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Spin Polarized Cold and Hot Dense Neutron Matter

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Abstract

The Thomas-Fermi (TF) model is used to calculate the equation of state of thermal polarized neutron matter. The properties of cold and hot neutron matter have been investigated. A realistic density-dependent M3Y effective nucleon-nucleon (NN) interaction where the Yukawa strengthes are based on the G-matrix of the Paris interaction has been used. Special attention is devoted to the effect of the spin excess parameter on these properties. Moreover, the effect of some realistic density dependence has been studied. The results obtained are in reasonable agreement with previous theoretical estimates using different methods of calculation.

Key Words: Neutron Matter, Equation of State, M3Y effective interaction

1. Introduction

One of the fundamental goals of theoretical nuclear physics is to explain the properties of nuclear matter. Another important topic is the properties of dense neutron matter which are useful in the study of supernova and neutron stars [1-3]. In the center of a supernova at the point where neutron star is formed, the temperature is on the order of $T \cong 10$ MeV. Also, among the high energy heavy ion (HI) collisions the temperature can be greater than 50 MeV [4]. This makes the calculation of thermal and dynamical properties of neutron matter, based on realistic NN interactions, a matter of interest for the discussion of HI and super heavy nuclei as well as for the calculation of equilibrium state of neutron stars. It is also of interest to study the magnetic properties of neutron matter. A quantity which is important in studying these properties is the magnetic susceptibility.

There are different theoretical models used to study the neutron matter, namely Brueckner's theory [5-6], variational methods [7], relativistic mean field theory [8-10] and the Thomas-Fermi (TF) model.

There are many neutron matter equations of state (EOS) derived from realistic interactions. Recently, Chabanat et al. [11] used the neutron EOS obtained variationally by Wiringa et al. [12] with a Skyrme parametrization from subnuclear to neutron star densities. Uma et al. [13], using Seyler-Blanchard potential investigated the ferromagnetic phase transition by calculating the energy per particle for neutron matter, minimising it with respect to spin polarization and compared this with that calculated for unpolarized neutron matter. Zuo et al. [14] applied the Bruckner Hartree Fock approximation extended to include ground state correlation, in the study of the single particle properties of neutron matter such as the effective mass.

In the present work, we generalize the density-dependent M3Y effective Paris interaction to include the spin component for the study of spin polarized cold and hot neutron matter. Actually, the purpose of this paper is two fold. First, we study the properties of spin polarized neutron matter such as energy per particle, pressure, effective mass and magnetic susceptibility applying the TF model in the T^2 approximation. Second, we investigate the effect of different types of the density dependence on these properties.

2. Theory and Model

In the present work we take an extended density dependent effective interaction based on the original M3Y Paris interaction with explicit form of the spin dependent component.

The direct and exchange parts of the central NN forces for neutron matter has the following general form:

$$V^{D(EX)}(r) = V_{\circ}^{D(EX)} + V_{\sigma}^{D(EX)}(r)\sigma_1 \cdot \sigma_2, \tag{1}$$

where

$$V_{\circ}^{D}(r) = 11061.625\frac{e^{-4r}}{4r} - 2537.5\frac{e^{-2.5r}}{2.5r}. \tag{2a}$$

$$V_{\circ}^{EX}(r) = -1524.25\frac{e^{-4r}}{4r} - 518.75\frac{e^{-2.5r}}{2.5r} - 7.8474\frac{e^{-0.7072r}}{0.7072r}; \text{ and} \tag{2b}$$

$$V_{\sigma}^{D}(r) = 938.875\frac{e^{-4r}}{4r} - 36\frac{e^{-2.5r}}{2.5r}; \tag{2c}$$

$$V_{\sigma}^{EX}(r) = -3492.75\frac{e^{-4r}}{4r} + 795.25\frac{e^{-2.5r}}{2.5r} + 2.615\frac{e^{-0.7072r}}{0.7072r}; \tag{2d}$$

These terms are obtained from the singlet and triplet even and odd components of the M3Y Paris nucleon force [15].

Recognizing the fact that the M3Y interaction is not adequate to reproduce the saturation properties of symmetric cold nuclear matter, realistic density dependence (types such as DDM3Y and BDM3Y in the form) have been introduced into the interaction [16] vs2mm

$$V^{D(EX)}(\rho, r) = f(\rho)V^{D(EX)}(r), \tag{3}$$

vs2mm
where

$$f(\rho) = \begin{cases} C(1 + \alpha\exp(-\beta\rho)) & \text{DDM3Y type} \\ C(1 - \alpha\rho^{\beta}) & \text{BDM3Y type} \end{cases} \tag{4}$$

The parameters C, α and β are chosen such as to reproduce the saturation of cold symmetric nuclear matter at density ρ_0 =0.17 fm^{-3}and a binding energy of 16 MeV.

