were used. The potential depth was adjusted to reproduce the binding energy of the last nucleon. The phases and normalization's of the spectroscopic amplitudes obtained from these wave functions have been carefully checked to make them consistent with the present DWBA calculations. In addition, the bound-state potential is expressed to have a Thomas-Fermi spin-orbit form [20]

Table 1. Optical model and bound state parameters.

Channel	Set	V (MeV)	r_o (fm)	a_o (fm)	W_V (MeV)	$4W_D$ (MeV)	r_i (fm)	a_i (fm)	$V_{s.o.}$ (MeV)	$r_{s.o.}$ (fm)	$a_{s.o.}$ (fm)	r_c (fm)
^{3}He	I	157.10	1.20	0.70					112.0	1.21	0.83	1.40
	II	179.02	1.11	0.75		25.91	1.19	0.85	4.0	1.11	0.45	1.40
									4.08	0.97	0.35	1.20
	III	217.60	1.15	0.63	32.6		1.31	0.98	6.49	1.07	0.60	1.40
n	I								7.0	1.16	0.75	1.20
	II	46.09	1.17	0.75	5.82	2.90	1.32	0.56	6.2	1.01	0.75	1.40
Bound state	I	(a)	1.25	0.65					$\lambda = 25$	1.25	0.65	1.40
	II		1.40	0.67								

a) The potential depth for bound states was adjusted to give the proper binding energies.

$$V_{s.o.} = \frac{\lambda}{45.2} \cdot \frac{1}{r} \frac{d}{dr} f(r, r_{s.o.}, a_{s.o.}) \ , \tag{6}$$

where $\lambda = 25$ and the well depth of the real potential was adjusted to give each nucleon separation energies using the usual separation energy method. The results obtained are normalized to give overall fit to the experimental data as shown by solid lines in Figures 1 and 2. In general, the shapes and magnitudes of the differential cross sections are quite satisfactory. It is found that the magnitudes of the DWBA cross sections depend strongly on the bound-state parameters in the case of a transition from highly bound-state to a loosely bound-state. The spectroscopic factors are determined on the assumption that the transitions for these are between the states of simple shell-model configurations [2].

Table 2. Spectroscopic Factors.

Reaction	Incident energy (MeV)	Excition energy (MeV)	J^π	Present work
^{12}C(^{3}He,n)	45.5	0.0	0^+	0.87
		5.20	1^-	0.84
^{26}Mg(^{3}He,n)	45.5	0.0	0^+	0.83
		1.78	2^+	0.89

4. Discussion and Conclusion

The present DWBA calculations using spin-orbit potentials have been performed successfully to describe the experimental data of $\left(^3He, n\right)$ reactions on ^{12}C and ^{26}Mg nuclei in the forward region at high-incident energy. The calculated differential cross sections for the transitions to the ground and first excitedstates are compared with the experimental data as shown in Figures 1 and 2. For $^{12}C\left(^3He, n\right)$ reactions, it has been found that neither cross section magnitudes nor angular distribution shapes depend strongly on the wave functions. As illustrated by the results that the angular distributions for $^{26}Mg\left(^3He, n\right)$ are found to be similar to those obtained for the $^{12}C\left(^3He, n\right)$ reaction. That is, despite the difference among the bound state wave functions, the data fits are equally good. For this reason, we employed the Millener-Kurath wave functions in the present calculations. Using these functions, the calculated angular distribution for the 2^+, 1.78 MeV state is found to be insensitive to the used bound-state parameters. However, the magnitude of the DWBA cross sections for a negative-parity state depend on the bound-state parameters. In general, the overall angular distributions shapes of the $^{12}C\left(^3He, n\right)^{14}O$ and $^{26}Mg\left(^3He, n\right)^{28}Si$ reactions at incident energy of 45.5 MeV are well reproduced by the DWBA calculations.
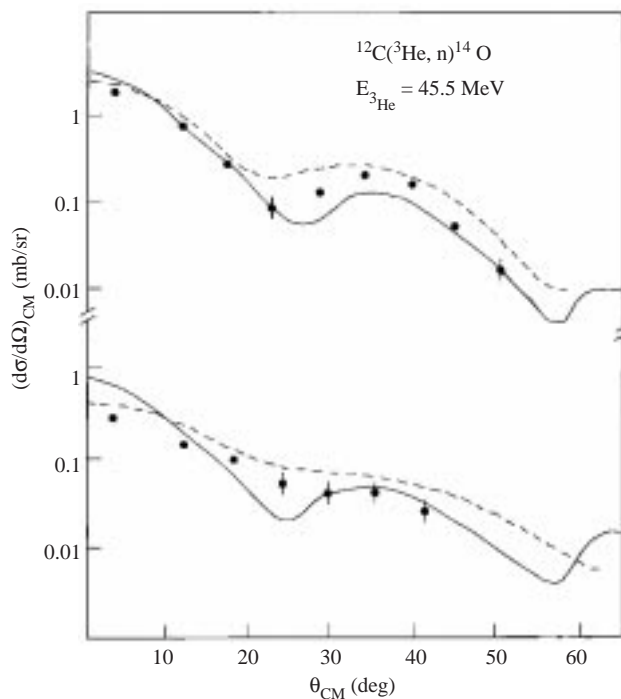
Figure 1. Differential cross sections of the ^{12}C(^{3}He,n)^{14}O reaction at $E_{3He} = 45.5$ MeV leaving the residual nucleus in various excited states. The solid curves are the present DWBA calculations. The dashed curves are the previous calculations and the dots are the experimental data taken from Ref. [6].
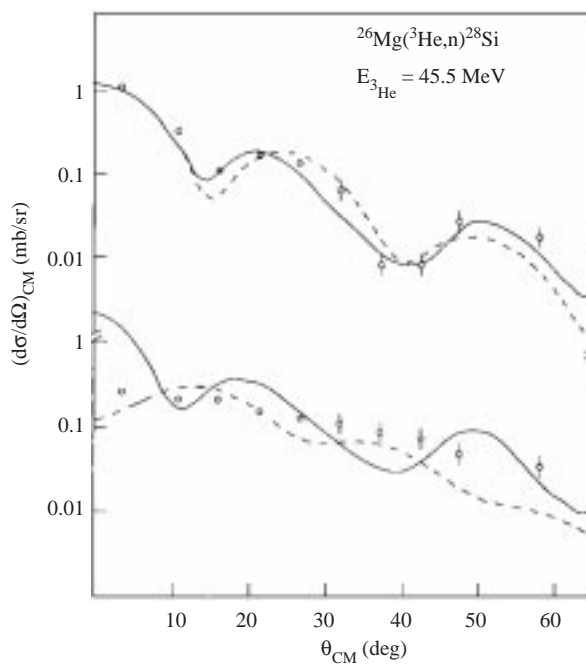

