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The $b \to sgg$ Decay in the General two Higgs Doublet Model

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Abstract

We study the decay width of the inclusive process $b \to sgg$ in the two Higgs doublet model with three-level flavor changing neutral currents (model III). We analyse the dependencies of the differential decay width to the s- quark energy E_s and model III parameters, charged Higgs mass $m_{H^\pm}$ and Yukawa coupling $\bar{\xi}^D_{N,bb}$. We observe that there exist a considerable enhancement in the decay width for the relevant process. This enhancement can be reduced by choosing C_7^{eff} as negative or increasing the lower bound of $m_{H^\pm}$ to the large values. This is an interesting result which gives an idea on the mass $m_{H^\pm}$ and sign of C_7^{eff}.

Key Words: Inclusive, model III, decay width, Yukawa coupling

1. Introduction

Rare B-meson decays are loop-induced processes and therefore they are sensitive to the theoretical models and the corresponding free parameters. With the forthcoming experiments at SLAC, KEK B-factories, HERA-B and possible future accelerators, the large number of events can take place and various branching ratios of events, CP-violating asymmetries, polarization effects, etc... can be measured [1, 2]. These measurements open a window to test the models under consideration. Among rare decays, the inclusive $b \to sg$ reaches a great interest since it is theoretically clean and sensitive to new physics beyond the SM, like two Higgs doublet model (2HDM) [3], minimal supersymmetric Standard model (MSSM) [4, 5], etc... . The Branching ratio Br of $b \to sg$ decay in the SM is $Br(b \to sg) \sim 0.2\%$ for on-shell gluon [6] and the enhancement of this ratio brings an advantage [7] to decrease the averaged cham multiplicity η_c [8] and to increase kaon yields [9]. This enhancement can be obtained by including the QCD corrections or looking for new models beyond the SM. In the literature, there are number of theoretical calculations on the Br of the corresponding process beyond the SM. In [10, 11] $Br\,(b \to sg)$ was calculated in the 2HDM (Model I and II) for $m_{H^\pm} \sim 200\,GeV$ and $tan\,\beta \sim 5$ and it was found that there was an enhancement less than one order. This decay was studied in the supersymmetric models [12] and further, the Br was calculated in the framework of the model III [13], resulting with the enhancement at least one order compared to the SM one. This make it possible to describe the results coming from experiments [14].

In the case of time-like gluon, namely a $b \to sg^*$ decay, the Br should be consistent with the CLEO data [15]

$$Br\,(b \to sg^*) < 6.8\,\% \tag{1}$$

and in [13], it was shown that the model III enhancement was not contradict with this data for light-like gluon case. Recently, $O(\alpha_s)$ virtiual corrections and additional $O(\alpha_s)$ bremstrahlung effects to the decay width of $b \to sg$ was calculated in the SM [16] and the enhancement in the Br was obtained as more than a factor of two larger of the previous SM results.

As a further process, g^* can decay into quark-antiquark $\bar{q}q$ or gluon-gluon (gg) pairs. Inclusive three body decay $b \to sgg$ is another interesting one which is studied in the literatur e extensively [17, 18, 19]. It becomes not only from the chain process $b \to sg^*$ followed by $g^* \to gg$ but also from the emission of on-shell gluons from the quark lines to obey gauge invariance. In [18], the complete calculation was done in the SM and the Br ratio was found at the order of 10^{-3}. In [13, 19] the additional contribution of gluon penguins in the Model III was estimated as negligible.

This work is devoted to the study of the complete calculation for $b \to sgg$ decay in the model III. It is found that the decay width (Γ) is strongly sensitive to the charged Higgs mass $m_{H^\pm}$. This leads to the possibility of getting a considerable enhancement in the Γ, even 2 orders larger compared to the SM case.

The paper is organized as follows: In Section 2, we give a brief summary for the model III. Further, we calculate the matrix element and decay width of the inclusive $b \to sgg$ decay in the framework of the model III. Section 3 is devoted to discussion and our conclusions. In Appendix, we present the form factors appearing in the SM.

2. The inclusive process $b \to sgg$ in the framework of the model III

The Yukawa interaction in the model III can be defined as

$$\mathcal{L}_Y = \eta^U_{ij} \bar{Q}_{iL} \tilde{\phi_1} U_{jR} + \eta^D_{ij} \bar{Q}_{iL} \phi_1 D_{jR} + \xi^U_{ij} \bar{Q}_{iL} \tilde{\phi_2} U_{jR} + \xi^D_{ij} \bar{Q}_{iL} \phi_2 D_{jR} + h.c. \ , \tag{2}$$

where L and R denote chiral projections with $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_i for $i = 1, 2$, are the two scalar doublets. The Yukawa matrices $\eta^{U,D}_{ij}$ and $\xi^{U,D}_{ij}$ have in general complex entries. With the choice of ϕ_1 and ϕ_2,