For polarized neutron matter one defines the neutron excess parameter as:

$$X = \frac{\rho_{N\uparrow} - \rho_{N\downarrow}}{\rho}, \tag{5}$$

where

$$\rho = \rho_{N\uparrow} + \rho_{N\downarrow}, \tag{6}$$

and

$$N = N\uparrow + N\downarrow, \tag{7}$$

In this case there are two Fermi momenta: $k_n(N\uparrow)$ and $\lambda_n(N\downarrow)$. These momenta are given by

$$k_n^3 = k_f^3(1+X), \tag{8}$$

$$\lambda_n^3 = k_f^3(1-X), \tag{9}$$

where k_f is the Fermi momentum of unpolarized neutron matter and is taken to be equal to 1.36 fm^{-1}.

The energy per neutron for polarized neutron matter can be written as:

$$E = E_V + \frac{1}{2}X^2 E_X. \tag{10}$$

$$E_V = \frac{3\hbar^2 k_f^2}{10m} + \frac{1}{2}\rho\, f(\rho)\left[J_\circ^D + \int \hat{j}_1^2(\eta)\, V_\circ^{EX}(r)\vec{dr}\right] \tag{11}$$

Terms higher than quadratic in X are neglected in Equation (11), and E_V and E_X are given by

$$E_X = \frac{\hbar^2 k_f^2}{3m} + \rho\, f(\rho)\left[J_\sigma^D + \int j_1^2(\eta)\, V_\circ^{EX} dr + \int j_\circ^2(\eta)\, V_\sigma^{EX}\vec{dr}\right] \tag{12}$$

where

$$J_i^D = \int V_i^D(r)\vec{dr}\,; \qquad i = \circ, \sigma \tag{13}$$

and

$$\hat{j}_1(\eta) = \frac{3j_1(\eta)}{\eta}\,; \qquad \eta = k_f r, \tag{14}$$

$j_n\ (\eta)$ being the n^{th} order spherical Bessel function and m is the nucleon mass.

The pressure at zero temperature ($P = \rho^2 \partial \mathrm{E}/\partial\rho$) is given by:

$$P = P_V + \frac{1}{2}X^2 P_X, \tag{15}$$

where

$$\begin{aligned} P_V \quad = \quad & \frac{\hbar^2 k_f^2 \rho}{5m} + \frac{1}{2}\Big\{\rho^2 a(\rho)\left[J_\circ^D + \int \hat{j}_1^2(\eta) V_\circ^{EX}\ \vec{dr}\right] \\ & + \rho^2 f(\rho)\int\left[\frac{2j_\circ(\eta)j_1(\eta)}{\eta} - \frac{4j_1(\eta)j_2(\eta)}{\eta} - \frac{6j_1^2(\eta)}{\eta^2}\right]\ V_\circ^{EX}\ \vec{dr}\Big\}; \end{aligned} \tag{16}$$

$$a(\rho) = \rho \frac{df(\rho)}{d\rho} + f(\rho) \tag{17}$$

and

$$\begin{aligned} P_X &= \frac{2\hbar^2 k_f^2}{9m} + \rho^2 a(\rho) \left[J_\sigma^D - \int j_1^2(\eta) V_\circ^{EX} \vec{dr} + \int j_\circ^2(\eta) V_\sigma^{EX} \vec{dr} \right] \\ &+ \frac{2}{9} \rho^2 f(\rho) \eta \left[\int (2 j_1(\eta) j_2(\eta) - j_\circ(\eta) j_1(\eta)) V_\circ^{EX} \vec{dr} - \int 3 j_\circ(\eta) j_1(\eta) V_\sigma^{EX} \vec{dr} \right]. \end{aligned} \tag{18}$$

The magnetic susceptibility in X^2 approximation (Equation (10)) is found to be [17]

$$\chi_i = \frac{\mu_n^2 \rho_n}{E_X}, \tag{19}$$

where μ_n is the neutron magnetic moment.

It is more convenient to consider the ratio of χ_i to the magnetic susceptibility of the Fermi gas χ_f of non interacting neutron [17]:

$$\chi_f = \frac{3\mu_n^2 \rho_n}{2\mu_\circ}. \tag{20}$$

$\mu_\circ$ is the Fermi energy of unpolarized neutron matter.

Thus one obtains

$$\frac{\chi_f}{\chi_i} = \frac{3E_X}{2\mu_\circ}. \tag{21}$$

To study the neutron matter at finite temperature one has to define the free energy which is given in terms of the density ρ and the temperature T:

$$F_i(\rho, T) = E_i(\rho, T) - T S_i(\rho, T) \qquad i = V, X, \tag{22}$$

S being the entropy of the system and E is the total energy.

According to the Fermi liquid approximation of Landau, the entropy can be calculated in terms of the Fermi integrals [18].

