Unsteady Couette Flow With Heat Transfer Considering Ion-Slip

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Abstract

The unsteady Couette flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel non-conducting porous plates is studied with heat transfer taking ion-slip into consideration. An external uniform magnetic field and a uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to a constant pressure gradient. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are included in the energy equation. The effect of ion-slip an uniform suction and injection on both the velocity and temperature distributions is examined.

Key Words: Hydromagnetic, fluid mechanics, heat transfer, Hall effect, ion-slip, numerical solution, Couette flow, parallel plates channel.

1.   Introduction

The magnetohydrodynamic flow between two parallel plates, known as Hartmann flow, is a classical problem whose solution has many applications in magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Hartmann and Lazarus [1] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary, and insulated plates. Then, a lot of research work concerning the Hartmann flow was conducted under different physical effects [2–10]. In most cases the Hall and ion-slip terms were ignored in applying Ohm's law, as they have no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable [5]. Under these conditions, the Hall current and ion slip are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. Tani [7] studied the Hall effect on the steady motion of electrically conducting and viscous fluids in channels. Soudalgekar et al. [8–9] studied the effect of the Hall currents on the steady MHD Couette flow with heat transfer. The temperatures of the two plates were assumed either


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to be constant [8] or to vary linearly along the plates in the direction of the flow [9]. Abo-El-Dahab [10] studied the effect of Hall current on the steady Hartmann flow subjected to a uniform suction and injection at the bounding plates. Later, Attia [11] extended the problem to the unsteady state with heat transfer, taking the Hall effect into consideration while neglecting ion-slip.

In the present study, the transient flow and heat transfer of an incompressible, viscous, electrically conducting fluid between two infinite non-conducting horizontal porous plates are studied with the consideration of both Hall current and ion-slip. The upper plate is moving with a constant velocity U_o while the lower plate is kept stationary. The fluid is acted upon by a constant pressure gradient, a uniform suction and injection and a uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4, 5], therefore, the uniform magnetic field B_o is considered as the total magnetic field acting on the fluid. The two plates are maintained at two different but constant temperatures: T_1 and T_2, respectively, where $T_1 < T_2$. The governing equations are solved numerically taking the Joule and the viscous dissipations into consideration in the energy equations. The effect of the magnetic field, Hall current, ion-slip, and suction and injection on both the velocity and temperature distributions is studied.

2. Description of the Problem

The two non-conducting plates are located at the $y = \pm h$ planes and extend from $x = -\infty$ to ∞ and $z = -\infty$ to ∞, as shown in Figure 1. The fluid flows between the two plates under the influence of a constant pressure gradient dP/dx in the x-direction, and a uniform suction from above and injection from below with constant velocity $\mathbf{v}_o$ which all are applied at $t = 0$. The whole system is subjected to a uniform magnetic field $\mathbf{B}_o$ in the positive y-direction. From the geometry of the problem, it is evident that all quantities apart from the pressure gradient dP/dx, which is assumed constant, do not depend on the coordinates x or z. The existence of the Hall term gives rise to a z-component of the velocity. Thus, the velocity vector of the fluid is
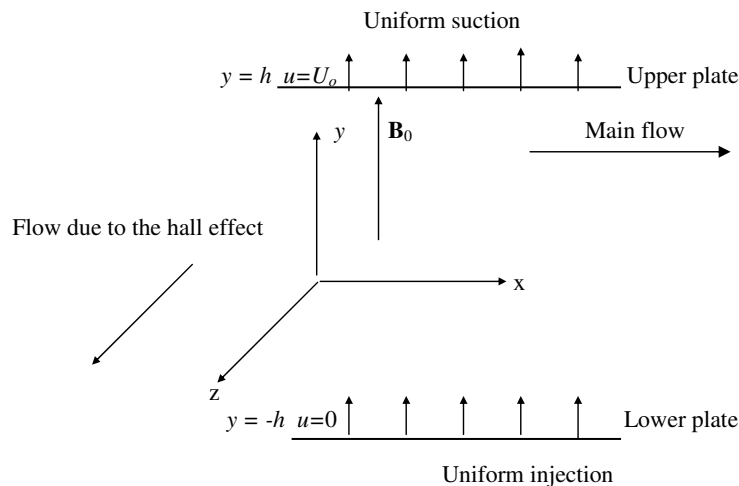
Figure 1. The geometry of the problem.

