

Collapse of Interstellar Molecular Clouds

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Abstract

In this paper we systematically investigate the length and time scales of an interstellar molecular cloud for collapse under the influence of self-gravity, magnetic field and Coriolis forces. We used Magnetohydrodynamic (MHD) equations in linearized form in order to explore the dynamical evolution of perturbations. We found that both the Lorentz force and the Coriolis force support the cloud against self contraction, *i.e.*, they introduce stabilizing effect against gravitational instability. Of the two cloud types with the same physical size, only those threaded by an interstellar magnetic field without rotation or those rotating without magnetic field will survive against gravitational collapse.

Key Words: Magnetohydrodynamics, interstellar clouds, self-gravity, Lorentz force, Coriolis force.

1. Introduction

Giant Molecular Clouds (GMCs) in the Milkyway and in other galaxies are believed to be the birth places of stars. A widely accepted view is that portions of molecular clouds go through gravitational collapse and initiate star formation. But, there is yet to be found actual kinematic evidence of collapse. It is essential to test the theoretical pictures with observational data. Some of the models, such as the “inside-out collapse model” [1, 2], make testable predictions about the density and velocity fields. Testing these predictions in simple clouds like small globules is a prerequisite for comprehending star formation in more massive clouds. Early optical studies based on star count techniques showed that many small globules have central density structures [3, 4]. These findings supported an early suggestion that gravitational collapse is going on in these globules [5]. Martin & Barret [6] suggested that molecular line studies indicate that some globules were unstable against gravitational collapse. Far infrared observations of B335 revealed direct evidence of star formation in this widely studied globule [7]. The IRAS data base has provided more evidence for star formation in globules [8, 9].

Typical masses of molecular clouds are in the range $10^3 - 10^4 M_{\odot}$. If these were free falling their lifetimes would have been unrealistically short, and the implied star formation rate would be much higher than observed [10, 11]. There must be some force or forces supporting these clouds against their self gravity. Mouschovias [12] showed that thermal pressure forces alone could not support these clouds for Bonnor – Ebert (\approx Jeans) critical mass is less than $10M_{\odot}$. Supersonic turbulence may be regarded as another mechanism supporting larger masses. And it is certainly true. However, supersonic turbulence dissipates in such a short time scale compared to cloud lifetimes to be of significance [13]. Magnetic fields, no matter how weak, have been shown to give support against gravity in dense, massive clouds [11, 14-16].

2. Self-Initiated Star Formation

We systematically present, with progressively more realistic physical inputs, the length and time scales of collapse in self-gravitating, isothermal, magnetically supported rotating molecular clouds in its initial state. In subsection 2.1 we review the well – known cloud collapse by self-gravity alone. For a homogeneous gas, an analysis due to J. H. Jeans can be found in Scheffler & Elsasser [17]. Subsection 2.2. deals with the same problem with a new physical ingredient, i.e., rotation. The effects of Coriolis forces to self-gravity is treated in subsection 2.2. Finally, in subsection 2.3. we investigate the more general problem of cloud collapse under the influence of self-gravity, rotation and the magnetic field. Summary and Conclusion is presented in Section 3.

2.1. Cloud Collapse by Self-Gravity

Dynamical evolution of the cloud is investigated by hydrodynamic equations given below. Equation (1) is the mass conservation equation; Equation (2) is the equation of motion taking into account the pressure gradient and the self gravitational potential of the cloud; Equation (3) is the Laplacian of the gravitational potential; and Equation (4) is the definition of sound speed in an isothermal homogeneous gas:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

$$\rho \frac{D \mathbf{v}}{Dt} \equiv \rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P - \rho \nabla \Phi \quad (2)$$

$$\nabla^2 \Phi = 4\pi \rho G \quad (3)$$

$$P = c_s^2 \rho. \quad (4)$$

Here, \mathbf{v} , ρ , P , c_s and Φ denote the velocity, density, pressure and speed of sound in an isothermal medium and gravitational potential respectively. $D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla$ is the convective derivative.