The entropy of the system can be calculated from

$$S(\rho, T) = S_V + \frac{1}{2} X^2 S_X, \tag{23}$$

where

$$S_V = \frac{\pi^2 m^*}{\hbar^2 k_f^2} k^2 T, \tag{24}$$

k being the Boltzman constant and is taken to be equal to unity:

$$S_X = -\frac{2}{9} \frac{\pi^2 m_x^*}{\hbar^2 k_f^2} T. \tag{25}$$

Here m^* is the effective mass and can be calculated from the single particle potential.

$$\frac{m^*}{m} = 1 + \frac{m}{3\hbar^2}\rho f(\rho)\int r^2 \hat{j}_1(\eta) V_\circ^{EX} \vec{dr} \tag{26}$$

and

$$\frac{m_X^*}{m} = 1 + \frac{m}{3\hbar^2}\rho f(\rho)\int r^2 j_\circ(\eta) V_\sigma^{EX} \vec{dr}. \tag{27}$$

The pressure can be obtained from

$$P_i^T = \rho^2 \frac{\partial F_i}{\partial \rho}. \tag{28}$$

3. Results and Discussion

The above described formalism has been applied to the study of cold and hot neutron matter. The calculations have been performed with explicit form of the spin dependent component of the NN Paris interaction. In the present analysis, the effect of the density dependent term, introduced into the M3Y Paris interaction, has been also considered. The values of the parameters C, α and β of *f(ρ)* are those given in Table 1 of reference (16) for the Paris interaction.

The volume energy is shown as a function of the density ρ in Fig. 1 which has the same shape as those obtained previously [19]. Comparison with the previous works of Arntsen et al. a [20], Owen et al. b [21], Friedman and Pandharipande [7], Lattimer and Ravenhall [22] and Mansour et al. [23], shows that our results are in good agreement with their calculations, particularly for the two types DDM3Y1 and BDM3Y0.

The pressure density isotherm at T=0, P_V is plotted in Fig. 2. The present results are compared with those of relativistic Hartree-Fock (HFI and HFII) of Weber and Weigel [24] and Friedman and Pandhanipande [7]. Again, the results obtained with all types of interaction have the same trend as those found previously.

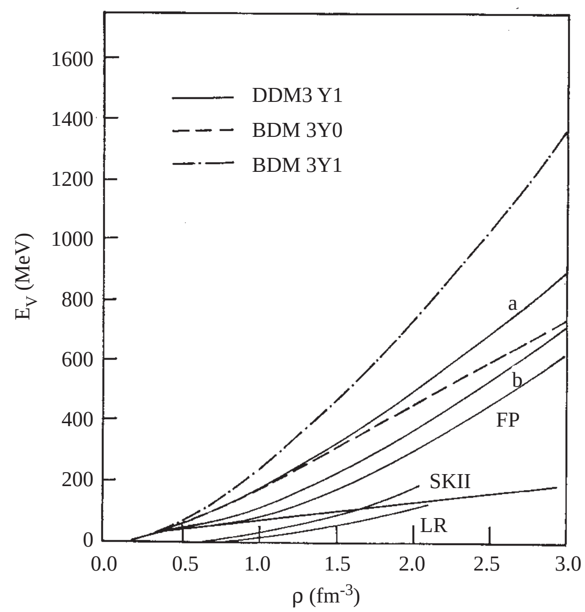

Figure 1. The energy per particle of neutron matter as a function of density using different density dependent M3Y interaction together with the results of previous calculations: a [20], b [21], FP [7],LR [22] and SKII [23].

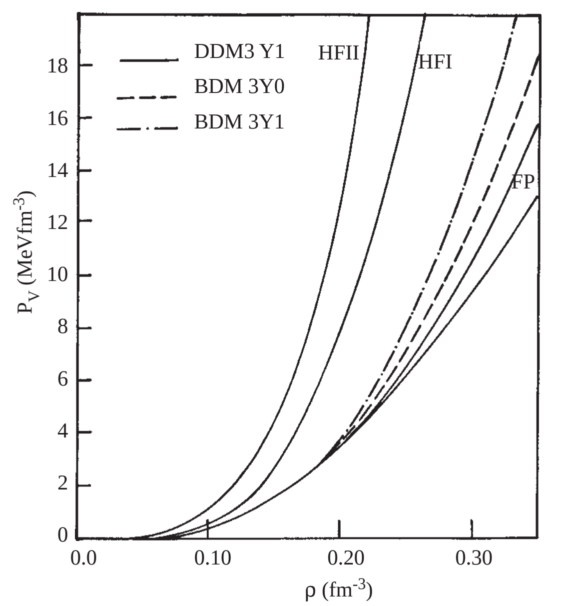

Figure 2. The pressure-density isotherm at T=0 of the volume term P_V, using different density dependent M3Y interaction, together with the results of previous calculations (HFI [24], HFII [24], FP [7]).

The symmetry energy E_X of cold neutron matter as a function of density is displayed in Fig. 3. This symmetry term does not differentiate between the different types of interaction. The present calculations show the same behavior as those obtained by Friedman and Pandharipande [7], who used the Urbana v_{14} potential plus TNI which denotes the sum of the effective repulsive three-body force called TNR and the phenomenological attractive three-body force (TNA) which are added by Friedman and Pandhanipande [7]. Also they used P1(Paris I) and P2(Paris II) in the same reference [7]. A plot of P_X as a function of the density ρ is presented in Fig. 4 corresponding to the three types of interaction. While P_x increases steadly with the density ρ for the DDM3Y1 type, it bends for large ρ ($\rho \geq 3$ fm^{-3}) for the BDM3Y1 type. This may be due to the fact that the exponential dependence on ρ does not change sign as ρ increases, while the situation is different for the power law dependence.