Figure 2. The solid curves are the present DWBA calculations. The dashed curves are the previous calculations and the dots are the experimental data taken from Ref. [3].

In conclusion, the present study shows that the one-step DWBA method is capable of producing accurate predictions for both the shapes and magnitudes of $\left(^{3}He, n\right)$ angular distributions. Therefore, we conclude

that the contribution of the two-step process in analyzing $\left(^{3}He, n\right)$ reactions are usually not important at least in the forward angle region.

References

[1] W. Bohne, H. Fuchs, K. Grabisch, M. Hagen, H. Homeyer, U. Janetzki, H. Lettau, K.H. Maler, H. Morgenstern, P. Pietrzyk, G. Röschert and J.A. Scheer, *Nucl. Phys.* **A131** (1969) 273.

[2] 2. W. Bohne, K.D. Bücks, H. Funchs, K. Grabisch, D. Hilscher, U. Jahnke, H. Kluge, T.G. Masterson, H. Morgenstern and B.H. Wildenthal, *Nucl. Phys.* **A378** (1982) 525.

[3] 3. K. Abe, K. Maeda, T. Ishimatsu, T. Kawamura, T. Furukawa, H. Orihara and C.D. Zafiratos, *Nucl. Phys.* **A462** (1987) 358.

[4] 4. M.R. Greenfield, C.R. Bingham, E. Newman and M.J. Saltmarsch, *Phys. Rev.* **C6** (1972) 1756.

[5] 5. D.H. Feng, K.N. Geller and N. Klein, *Phys. Rev.* **C18** (1978) 33.

[6] 6. K. Abe, K. Maeda, T. Ishimatsu, T. Kawamura, T. Furukawa, H. Orihara and C.D. Zafiratos, *Nucl. Phys.* **A466** (1987) 109.

[7] 7. H.M. Sen Gupta, J.B.A. England, F. Khazaie, E.M.E. Raways and G.T.A. Squier, *Nucl. Phys.* A517 (1990) 82.

[8] 8. M. Haque, S.K. Das, A.K. Basak and H.M.Sen Gupta, *IL Nuovo Cimento* **111A** (1998) 1131.

[9] 9. A. Osman and A.A. Farra, *J. Phys.* **G15** (1989) 871.

[10] 10. A. Osman and A.A. Farra, *IL Nuovo Cimento* **A107** (1994) 207.

[11] 11. I.S. Towner and J.C. Hardy, *Adv. Phys.* **18** (1969) 401.

[12] 12. G.R. Satchler, Nucl. Phys. **55** (1964) 1.

[13] 13. H. Nann, B. Hubert, R. Bass, *Nucl. Phys.* **A176** (1971) 553.

[14] 14. P. Schwandt, H.O. Meyer, W.W. Jacobs, A.D. Bacher, S.E. Vigdor and M.D. Kaitchuck, *Phys. Rev.* **C26** (1982) 55.

[15] 15. F.D. Becchetti, W. Makofske and G.W. Greenlees, *Nucl. Phys.* **A190** (1972) 437.

[16] 16. F.D. Becchetti, Jr and G.W. Greenlees, *Phys. Rev.* **182** (1969) 1190.

[17] 17. B.F. Bayman and A. Kallio, *Phys. Rev.* **156** (1967) 1121.

[18] 18. S. Cohen and D. Kurath, *Nucl. Phys.* **A101** (1967) 1.

[19] 19. D.J. Millener and D. Kurath, *Nucl. Phys.* **A255** (1975) 315.

[20] 20. A.K. Basak, A. Basher, A.S. Mondal, S. Bhattacherjee, A. Uddin, S.K. Das, M.Haque and H.M. Sen Gupta, *Phys. Rev.* **C56** (1997) 1983.