Now, let us imagine a homogeneous interstellar molecular cloud. In steady state, $(\partial/\partial t = 0)$, $\rho = \rho_0 = \text{const.}$; $P = P_0 = \text{const.}$; $\Phi = \Phi_0 = 0$; $\mathbf{v} = \mathbf{v}_0 = 0$. Let us suppose that the steady state is perturbed, say, by self-initiation of the accumulation of matter at certain point or points. Let us assign the subscripts “ $_0$ ” and “ $_1$ ” to denote the steady state and perturbed quantities, respectively. We treat the perturbation in linearized form, i.e., $\rho = \rho_0 + \rho_1$; $P = P_0 + P_1$; $\Phi = \Phi_0 + \Phi_1$; $\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1$. In linear approximation, it is assumed that $|\rho_1| \ll |\rho_0|$, $|P_1| \ll |P_0|$, *etc.*; on the other hand, the multiplication of two perturbed quantities are also assumed to be vanishingly small, i.e., $\rho_1 P_1 = \rho_1 \Phi_1 = P_1 v_1 = \Phi_1 v_1 = \text{etc.} = 0$. Under these approximations, the linearized forms of Equations (1) – (4) are given as follows:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = 0 \quad (5)$$

$$\frac{\partial \mathbf{v}_1}{\partial t} = -\frac{1}{\rho_0} \nabla P_1 - \nabla \Phi_1 \quad (6)$$

$$\nabla^2 \Phi_1 = 4\pi \rho_1 G \quad (7)$$

$$P_1 = c_s^2 \rho_1. \quad (8)$$

If perturbed quantities ρ_1, Φ_1, P_1, v_1 grow in time then the medium is said to be unstable. In an unstable medium the internal energy become free and drives the system to a new stable state. In what follows, we inquire about the conditions for marginal stability. To this end, we take the divergence of Equation (6); the value of P_1 taken from Equation (8) is also substituted into Equation (6):

$$\nabla \cdot \frac{\partial \mathbf{v}_1}{\partial t} = -\nabla^2 \left(c_s^2 \frac{\rho_1}{\rho_0} + \Phi_1 \right). \quad (9)$$

Now, the time derivative of Equation (5),

$$\frac{\partial^2 \rho_1}{\partial t^2} = -\rho_0 \nabla \cdot \frac{\partial \mathbf{v}_1}{\partial t}, \quad (10)$$

is also substituted into Equation (6). If the term $\nabla \cdot \partial \mathbf{v}_1 / \partial t$ is eliminated from Equations (9) and (10) and Equation (7) is used, we get the dispersion relation for the perturbation as follows:

$$\frac{\partial^2 \rho_1}{\partial t^2} = 4\pi \rho_0 \rho_1 G + c_s^2 \nabla^2 \rho_1. \quad (11)$$

Space and time variation of Fourier components of the perturbed quantities is assumed to be of the form, $\exp[i(\omega t - kx)]$ for a one dimensional case. With this assumption, the dispersion relation is reduced to:

$$\omega^2 = k^2 c_s^2 - 4\pi \rho_0 G. \quad (12)$$

If there is to be self-gravity, then ρ_1 should grow in time. A necessary condition for this is $\omega^2 < 0$. But it is not sufficient. At the same time, the purely imaginary ω should be negative, *i.e.*, $\omega_i < 0$. Because only then a multiplicative contribution from the phasor factor comes to the amplitude of the perturbation. The later condition ensures the growth of density perturbation in time. $\omega^2 < 0$ is equivalent to $k^2 < 4\pi \rho_0 G / c_s^2$. We define the threshold wave number k_g of the perturbation, $4\pi \rho_0 G / c_s^2 = k_g^2$. Put in other words, in order for perturbation to drive an instability, the perturbation wavelength should be larger than the threshold value, *i.e.*, $\lambda (= 2\pi/k) > \lambda_g (= 2\pi/k_g)$, where λ_g is the threshold value of the perturbation wavelength. The dimension L of the contracted region is half the perturbation wavelength, *i.e.* $L = \lambda / 2$ [17]. Thus the Jeans criterion for instability becomes

$$L_g > L_J = \sqrt{\frac{\pi c_s^2}{4\rho_0 G}} = \sqrt{\frac{\pi k_B T}{4\rho_0 G \bar{\mu} m_H}} = \frac{7.822}{\bar{\mu}} \sqrt{\frac{T}{N_H}} \text{ pc}, \quad (13)$$

where L_J is often called the Jeans length; k_B is the Boltzmann constant; T is the temperature; N_H is the number density of H atoms; m_H is the proton mass; and $\bar{\mu}$ is the mean molecular weight. Equation (13) indicates that, if a molecular cloud is to initiate a self collapse, its physical dimension should be greater than the Jeans length.