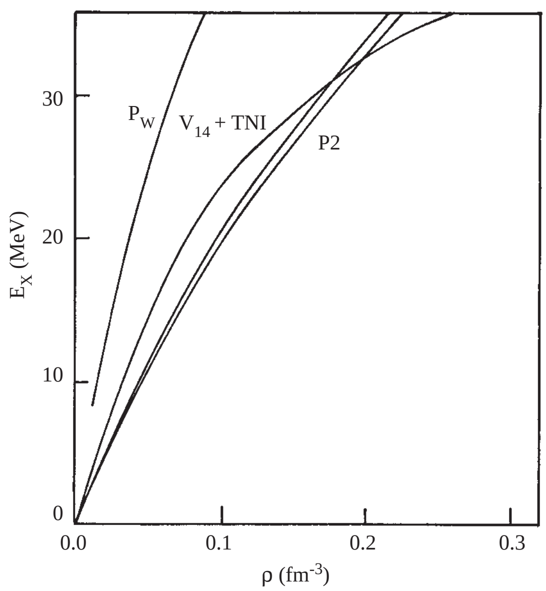

Figure 3. The spin-symmetry energy E_X as a function of the density (PW) together with the results of previous calculations (V_{14}+TNI, P_1, P_2)[7].

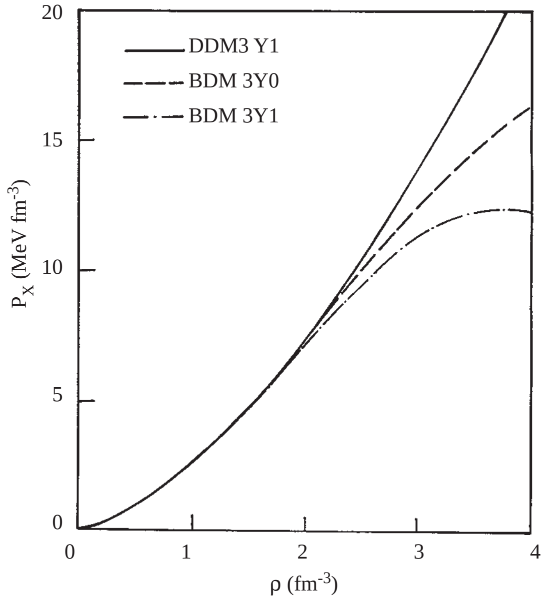

Figure 4. The spin symmetry pressure P_X as a function of the density using different density dependent M3Y interaction.

In Fig. 5 and Fig. 6 the ratio $\chi_f\ /\ \chi_i$ of the magnetic susceptibility of a Fermi gas of non-interacting neutrons to that of neutron matter is plotted as a function of $\rho/\rho_0$and k_f, respectively. The magnetic susceptiblity of a system characterizes the response of the system to a magnetic field. The condition $\chi_f\ /\ \chi_i$ =0 gives the onset of ferromagnetic transition. In general, the system prefers a ferromagnetic state for $\rho > 4\rho_0$. In previous calculations in the non-relativistic formalism, one does not find such a ferromagnetic transition [25]. However, in a relativistic framework, with an improved model [26] including mesons like ρ and π in addition to $\sigma + \omega$, one finds a transition at about $\rho \cong 3.5\ \rho_0$. But the incomperesssibility of the normal nuclear matter obtained in this model is too high (K ≅450 MeV). Fig. 5 shows that the DDM3Y type of interaction does not predict a ferromagnetic phase transition. However, one observes a ferromagnetic transition at $\rho \cong 6.8\ \rho_0$ and $\rho \cong 9.5\ \rho_0$ for the two types of interaction, namely BDM3Y1 and BDM3Y0, respectively [27]. Recently, Uma et al. [13], using a Seyler-Blanchard potential, found that increasing the incompressibility of normal nuclear matter will decrease the value of the density at which a ferromagnetic transition occurs. According to the present analysis, it was found that the incompressibility of normal nuclear matter is 270 MeV and 218 MeV for the two types of interaction, namely BDM3Y1 and BDM3Y0, respectively. This may partly explain the results obtained. Comparison with previous works is shown in Fig. 6 for $\chi_f\ /\ \chi_i$ as a function of k_f . The present analysis (PW) is in good agreement with what was found by Lattimer et al. [22] and SKII [23]. One also remarks that the three types of interaction give the same results. The results of the present calculations and other previous works for the effective mass are shown in Fig. 7. A steadly decrease of m^*/m with the increase of the density, for the DDM3Y1 and BDM3Y0 types, is observed while for the BDM3Y1 an increase of this ratio is obtained after passing by a minimum at $k_f \cong 2$ fm^{-3}.

