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Radial and Nonradial Oscillations of 63 Her (HD155514 = HR6391)

Rikkat CİVELEK, Nilgün KIZILOĞLU, Halil KIRBIYIK
Physics Department, Middle East Technical University
06531, Ankara-TURKEY
e-mail: civelek@newton.physics.metu.edu.tr

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Abstract

An attempt has been made toward explaining the observed frequencies in 63 Her. A sequence of evolutionary models have been calculated up to a point where stellar parameters match the observed luminosity and effective temperature of 63 Her. Radial and nonradial oscillations frequencies were obtained for a series of masses 1.85, 1.90 and 1.95 $M_\odot$ and eigth models which represent best the pulsations of 63 Her are given in this paper. Calculations are restricted to low harmonic degrees (l= 0,1,2,3). Six of the eigth observed frequencies quoted in literature were obtained. These we obtained for the model of mass 1.90 $M_\odot$. The observationally measured frequency 220.7 μHz, which is classified as a second harmonic radial oscillation, and five of the observed nonradial modes were obtained at the harmonic degree l=3. The difference between the observed and calculated frequencies are ($\nu_{n/l,c}$ -ν_{obs})= 0.3, 0.8, 0.7, 0.2, and 0.7 μHz with n/l= -15/3, -12/3, -9/3, -7/3, -3/3, respectively.

Key Words: Stars: 63 Her, pulsations, evolution

1. Introduction

The subject of this study is a δ Scuti star. It may therefore be a good idea to give a brief summary about the basic properties of these stars. δ Scuti stars are main sequence or sligthly post main sequence stars. They have spectral types A5 to F5. Their masses are in the range from 1.5 to 2.5 $M_\odot$. They burn hydrogen either in a convective core or in a shell outside a H-depleted core.

Most of the δ Scuti stars are thought to be radial pulsators. The observed period ratios show this. However, Dziembowski [1] has shown that models of main sequence stars which oscillate in radial modes also are found to be unstable towards nonradial modes. As opposed to the sun, we can not resolve the disk for δ Scuti stars; but we can only detect by photometry the low degree modes, l= 0, 1, 2, 3 which have intensities or radial velocities that do not average out over the disk (Mangeney et al. [2]). It should be stated that mode identification is problematic for δ Scuti stars (Guzik [3]). It is due to the rather advanced evolutionary status of the star 63 Her, the low order, low degree modes are more or less of mixed type (Dappen et al. [4]).

Apart from others, 63 Her (HD155514=HR6391=V620 Her) is a promising example for asteroseismology. It has been observed as a variable star and found to oscillate at certain frequencies. Its spectral type is A8V and has the following parameters: T_{eff}= 7800±200 K and M_v= 1.7±0.25 (Hauck & Mermilliod 1980 [5]). It was first observed by Breger [6]. He found it to pulsate with a period P$\simeq$ 0.077 days (150 μHz) and an amplitude of about 25 mmag. It was mentioned that there were also indications of multiperiodicity. Elliot [6] observed the same star and found it to pulsate approximately with P$\simeq$ 0.079 days (146.5 μHz) having a similar amplitude as in Breger's case. Others followed these observations (Baglin et al. [8]; Reed & Welch (RW) [9]; Belmonte et al. [10]; Breger et al. [11]). Reed & Welch, by analyzing 1983 data find that 63 Her is pulsating at at least two and possibly three distinct frequencies. These frequencies correspond to periods 0.088 (131.5 μHz), 0.052 (221 μHz) and 0.130 (89 μHz) days. RW concludes that 0.088 days or 131.5 μHz correspond to radial pulsation in the fundamental mode. 0.052 days or 221 μHz is suggested to represent the second radial overtone. The third frequency they associated with a nonradial mode.

Belmonte et al. [10], applying two different methods, determined 7 frequencies for 63 Her in the range 46-301 μHz. The data of 1987 observations done in two different observatories are used. They are 46.3, 73.7, 89.3, 131.0, 149.8, 232.3 and 301.1 μHz, obtained by the method of "iterative sine wave fitting". However, the same frequencies, within the frequency resolution $\sim$ 0.6 μHz of the observations, were found when the "clean" method was applied. With this method they found 46.4, 56.1, 73.6, 89.5, 131.0, 147.7 and 232.3 μHz. They note that the three frequencies 89.3, 131.0, 149.8 μHz are in good agreement with previous observations by Breger [6] and RW [9]. They found the spectral peak occurring at 131 μHz which possibly suggests gravity modes have also been observed. Thus the frequencies 46.3, 73.7, and 89.3 μHz, quoted above, are nonradial gravity-like modes. By arguing from the period ratio point of view, it is emphasized that the two high frequencies 149.8 and 232.3 μHz are also nonradial modes.