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\left(\begin{array}{c} 0 \\ v + H^0 \end{array} \right) + \left(\begin{array}{c} \sqrt{2}\chi^+ \\ i\chi^0 \end{array} \right) \right] , \phi_2 = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \sqrt{2}H^+ \\ H_1 + iH_2 \end{array} \right) \ , \tag{3}$$

and the vacuum expectation values,

$$< \phi_1 > = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v \end{array} \right) , < \phi_2 > = 0 \ , \tag{4}$$

the SM particles are collected in the first doublet and particles due to new physics in the second one. The part of Yukawa interaction which is responsible for physics beyond the SM is the Flavor Changing (FC) interaction and can be written as

$$\mathcal{L}_{Y,FC} = \xi^U_{ij} \bar{Q}_{iL} \tilde{\phi_2} U_{jR} + \xi^D_{ij} \bar{Q}_{iL} \phi_2 D_{jR} + h.c. \ , \tag{5}$$

where the couplings $\xi^{U,D}$ for the FC charged interactions are

$$\begin{array}{rcl} \xi^U_{ch} &=& \xi_N \, V_{CKM} \ , \\ \xi^D_{ch} &=& V_{CKM} \, \xi_N \ , \end{array} \tag{6}$$

and $\xi^{U,D}_N$ is defined by the expression (more details see [20])

$$\xi^{U,D}_N = (V^{U,D}_L)^{-1} \xi^{U,D} V^{U,D}_R \ . \tag{7}$$

Note that the "N" in $\xi^{U,D}_N$ denotes the word "neutral".

Now we start with the decay amplitude of the decay $b \to sgg$

$$M(b \to sgg) = i \frac{\alpha_s \, G_F}{\sqrt{2}\pi} \epsilon^\mu_a(k_1) \epsilon^\nu_b(k_2) \bar{s}(p') T^{a\,b}_{\mu\nu} \, b(p) \ , \tag{8}$$

where $\epsilon_a^\mu(k)$ are polarization vectors of the gluons with color a and momentum k. Using the same parametrization for $T_{\mu\nu}^{a\,b}$ as in [18], we have

$$T_{\mu\nu}^{a\,b} = T_{\mu\nu}\frac{\lambda^b}{2}\frac{\lambda^a}{2} + T_{\mu\nu}^E\frac{\lambda^a}{2}\frac{\lambda^b}{2} \,, \tag{9}$$

and $T_{\mu\nu}^E$ can be obtained by the replacements $k_1 \leftrightarrow k_2$ and $\mu \leftrightarrow \nu$ in the function $T_{\mu\nu}$. Here $\frac{\lambda^a}{2}$ are the Gell-Mann matrices. The functions $T_{\mu\nu}$ and $T_{\mu\nu}^E$, in general, contain masses of internal quarks, m_i, $i = u, c, t$ in the SM and $i = u, c, t, d, s, b$ in the model III, since the process underconsideration takes place at least at one loop level. Therefore, at this stage, we take into account two different possibilities,

- the mass of the internal quark is heavy (namely, t-quark),
- the mass of the internal quark is light (namely, d, s, b, u, c-quarks).

In the heavy internal quark case, the terms $k^2_{\mathsf{external}}/m_i^2$ and $k^2_{\mathsf{external}}/m_i^2\,(m_W^2,\, m_{H^\pm}^2)$ are neglected and the form factors are obtained as functions of $x_t = m_t^2/m_W^2$ and $y_t = m_t^2/m_H^2$ where $m_{H^\pm}$ is the mass of charged Higgs boson in the model III. Neglecting s-quark mass, $T_{\mu\nu}$ for the heavy internal quark is given by

$$\begin{aligned} T_{\mu\nu}^{\mathsf{heavy}} &= -i\,\lambda_t\,F_2^{\mathsf{2HDM}}\,\{\big(\frac{2\,p'_\nu+\gamma_\nu\,\not k_2}{2\,p'.k_2}\,\sigma_{\mu\alpha}k_1^\alpha+\sigma_{\nu\alpha}k_2^\alpha\,\frac{2p_\mu-\not k_1\gamma_\mu}{-2p.k_1}\big) \\ &+ \frac{1}{q^2}\big(2\,\sigma_{\alpha\beta}k_1^\alpha\,k_2^\beta g_{\mu\nu}+2\,\sigma_{\nu\alpha}k_{2\,\mu}\,q^\alpha-2\,\sigma_{\mu\alpha}k_{1\,\nu}\,q^\alpha+\sigma_{\mu\nu}q^2\big)\}\,m_b\,R\,. \end{aligned} \tag{10}$$