$$\mathbf{v}(y,\ t) = u(y,\ t)\mathbf{i} + v_o\mathbf{j} + w(y,\ t)\mathbf{k}$$

with the initial and boundary conditions $u = w \leq 0$ at $t \leq 0$, and $u = w = 0$ at $y = -h$ and $u = U_o$ and $w = 0$ at $y = h$ for $t > 0$. The temperature $T(y,\ t)$ at any point in the fluid satisfies both the initial and boundary conditions $T = T_1$ at $t \leq 0$, $T = T_2$ at $y = +h$, and $T = T_1$ at $y = -h$ for $t > 0$. The fluid flow is governed by the momentum equation

$$\rho \frac{D\mathbf{v}}{Dt} = \mu \nabla^2 \mathbf{v} - \nabla P + \mathbf{J} \times \mathbf{B}_o, \tag{1}$$

where ρ and μ are, the density and the coefficient of viscosity of the fluid respectively. If the Hall and ion slip terms are retained, the current density $\mathbf{J}$ is given by

$$\mathbf{J} = \sigma \left\{ \mathbf{v} \times \mathbf{B}_o - \beta(\mathbf{J} \times \mathbf{B}_o) + \frac{\beta Bi}{B_o}(\mathbf{J} \times \mathbf{B}_o) \times \mathbf{B}_o \right\},$$

where σ is the electric conductivity of the fluid, β is the Hall factor, $\mathbf{B}_i$ is the ion-slip parameter [4] and $B_o = |\mathbf{B}_o|$. This equation may be solved in $\mathbf{J}$ to yield

$$\mathbf{J} \times \mathbf{B}_o = -\frac{\sigma B_o^2}{(1 + BiBe)^2 + Be^2} \{[(1 + BiBe)\,u + Bew]\mathbf{i} + [(1 + BiBe)\,w - Beu]\mathbf{k}\}, \tag{2}$$

where $Be = \sigma\beta B_o$ is the Hall parameter [4]. Thus, in terms of Eq. (2), the two components of Eq. (1) read

$$\rho\frac{\partial u}{\partial t} + \rho v_o \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2}{(1 + BiBe)^2 + Be^2}((1 + BiBe)u + Bew), \tag{3}$$

$$\rho\frac{\partial w}{\partial t} + \rho v_o \frac{\partial w}{\partial y} = \mu\frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_o^2}{(1 + BiBe)^2 + Be^2}((1 + BiBe)w - Beu). \tag{4}$$

To find the temperature distribution inside the fluid, we use the energy equation [12]

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p v_o \frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{\sigma B_o^2}{(1 + BiBe)^2 + Be^2}(u^2 + w^2), \tag{5}$$

where c and k are, respectively, the specific heat capacity and the thermal conductivity of the fluid. The second and third terms on the right-hand side represent the viscous and Joule dissipations, respectively.

The problem is simplified by writing the equations in non-dimensional form. We define the following non-dimensional quantities:

$$\hat{x} = \frac{x}{h}, \hat{y} = \frac{y}{h}, \hat{z} = \frac{z}{h}, \hat{u} = \frac{u}{U_o}, \hat{w} = \frac{w}{U_o}, \hat{P} = \frac{P}{\rho U_o^2}, \hat{t} = \frac{tU_o}{h}, \hat{T} = \frac{T - T_1}{T_2 - T_1},$$

where

$Re = \rho h U_o/\mu$, is the Reynolds number,

$S = v_o/U_o$, is the suction parameter,

$Pr = \mu c_p/k$ is the Prandtl number,

$Ec = U_o^2/c_p(T_2 - T_1)$ is the Eckert number, and

$Ha^2 = \sigma B_o^2 h^2/\mu$ where Ha is the Hartmann number.