2.2. Magnetized Cloud Collapse by Self Gravity

Magnetic fields are shown to play a decisive dynamical role in regions of star formation (see [18] for theoretical arguments and [19] for observational data). Magnetic fields support the clouds against gravitational contraction in two ways. One is the pressure of static magnetic field perpendicular to the field lines [14]. The other magnetic support mechanism involves fluctuating fields associated with MHD waves at sub-Alfvénic

yet supersonic speeds. MHD waves may persist longer than purely hydrodynamic turbulence and supports the cloud in both parallel and perpendicular (to magnetic field) directions [13].

In this subsection we take the hitherto neglected Lorentz force into account and find the “modified” Jeans length. The linearized form of the equation is as follows:

$$\frac{\partial \mathbf{v}_1}{\partial t} = -\frac{1}{\rho_0} \nabla P_1 - \nabla \Phi_1 - \frac{1}{8\pi \rho_0} \nabla B^2 + \frac{1}{4\pi \rho_0} (\mathbf{B} \cdot \nabla) \mathbf{B}, \quad (14)$$

where the third and the fourth terms on the right hand side of Equation (14) come from the expansion of the Lorentz force, $\mathbf{J} \times \mathbf{B}$; and where \mathbf{J} is the current density and \mathbf{B} is the magnetic flux. The choice of uniform magnetic field is widely used in the literature (see, *e.g.*, Fig. 4 in [23]). If the magnetic field is assumed initially uniform, i.e. there is no variation of the field strength along the field lines, then $\mathbf{B} \cdot \nabla \mathbf{B} = 0$ and the equation (14) is reduced to

$$\frac{\partial \mathbf{v}_1}{\partial t} = -\frac{1}{\rho_0} \nabla P_1 - \nabla \Phi_1 - \frac{1}{8\pi \rho_0} \nabla B^2. \quad (15)$$

We assume that the magnetic field is “frozen” into the gas. Only a critical ionization level can justify this assumption. With now the assumption of the frozen-in condition, the magnetic induction equation can be written as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \quad (16)$$

The expansion of Equation (16) with the help of triple vector product,

$$\nabla \times (\mathbf{C} \times \mathbf{D}) = \mathbf{C} (\nabla \cdot \mathbf{D}) - \mathbf{D} (\nabla \cdot \mathbf{C}) + (\mathbf{D} \cdot \nabla) \mathbf{C} - (\mathbf{C} \cdot \nabla) \mathbf{D}, \quad (17)$$

gives the linearized form of the magnetic induction equation as below:

$$\frac{\partial \mathbf{B}_1}{\partial t} = \mathbf{B}_0 \cdot \nabla \mathbf{v}_1 - \mathbf{B}_0 (\nabla \cdot \mathbf{v}_1). \quad (18)$$

Equations (15), (5), (7), (8) and (18) form a closed set of equations. We may combine Equations (5) and (18) as

$$\frac{\partial \mathbf{B}_1}{\partial t} = \mathbf{B}_0 \cdot \nabla \mathbf{v}_1 + \frac{\mathbf{B}_0}{\rho_0} \frac{\partial \rho_1}{\partial t}. \quad (19)$$

In deriving Equation (15) we assumed that $\mathbf{B} \cdot \nabla \mathbf{B} = 0$. This implies that, due to the absence of the mirror force, particles cannot accelerate in space along magnetic field lines. This fact in turn implies that $\mathbf{B}_0 \cdot \nabla \mathbf{v}_1 = 0$. Elimination of this term from Equation (19) brings the induction equation into a new form:

$$\frac{\partial \mathbf{B}_1}{\partial t} = \frac{\mathbf{B}_0}{\rho_0} \frac{\partial \rho_1}{\partial t}. \quad (20)$$

Let us recall that space and time variation of Fourier components of the perturbed quantities is assumed to be of the form, $\exp[i(\omega t - kx)]$. Equation (20) becomes

$$B_1 = \frac{B_0}{\rho_0} \rho_1. \quad (21)$$

Now, assuming that the order of time $\partial/\partial t$ and space operators ($\nabla \cdot$) is interchangeable, let us take the divergence ($\nabla \cdot$) of Equation (15):

$$\frac{\partial}{\partial t} \nabla \cdot \mathbf{v}_1 = -\frac{1}{\rho_0} \nabla^2 P_1 - \nabla^2 \Phi_1 - \frac{1}{4\pi} \frac{B_0}{\rho_0} \nabla^2 B_1. \quad (22)$$