Here q is the momentum transfer, $q = k_1 + k_2$, λ_t is the CKM matrix combination $\lambda_t = V_{tb}V_{ts}^*$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and F_2^{2HDM} is the form factor

$$F_2^{\mathsf{2HDM}} = F_2^{\mathsf{SM}}\,(x_t) + F_2^{\mathsf{Beyond}}\,(y_t) \tag{11}$$

where $F_2^{\mathsf{SM}}\,(x_t)$ is the magnetic dipole form factor of $b \to sg^*$ vertex (see Appendix). $F_2^{\mathsf{Beyond}}\,(y_t)$ is the contribution coming from the charged Higgs boson in the model III:

$$\begin{aligned} F_2^{\mathsf{Beyond}}\,(y_t) &= \frac{1}{m_t^2}\,(\bar{\xi}^{*U}_{N,tt}+\bar{\xi}^{*U}_{N,tc}\frac{V_{cs}^*}{V_{ts}^*})\,(\bar{\xi}^{U}_{N,tt}+\bar{\xi}^{U}_{N,tc}\frac{V_{cb}}{V_{tb}})G_1(y_t)\,, \\ &- \frac{1}{m_t m_b}\,(\bar{\xi}^{*U}_{N,tt}+\bar{\xi}^{*U}_{N,tc}\frac{V_{cs}^*}{V_{ts}^*})\,(\bar{\xi}^{D}_{N,bb}+\bar{\xi}^{D}_{N,sb}\frac{V_{ts}}{V_{tb}})G_2(y_t)\,, \end{aligned} \tag{12}$$

and

$$\begin{aligned} G_1\,(y_t) &= \frac{y_t}{12\,(-1+y_t)^4}\big((-1+y_t)\,(-2-5\,y_t+y_t^2)+6\,y_t\,ln\,y_t)\big)\,, \\ G_2\,(y_t) &= \frac{1}{2\,(-1+y_t)^4}\,(y_t\,(3-4\,y_t+y_t^2)+2\,(-1+y_t)\,y_t\,ln\,y_t)\,. \end{aligned} \tag{13}$$

In Eq. (12) we used the redefinition

$$\xi^{U,D} = \sqrt{\frac{4G_F}{\sqrt{2}}}\;\bar{\xi}^{U,D}\,. \tag{14}$$

Note that we neglect the chiral partner of the form factor $F_2^{\mathsf{Beyond}}\,(y_t)$ and the neutral Higgs boson effects which should be very small due to the discussion given in [21] (see also Discussion part).

If the internal quark is light (u or c), the first additional contribution comes from m_i^2/m_W^2 and $m_i^2/m_{H^\pm}^2$ terms. In the approximation $m_i^2/m_W^2 \to 0$, it is enough to replace $F_2^{\mathsf{SM}}\,(x_t)$ with "$-F_2(0)$" since $\lambda_c = -\lambda_t$ by unitarity, namely $\sum_{i=u,c,t}\lambda_i = 0$. There is no additional term coming from a light quark for $F_2^{\mathsf{Beyond}}\,(y_t)$, since $F_2^{\mathsf{Beyond}}\,(0)$ almost vanishes. For light internal quark, the second contribution comes from $k^2_{external}/m_i^2$ term which can not be neglected as in the heavy internal quark case. This contribution ($T_{2\,\mu\nu}^{\mathsf{light}}$) was calculated in the literature [18] and we present its explicit form in Appendix. Therefore, the resulting amplitude can be written as

$$T_{\mu\nu} = T^{\mathsf{heavy}}_{\mu\nu} + T^{\mathsf{light}}_{1\,\mu\nu} + T^{\mathsf{light}}_{2\,\mu\nu} \,, \tag{15}$$

where $T^{\mathsf{heavy}}_{\mu\nu}$ is given in Eq. (10) and $T^{\mathsf{light}}_{1\,\mu\nu}$ is obtained from $T^{\mathsf{heavy}}_{\mu\nu}$ with the replacement $F_2^{\mathsf{2HDM}} \to -F_2^{\mathsf{SM}}(0)$.

The function $T^{a\,b}_{\mu\nu}$ can be parametrized by seperating color symmetric and antisymmetric parts [18] as

$$T^{a\,b}_{\mu\nu} = T^{+}_{\mu\nu}\{\frac{\lambda^b}{2}, \frac{\lambda^a}{2}\} + T^{-}_{\mu\nu}[\frac{\lambda^b}{2}, \frac{\lambda^a}{2}] \,, \tag{16}$$

with

$$\begin{aligned} T^{+}_{\mu\nu} &= \frac{1}{2}(T_{\mu\nu} + T^{E}_{\mu\nu}) \,, \\ T^{-}_{\mu\nu} &= \frac{1}{2}(T_{\mu\nu} - T^{E}_{\mu\nu}) \,. \end{aligned} \tag{17}$$