In terms of the above non-dimensional variables and parameters, the basic Eqs. (3)–(5) can be written as (the "hats" will be dropped for convenience)

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - \frac{Ha^2}{Re((1 + BiBe)^2 + Be^2)}((1 + BiBe)u + Bew), \tag{6}$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial y} = \frac{1}{Re}\frac{\partial^2 w}{\partial y^2} - \frac{Ha^2}{Re((1 + BiBe)^2 + Be^2)}((1 + BiBe)w - Beu), \tag{7}$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{Re\,Pr}\frac{\partial^2 T}{\partial y^2} + \frac{Ec}{Re}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{EcHa^2}{Re((1 + BiBe)^2 + Be^2)}(u^2 + w^2), \tag{8}$$

The initial and boundary conditions for the velocity become

$$u = w = 0, \quad t \leq 0, \quad u = w = 0, \quad y = -1, \quad u = 1, \quad w = 0, \quad y = 1, \quad t > 0 \tag{9}$$

and the initial and boundary conditions for the temperature are given by

$$t \leq 0 : T = 0, \quad t > 0 : T = 1, \quad y = +1, \quad T = 0, \quad y = -1. \tag{10}$$

3. Numerical Solution of the Governing Equations

Equations (6)–(8) are solved numerically using finite differences [13] under the initial and boundary conditions (9) and (10), to determine the velocity and temperature distributions for different values of the parameters Ha and S. The Crank-Nicolson implicit method is applied. The finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the y-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. Finally, the block tri-diagonal system is solved using Thomas' algorithm. All calculations have been carried out for $dP/dx = 5$, $Re = 1$, $Pr = 1$ and $Ec = 0.2$.

4. Results and Discussion

Figure 2 shows the profiles of the velocity components u and w and temperature T for various values of time t. The figure is plotted for $Ha = 3$, $Be = 3$, $Bi = 3$, and $S = 1$. It is observed that the velocity component u reaches the steady state faster than w which, in turn, reaches the steady state faster then T. This is expected, since u is the source of w, while both u and w act as sources for the temperature.

Figure 3 shows the time evolution of u, w, and T at the centre of the channel $y = 0$ for various values of the Hall parameter Be and the ion slip parameter Bi. In this figure, $Ha = 3$ and $S = 0$. It is clear from Figure 3a that increasing the parameter Be or Bi increases u. This is because the effective conductivity $(\sigma/\{(1 + BiBe)^2 + Be^2\})$ decreases with increasing Be or Bi, which reduces the magnetic damping force on u. In Figure 3b, the velocity component w increases with increasing Be, since w is a result of the Hall effect. On the other hand, increasing the ion-slip parameter Bi decreases w for all values of Be as a result of decreasing the source term of w $(BeHa^2u/\{(1 + BiBe)^2 + Be^2\})$ and increasing its damping term $(Ha^2w/\{(1 + BiBe)^2 + Be^2\})$. The influence of the ion slip on w becomes more pronounced for higher values of Be. An interesting phenomenon is observed in Figures 3a and 3b, specially when Be is large, where the components u and w overshoot. For some times, they exceed their steady state values and then go down towards steady state. It is interesting to see the effect of the ion-slip in the suppression of these overshoots.