Then take the time derivative ($\partial/\partial t$) of Equation (5)

$$\frac{\partial^2 \rho_1}{\partial t^2} = -\rho_0 \frac{\partial}{\partial t} \nabla \cdot \mathbf{v}_1. \quad (23)$$

Let us substitute Equations (23), (7), (21) and (8) into Equation (22), from which we get

$$-\frac{1}{\rho_0} \frac{\partial^2 \rho_1}{\partial t^2} = -\frac{1}{\rho_0} \nabla^2 c_s^2 \rho_1 - 4\pi \rho_1 G - \frac{B_0}{4\pi} \frac{1}{\rho_0} \nabla^2 B_1. \quad (24)$$

Applying the Laplacian operator to Equation (21), we get the following relation:

$$\nabla^2 B_1 = \frac{B_0}{\rho_0} \nabla^2 \rho_1. \quad (25)$$

If we substitute Equation (25) into Equation (24) we get an equation for ρ_1 :

$$-\frac{1}{\rho_0} \frac{\partial^2 \rho_1}{\partial t^2} = -\frac{1}{\rho_0} \nabla^2 c_s^2 \rho_1 - 4\pi \rho_1 G - \frac{1}{4\pi} \frac{B_0^2}{\rho_0^2} \nabla^2 \rho_1. \quad (26)$$

Let us recall once more that the perturbations vary as $\exp[i(\omega t - kx)]$; then we get the following equation for ρ_1 :

$$\frac{\omega^2}{\rho_0} = \frac{1}{\rho_0} c_s^2 k^2 - 4\pi G + \frac{1}{4\pi} \frac{B_0^2}{\rho_0^2} k^2. \quad (27)$$

We then finally get the dispersion relation for the perturbation:

$$\omega^2 = \left(c_s^2 + \frac{B_0^2}{4\pi \rho_0} \right) k^2 - 4\pi \rho_0 G. \quad (28)$$

Following the same line of reasoning as we did in subsection 2.1., we write the condition for marginal instability, as $\omega^2 < 0$. If this condition is fulfilled then the cloud will start collapsing under the force of self-gravity. The result is:

$$k^2 < \frac{4\pi \rho_0 G}{c_s^2 + \frac{B_0^2}{8\pi \rho_0}}. \quad (29)$$

Equation (29) gives us the Jeans length L_{g+B} , where subscripts denote the presence of both the gravitational potential and the magnetic field:

$$L_{g+B} > \sqrt{\frac{\pi (c_s^2 + v_A^2)}{4\rho_0 G}}, \quad (30)$$

where $B_0^2/8\pi \rho_0$ is the square of the Alfvén speed, *i.e.*, v_A^2 .

If we compare Equations (13) and (30) we see that $L_{g+B} > L_g$. This comparison clearly shows that magnetized clouds are supported by magnetic field against collapse by self-gravity. Indeed, any initiation of self contraction is resisted by magnetic pressure. In other words, of two interstellar clouds with exactly the same physical parameters, the one threaded by a uniform magnetic field requires greater dimensions (L) in order to collapse.

2.3. Rotating, Magnetized Cloud Collapse by Self Gravity

Observations have not yet revealed any massive cloud, as opposed to a core, rotating at such a rate that Coriolis forces would be important in its dynamical evolution. Even for the most rapidly rotating protostellar cores, the observed angular velocities imply angular momenta significantly smaller than those expected from angular momentum conservation. Moreover, rotating protostellar cores exhibit angular velocities consistent with theoretical predictions [20].

With the inclusion of Coriolis forces, the linearized version of the equation of motion becomes

$$\frac{\partial \mathbf{v}_1}{\partial t} = -\frac{1}{\rho_0} \nabla P_1 - \nabla \Phi_1 - \frac{1}{8\pi} B_0 \nabla B_1 - 2\boldsymbol{\Omega} \times \mathbf{v}_1, \quad (31)$$

where $\boldsymbol{\Omega}$ is the angular velocity. Now, Equations (5), (7), (8), (20) and (31) form a closed set of equations, by means of which self contraction of rotating and magnetized cloud may be investigated.