Finally we get the differential decay width of the process using the expression

$$\frac{d^2\,\Gamma}{dE_s\,dE_1} = \frac{1}{2\pi^3}\frac{1}{8\,m_b}|\bar{M}|^2 \,, \tag{18}$$

where E_s is the s-quark energy and E_1 is the energy of gluon with polarization $\epsilon^a_\mu(k_1)$. Here $\bar{M}$ is the average decay amplitude, $\bar{M} = \frac{1}{2\,J+1}\,\frac{1}{N_c}\,M$, and $J = \frac{1}{2}$, $N_c = 3$. Now, we divide the differential decay width into sectors as follows:

- Symmetric sector, (Γ^{Sym}),
- Antisymmetric sector, (Γ^{Asym}),

or

- Right sector, (Γ^{R}),
- Left sector, (Γ^{L}),
- Left-rigth mixed sector, (Γ^{LR}).

Antisymmetric and symmetric sectors do not mix and they enter into decay width as

$$\Gamma^{Sym\,(Asym)} \sim Tr(T^{+\,(-)}_{\mu\nu}\,(\not{p} + m_b))\,\bar{T}^{+\,(-)}_{\mu'\,\nu'}\;\;\not{p}')\,P^{\mu\mu'}\,P^{\nu\nu'} \,, \tag{19}$$

with the corresponding color factors $C_{+} = \frac{(N_c^2-1)(N_c^2-2)}{2\,N_c}$ and $C_{-} = \frac{N_c\,(N_c^2-1)}{2}$ respectively. Here we choose the polarization sum of the on-shell gluons as

$$P^{\mu\mu'} = -g^{\mu\mu'} + \frac{k_1^\mu\,k_2^{\mu'} + k_2^\mu\,k_1^{\mu'}}{k_1.k_2} \,,$$

and $\bar{T}^{+\,(-)}_{\mu'\nu'} = \gamma_0\,(T^{+\,(-)}_{\mu'\nu'})^\dagger\,\gamma_0$.

Right, left and right-left sectors can be extracted by using the following parametrization for $T_{\mu\nu}$ (see Eq. (15)):

$$T_{\mu\nu} = \alpha_R(T^{\mathsf{heavy}}_{\mu\nu} + T^{\mathsf{light}}_{1\,\mu\nu}) + \alpha_L T^{\mathsf{light}}_{2\,\mu\nu} \,, \tag{20}$$

Here α_R, α_L are real parameters to seperate the parts with factors R and L in the fuction $T_{\mu\nu}$. With this parametrization Γ can be written as

$$\Gamma = \alpha_R^2\Gamma^R + \alpha_L^2\Gamma^L + \alpha_R\alpha_L\Gamma^{LR}|_{\alpha_L\to 1,\alpha_R\to 1} \,. \tag{21}$$

Γ^R contains form factors which are functions of $x_i = m_i^2/m_W^2$ and $y_i = m_i^2/m_{H^\pm}^2$, where $i = u, c, t$. Γ^L have the form factors which are created by the nonvanishing $k_{external}^2/m_{light}^2$ terms. Γ^{LR} contains mixed terms and its contribution is negligible compared to the other sectors.

In the calculation of Γ, there appear infrared divergences at the boundary of the kinematical region and to overcome this difficulty we follow the procedure given in [18], namely taking a cutoff c in the integration over phase space:

$$\frac{m_b}{2} - E_s \leq E_1 \leq \frac{m_b}{2}(1-c) \ , \tag{22}$$

and

$$c\frac{m_b}{2} \leq E_s \leq \frac{m_b}{2}(1-c) \ , \tag{23}$$

with $c = 0.1$.

3. Discussion

There are many free parameters in the model III such as Yukawa couplings, $\xi_{ij}^{U,D}$ where i, j are flavor indices, masses of charged and neutral Higgs bosons. The procedure is to restrict these parameters using the experimental measurements. Since the contributions of the neutral Higgs bosons h_0 and A_0 to the Wilson coefficient C_7^{eff} should not contradict with the CLEO measurement [22],

$$Br(B \to X_s\gamma) = (3.15 \pm 0.35 \pm 0.32)\, 10^{-4} \ , \tag{24}$$

the couplings $\bar{\xi}_{N,is}^D (i = d, s, b)$ and $\bar{\xi}_{N,db}^D$ should be negligible (see [21] for details). In addition, the constraints, coming from the $\Delta F = 2$ mixing, the ρ parameter [23], and the measurement by CLEO Collaboration results in the following restrictions: $\bar{\xi}_{N,tc} << \bar{\xi}_{N,tt}^U$, $\bar{\xi}_{N,bb}^D$ and $\bar{\xi}_{N,ib}^D \sim 0\,, \bar{\xi}_{N,ij}^D \sim 0$, where the indices i, j denote d and s quarks. Therefore, we can neglect all the couplings except $\bar{\xi}_{N,tt}^U$ and $\bar{\xi}_{N,bb}^D$. This leads to the cancellation of the contributions coming from the neutral Higgs bosons h_0 and A_0, having interactions which include the Yukawa vertices with the combinations of $\bar{\xi}_{N,sb}^D$ and $\bar{\xi}_{N,ss}^D$. Finally, we only take into account the multiplication of Yukawa couplings, $\bar{\xi}_{N,tt}^U\, \bar{\xi}_{N,bb}^{*D}$ and $|\bar{\xi}_{N,tt}^U|^2$ in our expressions.