(a)

(b)

(c)
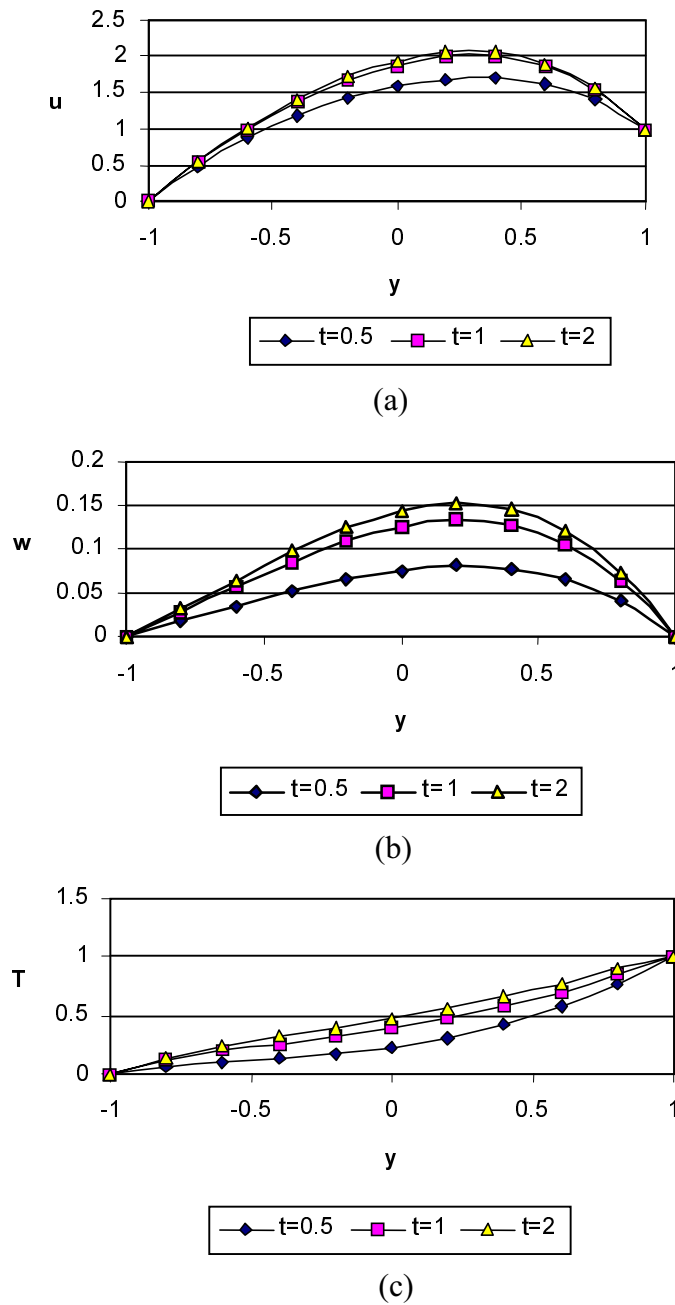
Figure 2. Time variation of the profile of: (a) u; (b) w; and (c) T. ($Ha = 3$, $Be = 3$, $Bi = 3$, and $S = 1$).