Take the divergence ($\nabla \cdot$) of Equation (31):

$$\nabla \cdot \frac{\partial \mathbf{v}_1}{\partial t} = -\frac{1}{\rho_0} \nabla \cdot \nabla P_1 - \nabla \cdot \nabla \Phi_1 - \frac{1}{4\pi} B_0 \nabla \cdot \nabla B_1 - 2\nabla \cdot (\boldsymbol{\Omega} \times \mathbf{v}_1). \quad (32)$$

Expand the last term on the right hand side of Equation (32) in accordance with the vector relation

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \quad (33)$$

so that it may be written as

$$\nabla \cdot \frac{\partial \mathbf{v}_1}{\partial t} = -\frac{1}{\rho_0} \nabla \cdot \nabla P_1 - \nabla \cdot \nabla \Phi_1 - \frac{1}{4\pi} B_0 \nabla \cdot \nabla B_1 - 2[\mathbf{v}_1 \cdot (\nabla \times \boldsymbol{\Omega}) - \boldsymbol{\Omega} \cdot (\nabla \times \mathbf{v}_1)]. \quad (34)$$

At the initial state, for the sake of simplicity, we assume that the cloud shows no differential rotation, but instead rotates like a solid body. This implies that $\nabla \times \boldsymbol{\Omega} = 0$. If we substitute Equations (23) and (8) into Equation (34) we get

$$\frac{\partial^2 \rho_1}{\partial t^2} = c_s^2 \nabla^2 \rho_1 + \rho_0 4\pi \rho_1 G + \frac{B_0^2}{4\pi \rho_0} \nabla^2 \rho_1 - 2\rho_0 \boldsymbol{\Omega} \cdot \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_{1x} & v_{1y} & v_{1z} \end{vmatrix}. \quad (35)$$

In the previous subsection we showed that the magnetic field resists contraction through magnetic pressure. Here, we are interested in the perturbation \mathbf{v}_1 velocity of which has a perpendicular component to the magnetic field (see below). We work in Cartesian coordinates, such that the axis of rotation is along the z direction, $\boldsymbol{\Omega} = \Omega \hat{z}$ and the uniform magnetic field is in the x direction $\mathbf{B} = B \hat{x}$ where \hat{x} and \hat{z} are the unit vectors of the x and z directions, respectively. There is no harm in repeating that the choice of uniform magnetic field is widely used in the literature. In order that the Coriolis force be effective, the perturbation velocity should have components in x and y direction, *i.e.*, $\mathbf{v}_1 = v_{1x} \hat{x} + v_{1y} \hat{y}$; where \hat{y} is the unit vector of the y direction. In that case, the last term on the right hand side of Equation (35) becomes $-2\rho_0 \Omega (\partial v_{1y}/\partial x - \partial v_{1x}/\partial y)$. Since perturbation quantities are assumed to vary with x only, then $\partial v_{1x}/\partial y$ vanishes. On the other hand, the linearized form of the mass conservation equation is

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = 0 \Rightarrow i\omega \rho_1 + \rho_0 \frac{\partial v_{1x}}{\partial x} = 0. \quad (36)$$

With this in mind, we should investigate as to whether the derivative $\partial v_{1x}/\partial x$ could be expressed in terms of v_{1y} . For this, the reader is asked to refer to Figure 1. A spherical cloud is viewed along the

rotation axis (z). The uniform magnetic field is along the x direction. The velocity vector with its x and y components is drawn from a fiducial point which is involved in a corotation together with the rest of the cloud. The line joining the center of the circle to the tangent point of the velocity vector makes an angle α with the x axis. In that case,

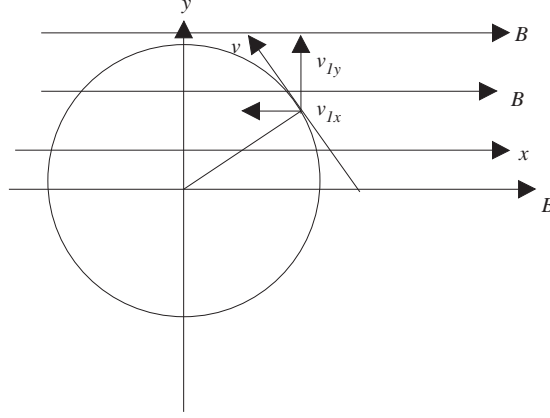


Figure 1. The projection of a spherical cloud onto xy plane.