In this section, we study the s quark energy E_s, Yukawa coupling $\bar{\xi}_{N,bb}^D$ and charged Higgs mass $m_{H^\pm}$ dependencies of the differential decay width $\frac{d\,\Gamma}{d\,E_s}$ for the inclusive decay $b \to sgg$. In our analysis, we restrict the parameters $\bar{\xi}_{N,tt}^U$, $\bar{\xi}_{Nbb}^D$ using the constraint for $|C_7^{eff}|$, $0.257 \leq |C_7^{eff}| \leq 0.439$ [22], where the upper and lower limits were calculated in [24] following the procedure given in [25]. Here C_7^{eff} is the effective magnetic dipole type Wilson coefficient for $b \to s\gamma$ vertex (see [24]). Throughout these calculations, we take the charged Higgs mass $m_{H^\pm} = 400\, GeV$, and we use the input values given in Table (1).

Table 1. The values of the input parameters used in the numerical calculations.

Parameter	Value
m_c	1.4 (GeV)
m_b	4.8 (GeV)
λ_t	0.04
m_t	175 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
Λ_{QCD}	0.214 (GeV)
$\alpha_s(m_Z)$	0.117
c	0.1

In Fig. 1 we plot $\frac{d\,\Gamma}{d\,E_s}$ with respect to the s quark energy E_s, for $\bar{\xi}^{D}_{N,bb} = 40\, m_b$, and $|r_{tb}| = |\frac{\bar{\xi}^{U}_{N,tt}}{\bar{\xi}^{D}_{N,bb}}| < 1$. $\frac{d\,\Gamma}{d\,E_s}$ is restricted in the region bounded by dotted (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). Solid line represents the SM contribution. There is a considerable enhancement in the differential decay width especially for $C_7^{eff} > 0$ case. Besides, the allowed region becomes larger for $C_7^{eff} < 0$.

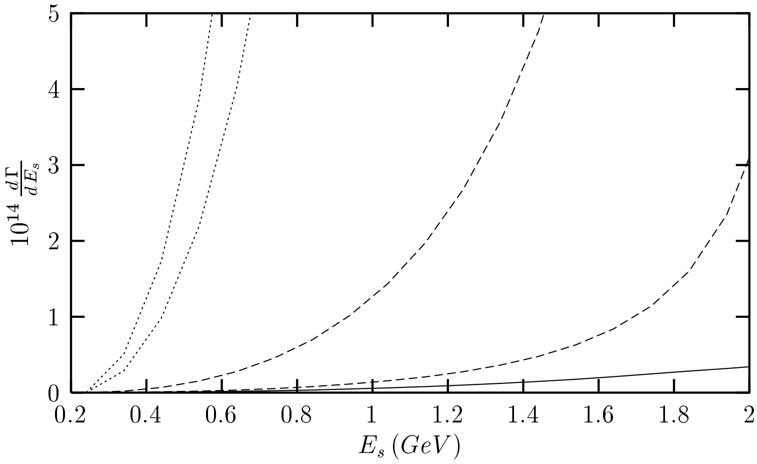

Figure 1. $\frac{d\,\Gamma}{d\,E_s}$ a function of E_s for fixed $\bar{\xi}^{D}_{N,bb} = 40\, m_b$ and $|r_{tb}| = |\frac{\bar{\xi}^{U}_{N,tt}}{\bar{\xi}^{D}_{N,bb}}| < 1$. Here $\frac{d\,\Gamma}{d\,E_s}$ is restricted in the region bounded by dotted (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). Solid line represents the SM contribution.

Fig. 2 is devoted to the E_s dependence of color antisymmetric and symmetric part of $\frac{d\,\Gamma}{d\,E_s}$. The color antisymmetric part lies in the region bounded by dotted (small dashed) lines and the color symmetric part by dashed (solid) lines, for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). This figure shows that, for $C_7^{eff} > 0$, the contribution of the color antisymmetric part is greater than that of color symmetric one. This is true also for $C_7^{eff} < 0$ case. However, the contribution of the color symmetric part for $C_7^{eff} > 0$ exceeds that of the color antisymmetric one for $C_7^{eff} < 0$. The allowed region becomes narrower for $C_7^{eff} > 0$ (see dotted and dashed lines). Note that the contributions due to the SM is presented by the dot-dashed and 3-dotted lines which almost coincide with the x-axis.