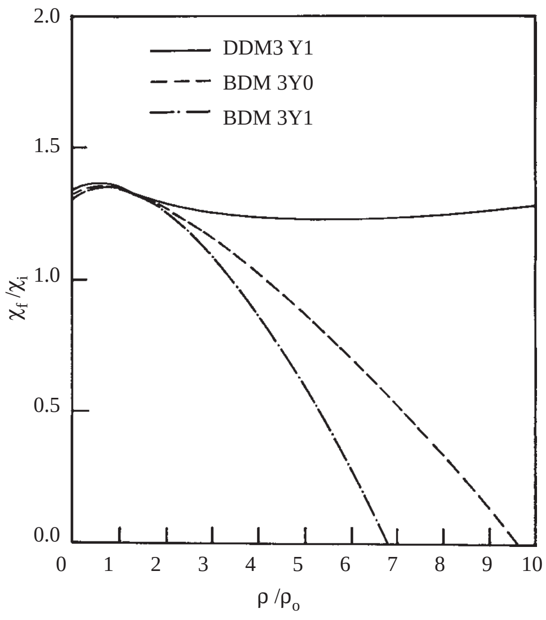

Figure 5. The ratio $\chi_f \ / \chi_i$ as a function of the relative density using different density dependent M3Y interaction.

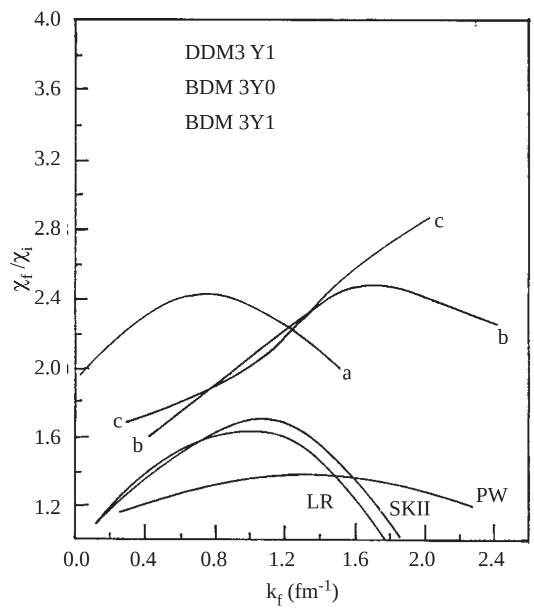

Figure 6. The ratio $\chi_f / \ \chi_i$as a function of the Fermi momentum k_f (PW) together with the results of the previous calculations (a [19], b [28], c [29], LR [22], SkII [23]).

For the study of hot neutron matter, the entropy is the key quantity. So the entropy S is plotted as a function of the density in Fig. 8. The present calculations reproduce the same feature as the corresponding ones obtained by Friedman et al. [7]. The free energy F_V, at temperature T=6 MeV and 20 MeV is presented in Fig. 9 as a function of $\rho \ /\rho_0$ (PW) . Comparison with previous works shows a good agreement. Figure 10 presents the dependence of the free energy F_V on the temperature T up to T=50 MeV. The pressure-density isotherms at T=6 MeV and T=20 MeV are plotted in Fig. 11 with comparison with the results of Friedman et al. [7]. The dependence of the pressure on the temperature is displayed in Fig. 12. The symmetry free energy F_X, as a function of ρ is plotted in Fig. 13 for different types of interaction at T=6 MeV. Figure 14 shows the dependence of this term on the temperature. While F_V steadly decreases with temperature, the behaviour of F_X is reversed as can be observed by comparison between Figs. 10 and 14. This may be attributed to the different sign of the entropy for these two terms. The symmetry term of the pressure P_X as a function of ρ is displayed in Fig. 15 for different types of the interaction, at T=6 MeV. While P_V steadly increases with temperature, P_X decreases as the temperature increases as can be observed when comparing between Figs. 12 and 16.

In summary, one has calculated the binding energy, pressure and several thermal and magnetic properties of pure polarized neutron matter, using TF model. The potential model employed is a generalization of the M3Y Paris interaction where realistic explicit spin dependence with density dependence has been introduced. In general it is gratifying to see that our results are quite similar to those obtained previously. However, one may hope to improve the results by considering other effects such as three-body forces and /or relativistic effects which are not taken into account in the present calculations.

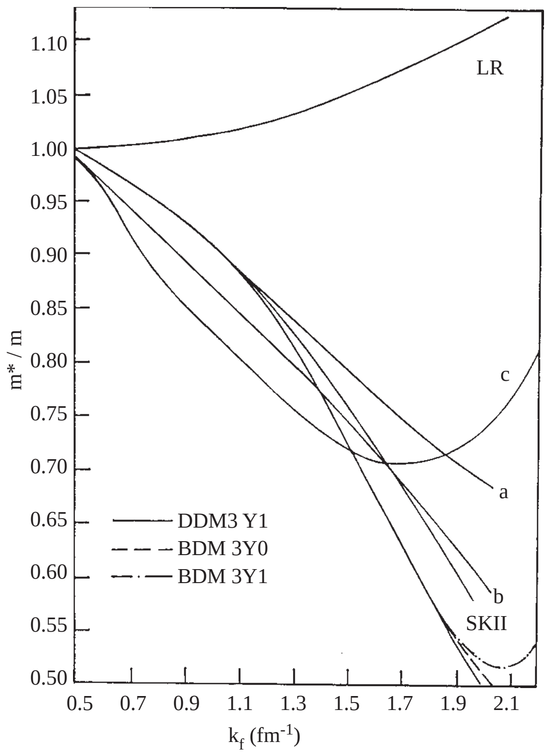

Figure 7. The effective mass m^*/m as a function of k_f using different density dependent M3Y interaction together with the results of the previous calculations (a [19], b [30], c [31], LR [22], SKII [23]).