$$\begin{aligned} v_{1x} &= v \sin \alpha \\ v_{1y} &= v \cos \alpha. \end{aligned} \quad (37)$$

At the initial state the cloud is supposed to rotate like a solid body. Therefore the relation between the instantaneous velocity, radial vector and the angular velocity is,

$$\mathbf{v} = \boldsymbol{\Omega} \times \mathbf{r}. \quad (38)$$

Or, due to the solid body rotation, $v = \Omega r$. Since the projection of the spherical cloud onto the xy plane is a circle, $x^2 + y^2 = r^2$ and $v = \Omega \sqrt{x^2 + y^2}$. Substituting into Equations (37) we get

$$\begin{aligned} v_{1x} &= \Omega \sqrt{x^2 + y^2} \sin \alpha \\ v_{1y} &= \Omega \sqrt{x^2 + y^2} \cos \alpha \end{aligned}, \quad (39)$$

from which the spatial derivatives are

$$\begin{aligned} \frac{\partial v_{1x}}{\partial x} &= \Omega \sin \alpha \frac{x}{\sqrt{x^2 + y^2}} \\ \frac{\partial v_{1y}}{\partial x} &= \Omega \cos \alpha \frac{x}{\sqrt{x^2 + y^2}} \end{aligned}. \quad (40)$$

Squaring them and taking the average during one rotational period, we get

$$\begin{aligned} \left(\frac{\partial v_{1x}}{\partial x} \right)^2 &= \Omega^2 \frac{x^2}{x^2 + y^2} \sin^2 \alpha \\ \left(\frac{\partial v_{1y}}{\partial x} \right)^2 &= \Omega^2 \frac{x^2}{x^2 + y^2} \cos^2 \alpha \end{aligned} \quad (41)$$

If $A = \frac{x^2}{x^2 + y^2}$ then, $\int_0^{2\pi} A \sin^2 \alpha \, d\alpha = \frac{A}{2}$ and, similarly, $\int_0^{2\pi} A \cos^2 \alpha \, d\alpha = \frac{A}{2}$. Thus

$$\left| \frac{\partial v_{1x}}{\partial x} \right| = \left| \frac{\partial v_{1y}}{\partial x} \right| \Rightarrow \frac{\partial v_{1x}}{\partial x} = \mp \frac{\partial v_{1y}}{\partial x}. \quad (42)$$

Variations of v_{1x} and v_{1y} with x show just the opposite tendency, that is to say, in all four quadrants of Fig. 1. v_{1y} , for example, gets smaller and smaller while v_{1x} becomes larger and larger. Therefore, in Equation (42) the one with the minus sign is the real solution. Now, if we substitute Equation (42) bearing the minus sign into Equation (36), we get,

$$i\omega \rho_1 - \rho_0 \frac{\partial v_{1y}}{\partial x} = 0. \quad (43)$$

Now, the equation of motion can be written as

$$\frac{\partial^2 \rho_1}{\partial t^2} = c_s^2 \nabla^2 \rho_1 + \rho_0 4\pi \rho_1 G + \frac{B_0^2}{4\pi \rho_0} \nabla^2 \rho_1 - 2\rho_0 \Omega \frac{\partial v_{1y}}{\partial x}. \quad (44)$$

Substitute Equation (43) into (44) and take the space and time derivatives,

$$(i\omega)^2 = c_s^2 (ik)^2 + \rho_0 4\pi G + v_A^2 (ik)^2 - 2i\omega \Omega, \quad (45)$$

and finally rearrange (45) to get the dispersion relation for the perturbation as follows:

$$\omega^2 = c_s^2 k^2 + v_A^2 k^2 - \rho_0 4\pi G + 2i\omega \Omega. \quad (46)$$

As above, the necessary condition for the perturbation to grow in time is $\omega^2 < 0$.