Figure 3c indicates that increasing Be decreases T for all times. For small times, increasing Bi decreases T. For large times, increasing Bi decreases T, but increasing Bi more increases T. This can be attributed to the fact that, for small times, u and w are small and an increase in Be or Bi decreases the Joule dissipation which is also proportional to $1/\{(1 + BiBe)^2 + Be^2\}$. For large times, increasing Be increases both u and w and, in turn, increases the Joule and viscous dissipations. This accounts for crossing the curves of T for all values of Be and Bi.
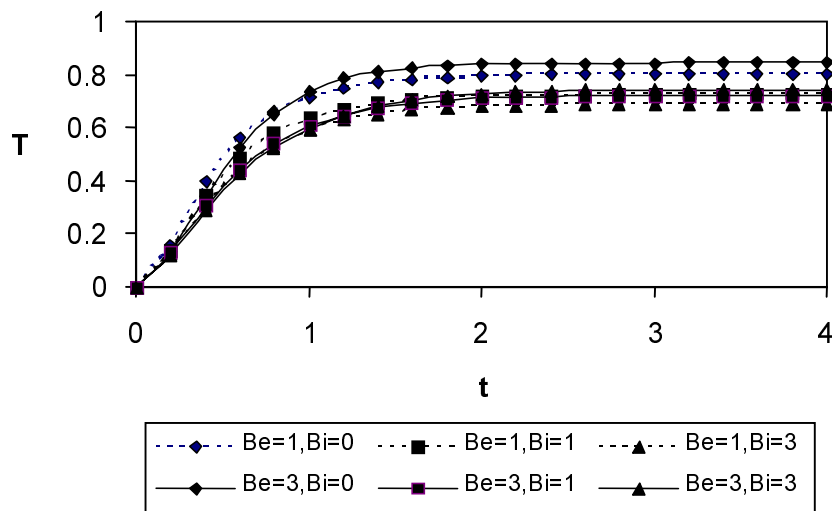