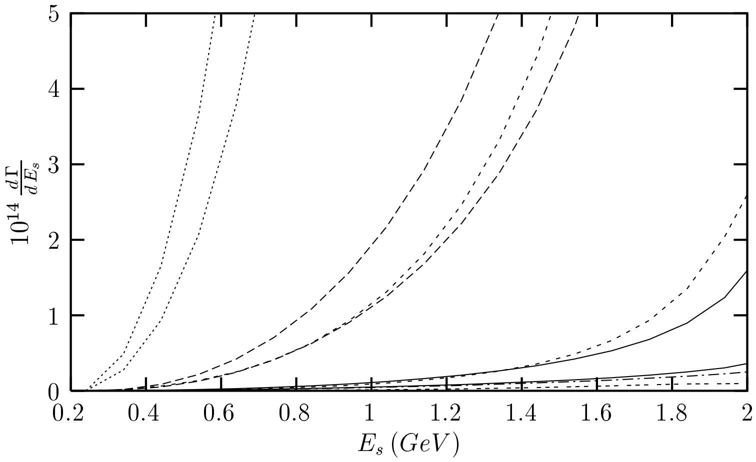

Figure 2. The color antisymmetric and symmetric part of $\frac{d\,\Gamma}{d\,E_s}$ as a function of E_s for fixed $\bar{\xi}^{D}_{N,bb} = 40\, m_b$, $m_{H^\pm} = 400\, GeV$ and $|r_{tb}| < 1$. Here the color antisymmetric part lies in the region bounded by dotted (small dashed) lines and the color symmetric part by dashed (solid) lines, for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). The antisymmetric (symmetric) SM contribution is represented by dot-dashed (3-dashed) lines.

Fig. 3 shows the E_s dependence of right, left and left-right mixed parts of $\frac{d\,\Gamma}{d\,E_s}$ in the SM. Solid line represents right, dashed line left and dotted line left-right contributions. The left one exceeds the right one up to almost $E_s = 2\,GeV$ since the $k^2_{external}/m^2_{light}$ contribution, responsible for the left part, is comparable with the heavy internal quark, namely m_t, contribution. Left-right mixed part is very small and has also negative values. For the model III we have no additional contribution to the left part beyond the SM in our approximation.

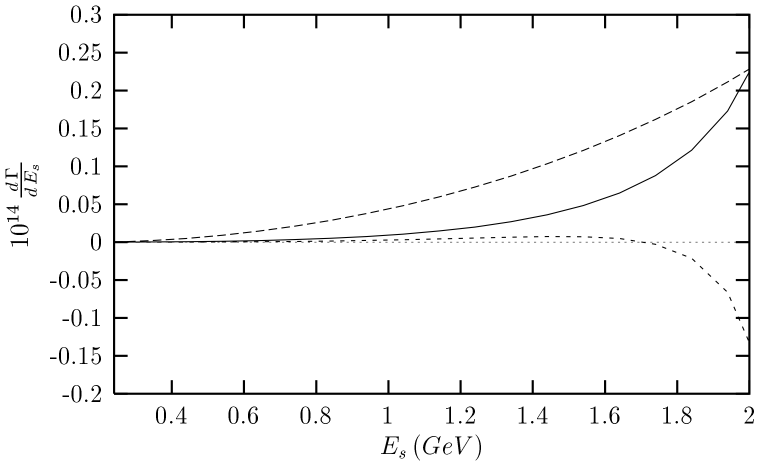

Figure 3. Right, left and left-right mixed parts of $\frac{d\,\Gamma}{d\,E_s}$ a function of E_s for fixed $\bar{\xi}^D_{N,bb} = 40\,m_b$, $m_{H^\pm} = 400\,GeV$ and $|r_{tb}| < 1$. Here solid line represents right, dashed line left and dotted line left-right contributions.

In Fig. 4, we present the $\bar{\xi}^D_{N,bb}$ dependence of $\frac{d^2\,\Gamma}{d\,E_1\,d\,E_s}$ for fixed values of $E_1 = 2\,GeV$ and $E_s = 1\,GeV$. It is seen that there is almost no dependence on the parameter $\bar{\xi}^D_{N,bb}$ especially for its large values.