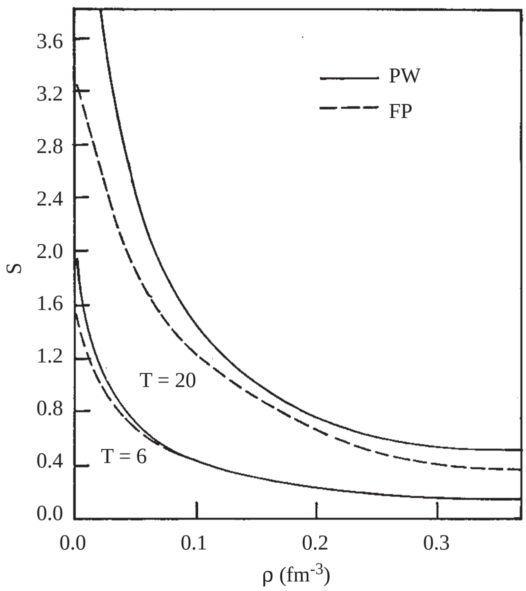

Figure 8. The entropy as a function of the density (PW) at T=6 and 20 MeV, together with the results of Friedman and Pandhanipande FP [7].

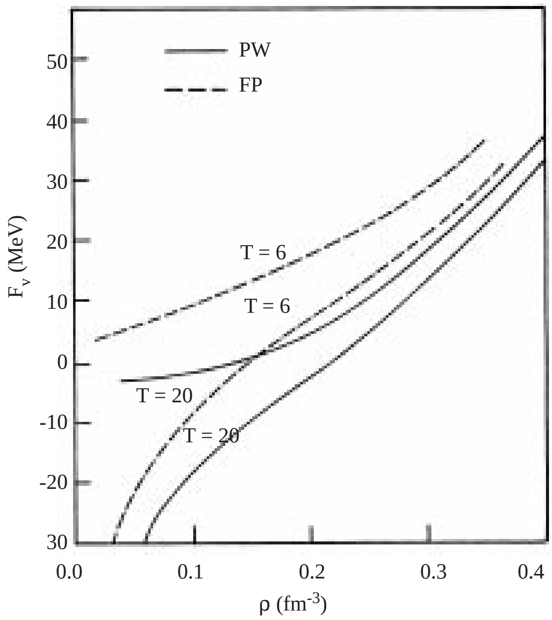

Figure 9. The free energy F_V as a function of the density (PW) at T=6 and 20 MeV together with the results of Friedman and Pandhanipande FP [7].

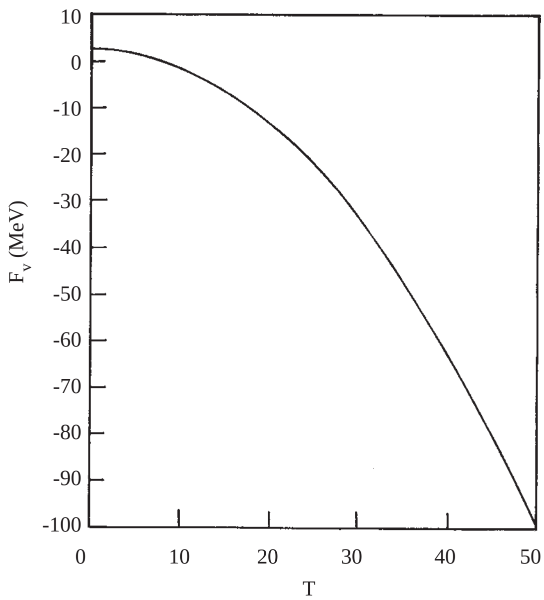

Figure 10. The free energy F_V as a function of the temperature.

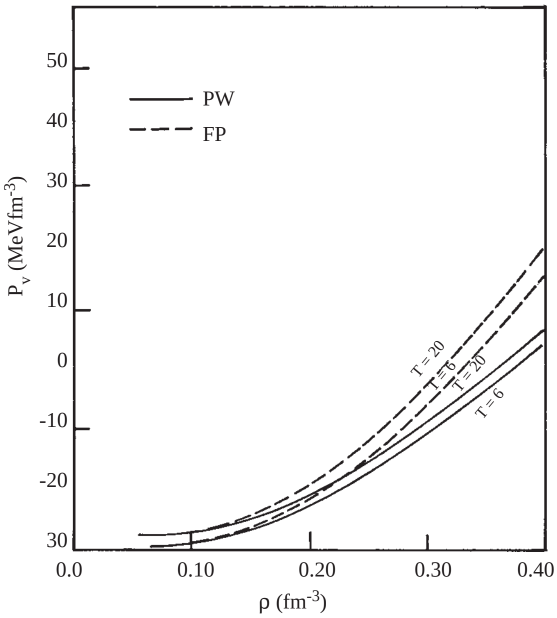

Figure 11. The pressure- density isotherms at T=6 and 20 MeV together with the results of Friedman and Pandhanipande FP [7].