Figure 4 shows the time evolution of u, w, and T at the centre of the channel $y = 0$ for various values of the Hartmann number Ha and ion slip parameter Bi. In this figure, $Be = 3$ and $S = 0$. Figure 4a indicates that the effect of Bi on u depends on Ha. For small values of Ha, slightly increasing Bi decreases u as a
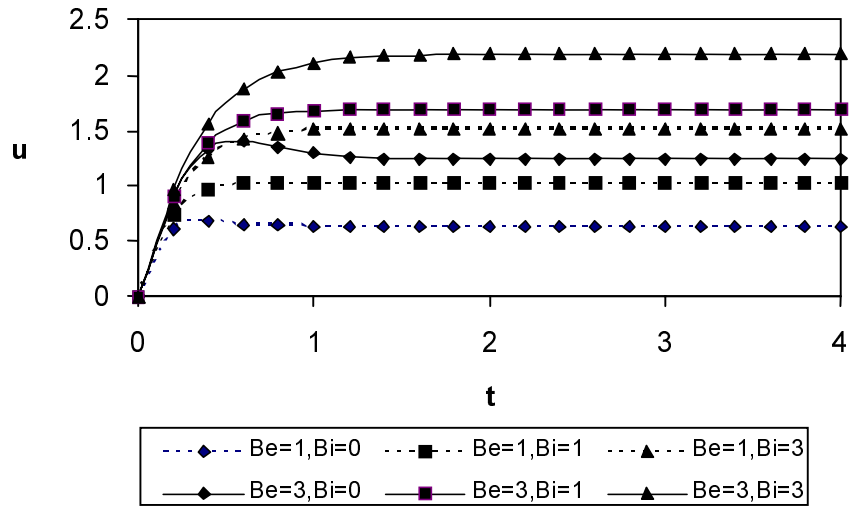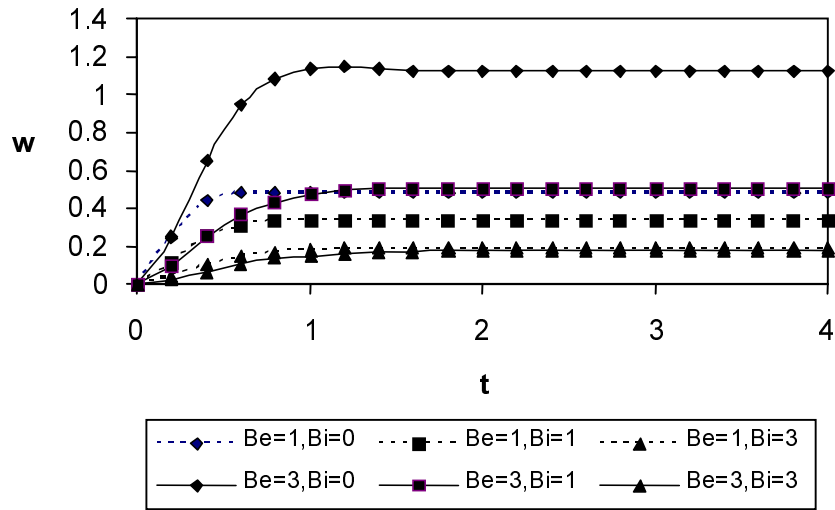
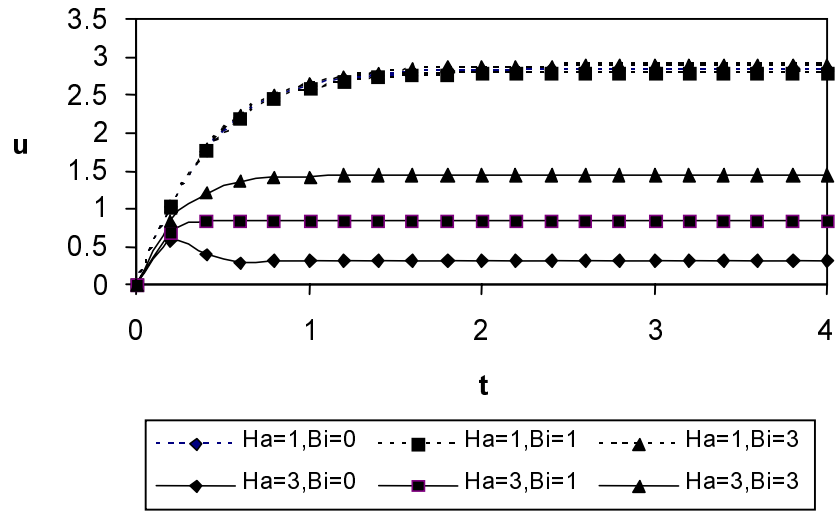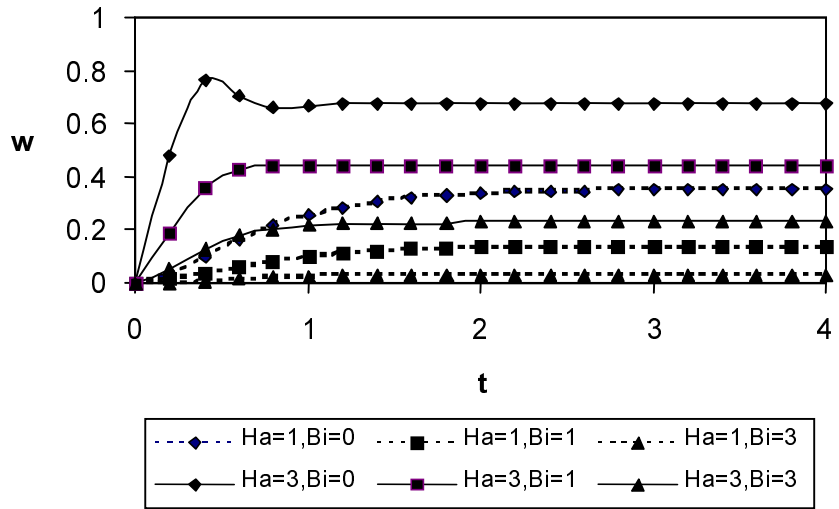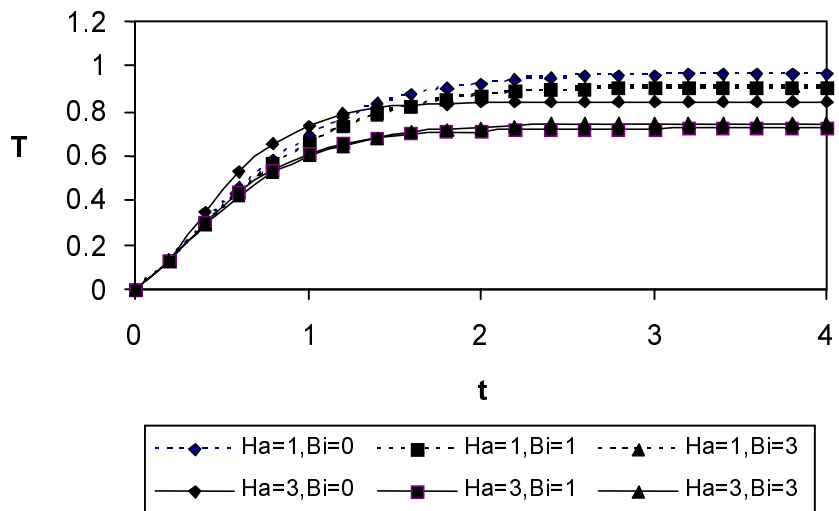**Figure 3.** Effect of the parameters Be and Bi on: (a) u at $y = 0$; (b) w at $y = 0$ and (c) T at $y = 0$. ($Ha = 3$ and $S = 0$).