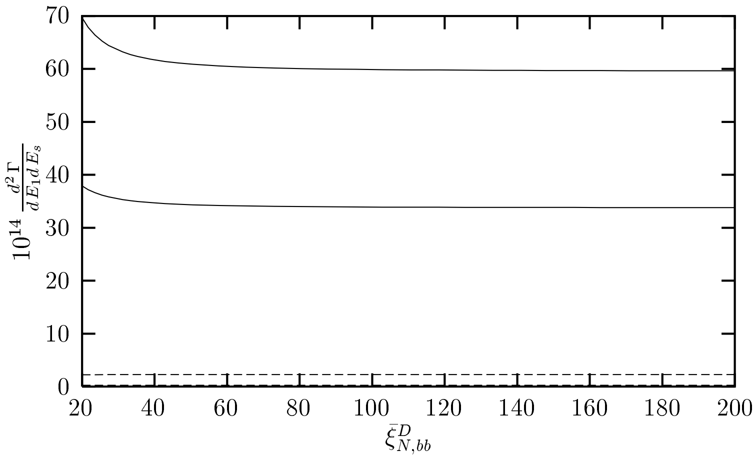

Figure 4. $\frac{d^2\,\Gamma}{d\,E_1\,d\,E_s}$ as a function of $\bar{\xi}^D_{N,bb}$ for $m_{H^\pm} = 400\,GeV$, $|r_{tb}| < 1$, $E_1 = 2\,GeV$ and $E_s = 1\,GeV$. Here $\frac{d^2\,\Gamma}{d\,E_1\,d\,E_s}$ lies in the region bounded by solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). Here the SM contribution almost coincides with the horizontal axis.

For completeness, we also present $m_{H^\pm}$ dependence of $\frac{d^2\,\Gamma}{d\,E_1\,d\,E_s}$ for fixed values of $\bar{\xi}^D_{N,bb} = 40\,m_b$, $E_1 = 2\,GeV$ and $E_s = 1\,GeV$, for $C_7^{eff} < 0$ (Fig. 5). Here the restricted region is bounded by solid lines. This figure shows that there may be a strong dependence on the mass $m_{H^\pm}$.

Now we would like to give some numerical results for our calculations. The total decay width for $b \to sX$

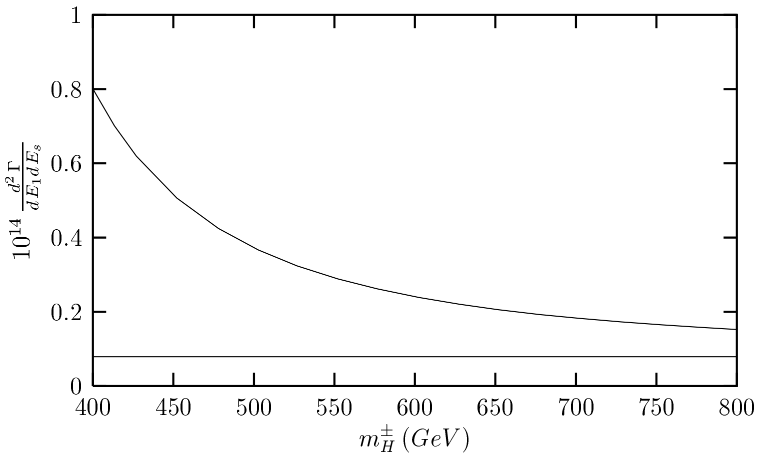

Figure 5. The same as Fig 4, but $\frac{d^2\,\Gamma}{d\,E_1\,d\,E_s}$ as a function of $m_{H^\pm}$, for $C_7^{eff} < 0$ and $\bar{\xi}^D_{N,bb} = 40\, m_b$.

transition is

$$\Gamma_{tot} = (r\,|V_{ub}|^2 \,+ s\,|V_{cb}|^2)\Gamma_0 \;, \tag{25}$$

where $\Gamma_0 = \frac{m_b^5\, G_F^2}{192\pi^3}$ and r, s are QCD sensitive parameters [26]

$$\begin{aligned} & 6.46 \leq r \leq 7.55 \;, \\ & 2.38 \leq s \leq 2.92 \end{aligned}$$

for $\alpha_s = 0.2$ and the total decay width reads as $\Gamma_{tot} = 3.50 \pm\, 1.50\; 10^{-13}\, GeV$.