But it is not sufficient; $\omega_i < 0$ condition should also be satisfied. We rearrange Equation (46) in ascending order of ω ,

$$\omega^2 - 2i\omega \Omega - c_s^2 k^2 - v_A^2 k^2 + \rho_0 4\pi G = 0. \quad (47)$$

If the solution to (47) is to yield a root of ω as $\omega = i\omega_i$, then the determinant of the quadratic should be zero:

$$\Delta = b^2 - 4ac = -4\Omega^2 - 4(-c_s^2 k^2 - v_A^2 k^2 + \rho_0 4\pi G) = 0. \quad (48)$$

From this condition we get a relation between the wave vector of the propagating perturbation and the physical parameters of the cloud, *i.e.*,

$$k^2 = \frac{\rho_0 4\pi G + \Omega^2}{c_s^2 + v_A^2}, \quad (49)$$

where $k = 2\pi / \lambda$, by definition. Substituting this definition into Equation (49),

$$\frac{4\pi^2}{\lambda^2} < \frac{\rho_0 4\pi G + \Omega^2}{c_s^2 + v_A^2}, \quad (50)$$

and remembering that $L = \lambda / 2$, we get the relation below:

$$\left(\frac{\lambda}{2}\right)^2 = L_{g+B+\Omega}^2 > \frac{\pi^2 (c_s^2 + v_A^2)}{\rho_0 4\pi G + \Omega^2}, \quad (51)$$

where $L_{g+B+\Omega}$ is the Jeans length in the presence of self gravity, magnetic field and rotation. And finally, Jeans length for marginal instability is found as

$$L_{g+B+\Omega} > \sqrt{\frac{\pi (c_s^2 + v_A^2)}{\rho_0 4\pi G + \Omega^2}}. \quad (52)$$

Recall that $\omega^2 < 0$ is only a necessary but not a sufficient condition for gravitational instability. In addition, the solution to the Equation (47) should yield a negative value for ω , which cannot! To see this, we get back Equation (48) which gives $-\Omega^2 = -c_s^2 k^2 - v_A^2 k^2 + \rho_0 4\pi G$. If we put this relation back into Equation (47) we get $\omega^2 - 2i\omega\Omega - \Omega^2 = 0$ which yields a solution as $\omega = i\Omega$. For Ω is always a positive quantity, we may conclude that, in the presence of rotation, there can be no gravitational instability. Therefore Equation (52), as derived from the necessary condition $\omega^2 < 0$, itself has no meaning at all. In such clouds with even a small amount of rotation, gravitational contraction is severely limited by a centrifugal barrier [21]. This result of ours is also supported by Shu's [22] conclusion that no gravitational instability can arise in the presence of wave propagation perpendicular to the rotation axis.

2.4. Time Scales for Collapse

In order to find the time scales for collapse we refer to Equations (13) and (30). For L_J is the length scale for marginal stability, c_s^2 is the square of the ratio of the length scale to the time scale for the same stability, *i.e.*, $c_s^2 = L_J^2/\tau_J^2$ where τ_J is the time scale for the gravitational collapse. Substituting this into Equation (13), we get $\tau_J \cong 1/\sqrt{G\rho_0}$. For a cloud whose dynamical properties are determined by gravitational potential and Lorentz forces, we have to inquire about two extremes, that is, if the stresses within the cloud are predominantly acoustic, then the $c_s^2 \gg v_A^2$ approximation applies; if the reverse is true, *i.e.*, the dominant stresses are magnetic, the $v_A^2 \gg c_s^2$ approximation is justified. With any one of these approximations we go to Equation (30) and find the collapse time scale for the cloud.

3. Summary and Conclusion

Molecular clouds with masses $\sim 10^3 - 10^4 M_\odot$ cannot go through free fall, otherwise star formation rate would be higher than observed. Forces resisting the initiation of self contraction may be listed as thermal pressure forces which balance gravity along field lines; while magnetic, Coriolis and thermal pressure forces do so perpendicular to the field lines.

We reviewed the case in subsection 2.1. wherein self-gravity and thermal pressure forces determine the dynamical evolution of a cloud. Inclusion of Lorentz forces clearly showed that the magnetic pressure forces support the cloud against self contraction. This fact manifested itself through the Jeans length. Comparison of marginal stability length scales showed that of two identical clouds the one threaded by a uniform magnetic field requires greater dimensions to collapse. Rotating clouds are even more strongly stable against collapse by self-gravity. Jeans length derived in the presence of self-gravity, magnetic field and rotation should be viewed critically. Because, the term introduced by Coriolis force into the dispersion relation ensures the stability of the cloud almost permanently. One should be extra cautious in interpreting the case wherein Coriolis force is taken into account. In later case, both the necessary and sufficient conditions are to be considered and due assessment is to be given.

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