(a)
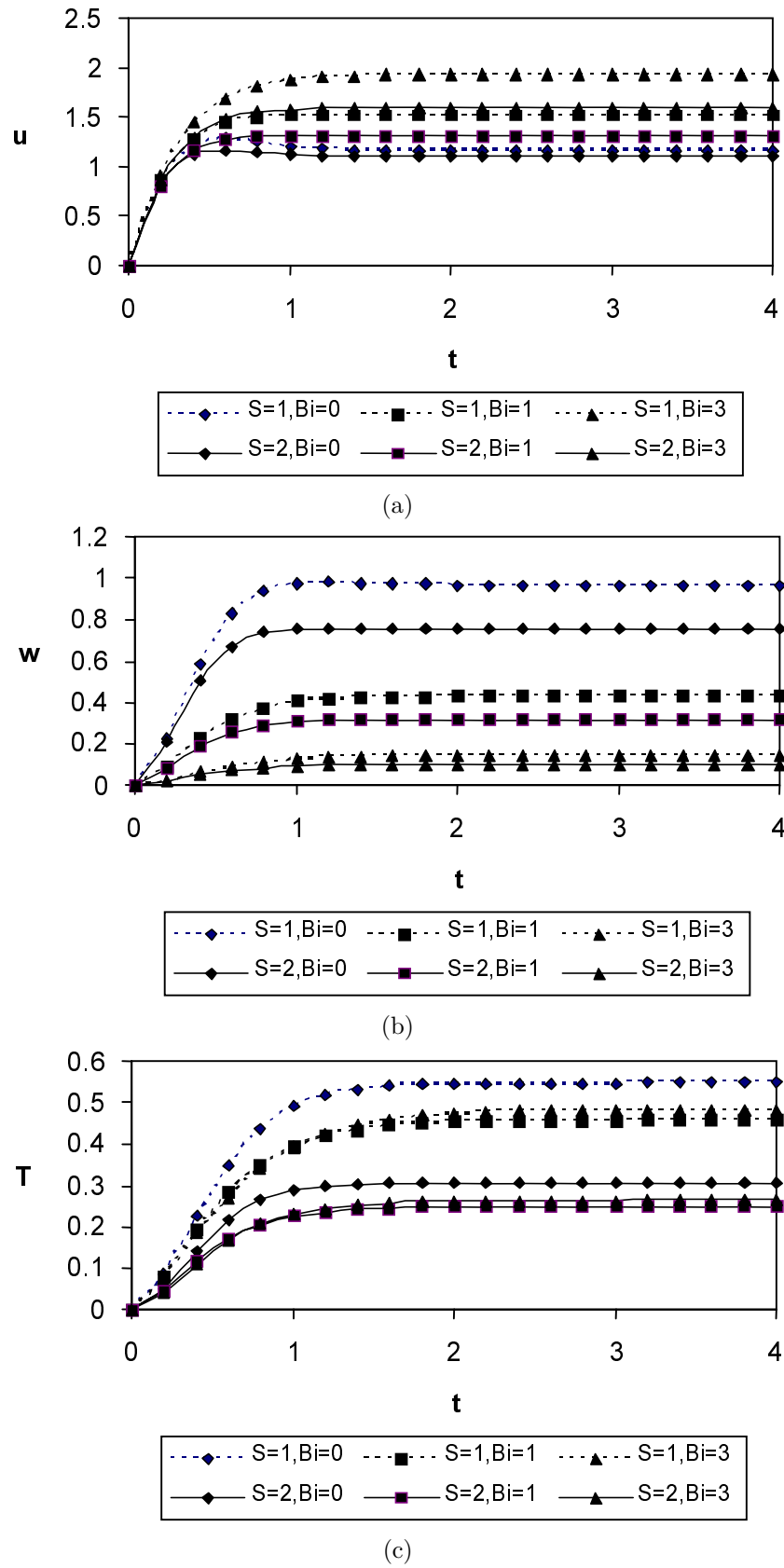

(b)

(c)

Figure 4. Effect of the parameters Ha and Bi on: (a) u at $y = 0$; (b) w at $y = 0$ and (c) T at $y = 0$ ($Be = 3$ and $S = 0$).

(a)

(b)

(c)

Figure 5. Effect of the parameters S and Bi on: (a) u at $y = 0$; (b) w at y $= 0$ and (c) T at $y = 0$ (Be=3 and $Ha = 3$).

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result of increasing the damping force on u, which is proportional to Bi. Increasing Bi more increases the effective conductivity and, in turn, decreases the damping force on u, which increases u. On the other hand, for larger values of Ha, u becomes small, and increasing Bi always decreases the effective conductivity and therefore increases u. It is also clear that the effect of Bi on u becomes more apparent for higher values of Ha. Figure 4b ensures that increasing the ion-slip parameter Bi decreases w for all values of Ha and that its effect is more apparent for higher values of Ha. Figures 3a and 3b indicate the effect of Bi in the suppression of the overshoots in u and w that occur for large values of Ha. Figure 4c shows that the parameter Bi has a negligible effect on T for weak magnetic fields. For large values of Ha, increasing Bi decreases T but increasing Bi more increases T.

Figure 5 shows the time evolution of u, w, and T at the centre of the channel $y = 0$ for various values of the suction parameter S and the ion-slip parameter Bi. In this figure, $Ha = 3$ and $Be = 3$. Figures 5a and 5b show that increasing the suction decreases both u and w due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. It is also clear from Figures 5a and 5b that the effect of the suction parameter on u becomes more pronounced as Bi increases, while its