In our calculation, we obtain the decay width for the SM as $\Gamma_{SM} = 2.37\; 10^{-15}\, GeV$. For $m_{H^\pm} = 400\, GeV$ and $\bar{\xi}^D_{N,bb} = 40\, m_b$, the model III result is four (three) orders larger for $C_7^{eff} > 0$ ($C_7^{eff} < 0$) compared to the SM result. This is a strong enhancement contradict with the total decay width given above. This forces us to choose the sign of C_7^{eff} as negative ($C_7^{eff} < 0$) and also to take large values of charged Higgs mass, $m_{H^\pm}$. For $m_{H^\pm} = 400\, GeV$, $\bar{\xi}^D_{N,bb} = 40\, m_b$ and $C_7^{eff} < 0$ we get:

$$\begin{aligned} & 1.64\, 10^{-14}\, GeV \leq \Gamma \leq 1.43\, 10^{-13}\, GeV \;, \\ & 2.40\, 10^{-15}\, GeV \leq \Gamma^{Sym} \leq 1.30\, 10^{-14}\, GeV \;, \\ & 1.40\, 10^{-14}\, GeV \leq \Gamma^{ASym} \leq 1.30\, 10^{-13}\, GeV \;, \\ & 1.29\, 10^{-14}\, GeV \leq \Gamma^{R} \leq 1.48\, 10^{-13}\, GeV \;, \\ & 1.74\, 10^{-15}\, GeV \leq \Gamma^{L} \leq 1.74\, 10^{-15}\, GeV \;, \\ & 1.70\, 10^{-15}\, GeV \leq |\Gamma^{LR}| \leq 6.74\, 10^{-15}\, GeV \;. \end{aligned} \tag{26}$$

In conclusion, we get a considerable enhancement in the decay width of the process $b \to sgg$ in the model III compared to the SM case. The enhancement can be suppressed by choosing $C_7^{eff} < 0$ and increasing lower bound of charged Higgs mass, $m_{H^\pm}$. Further, the decay width of the process under consideration is not sensitive to the parameter $\bar{\xi}^D_{N,bb}$. Therefore, these observations can give important clues in the prediction of the sign of C_7^{eff} and the mass value of the charged Higgs boson $H^\pm$.

Appendix

A. The form factors in the SM for $b \to sg^*$ decay

Here we present the magnetic dipole form factor $F_2^{\mathsf{SM}}(x_t)$ and the additional form factors due to the non-vanishing $k^2_{external}/m^2_{light}$ terms. (for details see [18]). The vertex function for $b \to sg^*$ decay with on-shell quarks can be written as

$$\Gamma_\mu(p,p',q) = F_1(x_t)\,(q^2\gamma_\mu - q_\mu \not{q})\,L - F_2(x_t)\,i\,\sigma_{\mu\nu}\,q^\nu\,(m_b\,R + m_s\,L)\ , \tag{27}$$

where p, p' and q are four-momentum of b-quark, s-quark and gluon respectively. The magnetic dipole form factor $F_2^{\mathsf{SM}}(x_t)$ in the SM is

$$F_2^{\mathsf{SM}}(x_t) = \frac{-8 + 38\,x_t - 39\,x_t^2 + 14\,x_t^3 - 5\,x_t^4 + 18\,x_t^2\,ln\,x_t}{12\,(-1+x_t)^4}\ , \tag{28}$$

and $x_t = m_t^2/m_W^2$. The non-vanishing $k^2_{external}/m^2_{light}$ terms for light quarks bring new additional contributions, $\Delta\,F_1$, $\Delta\,i_2$, and $\Delta\,i_5$ (See [18] for details):

$$\begin{aligned}
\Delta F_1 &= -\frac{2}{9} - \frac{4}{3}\frac{Q_0(z)}{z} - \frac{2}{3}Q_0(z)\ ,\\
\Delta i_2 &= -\frac{5}{9} - 2\frac{Q_-(z)}{z} + \frac{8}{3}\frac{Q_0(z)}{z} - \frac{2}{3}Q_0(z)\ ,\\
\Delta i_5 &= -1 - 2\frac{Q_-(z)}{z}\ ,
\end{aligned} \tag{29}$$

where

$$\begin{aligned}
Q_0(z) &= -2 - (u_+ - u_-)(ln\frac{u_-}{u_+} + i\pi)\ ,\\
Q_-(z) &= \frac{1}{2}(ln\frac{u_-}{u_+} + i\pi)^2\ ,
\end{aligned} \tag{30}$$

with

$$u_\pm = \frac{1}{2}(1 \pm \sqrt{1 - \frac{4}{z}})\ , \tag{31}$$

and

$$z = \frac{q^2}{m_i^2},\ \ i = u, c\ . \tag{32}$$

Finally, the contributions due to the non-vanishing $k^2_{external}/m^2_{light}$ terms are

$$\begin{aligned}
T^{\mathsf{light}}_{2\,\mu\nu} &= -\lambda_t\,\{(\Delta\,i_2 - \Delta\,F_1)(\not{k}_1 - \not{k}_2)\,g_{\mu\nu}\,L + \Delta\,i_5\,i\,\epsilon_{\alpha\mu\nu\beta}\gamma^\beta(k_1^\alpha - k_2^\alpha)\,L\\
&- 2\Delta\,F_1\,(\gamma_\nu\,k_{2\,\mu} - \gamma_\mu\,k_{1\,\nu})\,L\}
\end{aligned} \tag{33}$$

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