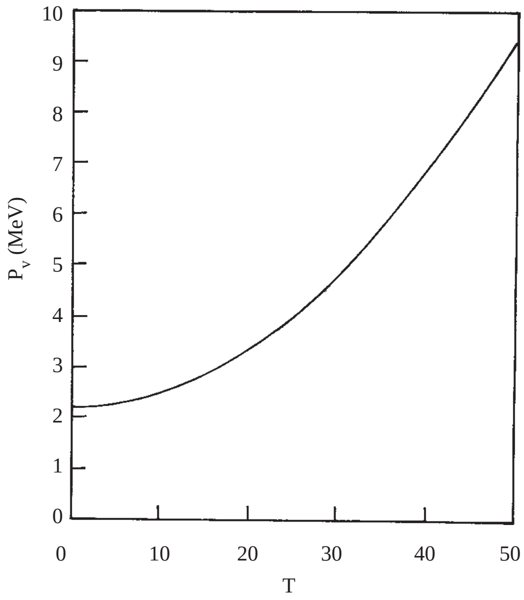

Figure 12. The pressure P_V as a function of the temperature.

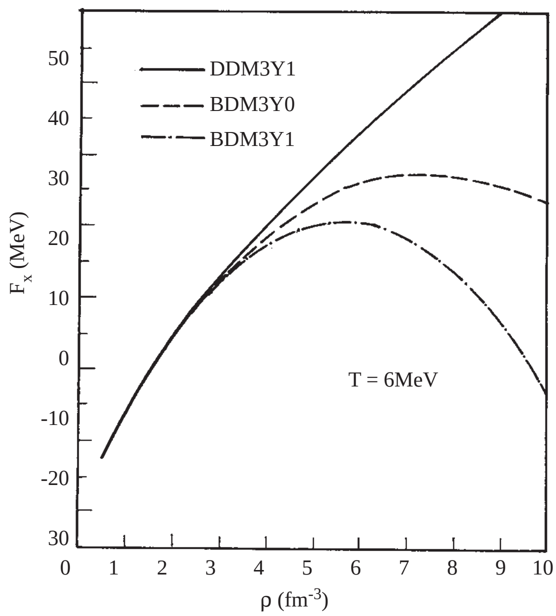

Figure 13. The spin-symmetry free energy as a function of the density, at T=6 MeV, using different density dependent M3Y interaction.

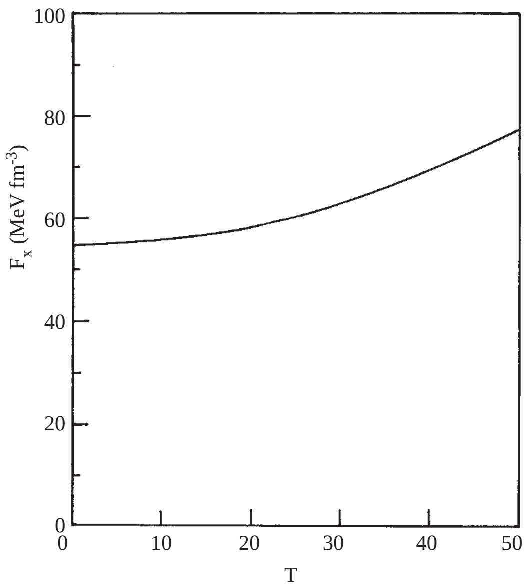

Figure 14. The spin-symmetry free energy as a function of the temperature.

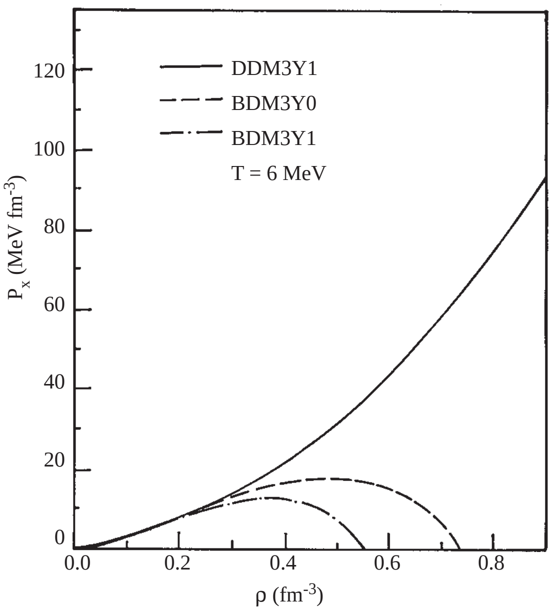

Figure 15. The spin-symmetry pressure as a function of the density, at T=6 MeV, using different density dependent M3Y interaction.