effect on w decreases as Bi increases. Figure 5c shows that increasing S decreases the temperature at the centre of the channel. This is due to the influence of convection in pumping the fluid from the cold lower half towards the centre of the channel. Figure 5c indicates that the influence of the parameter Bi on T depends on t for all values of S.

5.   Conclusion

The unsteady Couette flow of a conducting fluid under the influence of an applied uniform magnetic field has been studied, considering the Hall and ion-slip effects in the presence of uniform suction and injection. The effect of the magnetic field, the Hall parameter, the ion slip parameter, and the suction and injection velocity on the velocity and temperature distributions has been investigated. It is found that the effect of the ion slip on the main velocity u depends upon the magnetic field. For large values of the magnetic field, increasing the ion slip increases u. For small values of the magnetic field, increasing the ion slip slightly decreases u, but increasing it more increases u. It is also shown that increasing the Hall parameter increases the velocity component w, while increasing the ion-slip decreases w. The influence of the Hall current on w decreases greatly as the ion slip increases. The effect of the suction and injection velocity on u increases as the ion slip increases while its effect on w decreases when increasing the ion slip. The influence of the ion slip on the temperature T depends on time and the magnetic field. Increasing the ion slip decreases T for small time and increases it for large time. The effect of the ion slip on T can be neglected for small values of the magnetic field.

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