Reed & Welch [9] indicated the possible presence of gravity modes corresponding to the smaller frequencies for 63 Her. RW compared their results obtained from 1983 data with Breger's [6] results and concluded the existence of some discrepancies. However, Breger et al. [11] argue that the 1969 data that they used are consistent with the frequency solutions now known from more extensive observations. Thus a further observational study was planned for 63 Her, and a multisite campaign of photoelectric photometry was carried out at three observatories on three continents.

By using 1990 data, obtained from the multisite observations, Breger et al. [11] measured three frequencies 131.0, 232.3 and 89.3 μHz. They also indicated that there may possibly be a fourth frequency 52.3 μHz, but it was discarded afterwards. In relation to 1983 data, on which RW calculations were based, Breger et al. [10] admit that it was the first of its kind for 63 Her to have extensive data suitable for analysis of the star's multiperiods. Two of the frequencies (131.0 and 89.3 μHz) are identical to the frequencies found in RW. They argued that the third frequency that they measured from the data of multisite campaign (232.3 μHz) should be preferred when compared with the frequency 221.0 μHz that RW found. They also reported the pulsation frequency of 149.8 μHz.

Mangeney et al. [2] gave a theoretical interpretation of the δ Scuti type pulsation of the star 63 Her. Several models representing 63 Her are computed and an attempt is made to identify the observed frequencies. For the theoretical calculations of observed effective temperature, absolute visual magnitude (Hauck & Mermilliod, [5]) is used. The chemical composition is solar. Oscillations are treated as adiabatic and linear. They have computed two evolutionary sequences for M= 1.80 $M_\odot$ and M= 1.85 $M_\odot$ with Cox-Stewart opacities [12] and solar composition. The best fit model for 63 Her has the following properties: M=1.85 $M_\odot$, L= 17.390 $L_\odot$, R= 2.336 $R_\odot$ and T_{eff}= 7674 K. This model gives 131.3 μHz for the radial fundamental mode. Some other frequencies for different n/l combinations also are calculated. They obtained several nonradial frequencies for low degrees which approximately agree with the observed ones. They note the difficulty in deciding which of the possible frequencies, 131.0 or 149.8 μHz, correspond to the radial fundamental mode. Although the first frequency has four times the observed amplitude of the second one, it is concluded that both choices are possible.

In this paper, by taking the parameters of the star 63 Her within the observational limits a sequence of models are calculated. Calculations are done for radial and nonradial oscillations. We aim at finding the best model that represents the observed 63 Her. It must be noted that rotational splitting is not considered in this work. We plan to include rotational effect in a seperate study. In Section 2 the properties of theoretical models are given. Radial and nonradial adiabatic oscillations are described in Section 3, and results and discussion are given in Section 4.

2. The Model

Evolutionary sequences of 1.85, 1.90 and 1.95 solar mass stars have been computed with initial solar composition of (X,Y,Z)= (0.70, 0.28, 0.02) and the mixing length parameter α= 1.75. Models were evolved up to the point where the luminosity and the effective temperature were almost satisfying the observed counterparts in 63 Her. Hauck & Mermilliod [5] reported that the absolute magnitude of 63 Her is 1.70±0.25 and effective temperature is 7800±200 K.

In our evolutionary code (based on Ezer & Cameron [13], Yıldız & Kızıloğlu [14],with modification) OPAL opacity tables (Iglesias et al., 1992) are used together with the
Alexander & Ferguson [16] opacities for low temperatures. Equation of state of MHD (Mihalas et al. [17] and references therein., Yıldızi [18]) is applied to the stellar models. The Caughlan & Fowler [19] nuclear reactions and the classical mixing theory are used. The other details are discussed in the study of Yıldız & Kızıloğlu [14]. Typically, 5000 mass zones were used to obtain the best possible radial and nonradial modes.