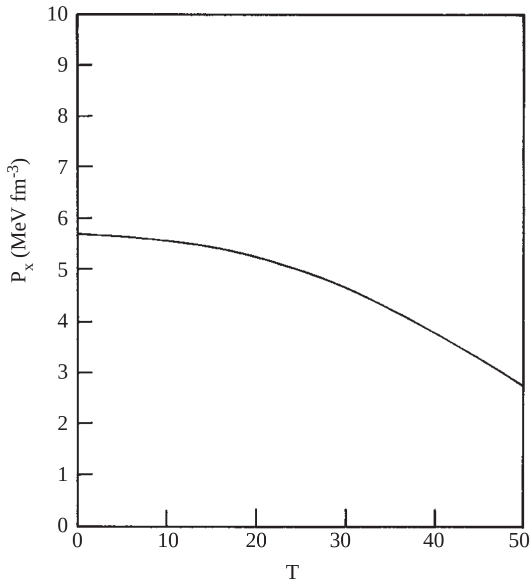

Figure 16. The spin-symmetry pressure as a function of the temperature.

References

[1] N.K. Glendenning; *Nucl. Phys.*, **A493**, (1989), 521.

[2] F. Weber and M.K. Weigel; *Nucl. Phys.,* **A493**, (1989), 549.

[3] S. Kahana; *Annu. Rev. Nucl. Part. Sci.,* **39**, (1989), 231.

[4] M. Abd-Alla, H.S. Ragab and M.Y.M. Hassan; *Egypt.J.Phys..* **25**, (1994), 1.

[5] J. Cugnon, P. Deneye, and A. Lejeune; *Z.Phys. A Atomic Nuclei,* **328**, (1987), 409.

[6] G.Q. Li, R. Machleidt and R. Brockmann; *Phys. Rev.*, **C45**, (1992), 2782.

[7] B. Friedman and V.R. Pandhanipande; *Nucl. Phys.*, **A361**, (1981), 502.

[8] J.D. Walecka; *Ann. of Phys.*, **83**, (1974), 49.

[9] B.D. Serot, J.D. Walecka; *Phys. Lett.*, **87B**, (1980), 2027.

[10] B.D. Serot, J.D. Walecka; "*Advances in Nuclear Physics*" **16**, (1986), 1.

[11] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, R. Schaeffer; *Nucl. Phys.,* **A627**, (1997), 710.

[12] R.B. Wiringa,V. Fiks and A. Fabrocini; *Phys. Rev.,* **C38**, (1986), 1010, R.B.Wiringa; *Rev. Mod. Phys.,* **65**, (1993), 231.

[13] V.S. Uma Maheswari, D.N. Basu, J.N. De, S.K. Samaddar; *Nucl. Phys.,* **A615**, (1997), 516.

[14] W. Zuo,G. Giansirocusa, U. Lombardo, N. Sandulesen, H.J.Schulze, *Phys. Lett.,* **B421**, (1998), 1.

[15] N. Anantaraman, H. Toki and G.F. Bertsch; *Nucl. Phys.*, **A398**, (1983), 269.

[16] Dao T. Khoa, W.von Oertzen and A.A.Oglobin; *Nucl. Phys.,* **A602**, (1996), 98.

[17] P. Haensel; *Phys. Rev.,* C11, (1975), 822.

[18] M. A. Abd-Alla, S. Ramadan and M.Y.M. Hassan; *Phys. Rev.,* **C36**, (1987), 1565.

[19] B. Behera and R.K. Satpathly; *J. Phys. G : Nucl. Phys.*, **5**, (1979), 1085.

[20] B. Arntsen and E. Ostgoard; *Phys. Rev.,* **C30**, (1984), 335.

[21] J.C. Owen, R.F. Bishop and J.M. Frivine; *Ann.Phys.,* **102**, (1976), 107, J.C. Owen; *Nucl. Phys.,* **A328**, (1979), 143.

[22] J.M. Lattimer and D.G. Ravenhall; *Astro. Phys. J.*, **223**, (1978), 314.

[23] H.M.M. Mansour, M.Y.M. Hassan and S. Ramadan; *Nucl. Sci. J.,* **25**, (1988), 1.

[24] F. Weber and M. K. Weigel; *Nucl. Phys.,* **A505**, (1989), 779.

[25] O. Forseth and E. Ostgaard; *Phys. Scripta,* **24**, (1981), 519.

[26] S. Marcos, R. Niewbro, M.L. Quelle and J. Navarro; *Phys. Lett,* **B271**, (1991), 271.

[27] S. A. Moharram to be published.

[28] S. A. Moszkowski; *Phys. Rev.,* **D9**, (1974), 1613.

[29] S. A. Moszkowiski; *Phys. Rev.,* **C2**, (1970), 402.

[30] J. Nitsch; *Z. Phys.,* **25**, (1972), 141.

[31] S. Ramadan, S.S. Montasser and M.Y.M. Hassan; *Acta Physica Polonica,* **B19**, (1988), 79.