3. Radial and Nonradial Adiabatic Oscillations

The radial adiabatic oscillations are described by the application of classical linear theory (Ledoux & Walraven, [20]). The basic equations governing nonradial linear adiabatic oscillations are given in Cox [21]. Oscillations are treated both radially and nonradially for different models in Kırbıyık [22] and Al-Murad&Kırbıyık [23]. We adopt Equations (1)-(3) in Al-Murad and Kırbıyık's paper. The usual boundary conditions are used. These equations must satisfy the following boundary conditions (Cox, [21]): at r= 0 (center), δr has to remain finite, and at r= R (surface) (P$\prime$+δr dP/dr) has to remain finite. The primes denote the Eulerian perturbations and δ indicates the Lagrangian variation. The other symbols have their usual meanings. The equations are solved subject to the boundary conditions. The equations give rise to an eigen value problem with the dimensionless frequency $\omega^2= \sigma^2$ R^3/GM is where R is the surface radius; M, the total mass; G is the gravitational constant and σ is the frequency of oscillations. In solving the Equations the standard iteration technique is used. Oscillation program developed by Al-Murad [23] is used to obtain radial and nonradial frequencies.

4. Results and Discussion

More than thirty models for which the observed temperature and luminosity are within the specified error range radial and nonradial frequencies are calculated for the above indicated masses. Eight of them have been selected to present in this work for their close agreement of the calculated fundamental frequencies with the observed values. The results for radial oscillations are given in Table 1.

Table 1. The characteristics of equilibrium models and the pulsational quantities with the fundamental mode, the first harmonic, and the second harmonic of radial oscillations. ν_{nl} is the frequency for order n and harmonic degree l combinations. Other symbols have their usual meanings.

Model No	M ($M_\odot$)	Time (Gy)	L ($L_\odot$)	R ($R_\odot$)	T_e (K)	X_c (center)	ν_{00} (μHz)	ν_{10} (μHz)	ν_{20} (μHz)
1	1.85	0.714	15.13	2.12	7829	0.322	157.8	204.4	252.2
2	1.85	0.729	14.57	2.19	7615	0.310	**149.7**	193.5	239.5
3	1.90	0.663	16.66	2.24	7783	0.288	146.0	189.0	233.8
4	1.90	0.669	16.79	2.25	7791	0.284	145.5	188.5	233.2
5	**1.90**	**0.709**	**16.70**	**2.34**	**7636**	**0.251**	**137.6**	**178.1**	**220.7**
6	1.95	0.614	18.34	2.33	7839	0.294	141.3	184.3	**231.9**
7	1.95	0.656	17.77	2.34	7760	0.254	140.1	181.5	225.6
8	1.95	0.657	17.43	2.37	7666	0.253	137.0	177.4	**220.0**

The results for model numbers 2 and 5 are interesting to note. There are two frequencies which fit the observed frequencies and mode identifications. Therefore low order and low degree nonradial mode calculations also are performed. The calculated frequencies are given in Table 2 and Table 3 for models 2 and 5, respectively.

In Table 4 frequencies so far observed by different observers and the calculated frequencies in this work together with periods and Q values in days are given for an easy comparison.

In Table 1, model 2 gives the fundamental frequency 149.7 μHz which is in excellent agreement with the observations by Breger [6] and Belmonte et al. [10]. It must be noted that the parameters of the model such as luminosity and effective temperature are compatible with the observed stellar parameters of 63 Her, but within larger error ranges. The parameters of model 5, however, are in better agreement with the observed ones of 63 Her. Luminosity, radius and effective temperature are compatible with the observed counterparts which are quoted in Section 1. Thus the most promising model is taken as model 5 in Table 1. It is seen in the Table that we have obtained ν_{20}= 220.7 μHz which corresponds to the second harmonic which was observed by RW [9]. Their identification of the mode, related to this frequency, is exactly the same as we have obtained. However the frequency for the fundamental radial mode that we obtained is 6.1 μHz bigger than the persistently observed frequency of 131.5 μHz which has been found to have the largest amplitude thus interpreted as the fundamental frequency. It is interesting that the very same frequency is obtained in model 7 at n/l= -3/2 as a nonradial mode. For comparison, note the Q values computed (Petersen [26]; Andreasen et al. [26]) for the fundamental radial modes and the second harmonic: 0.031$\leq$ $Q_0$$\leq$ 0.034 and $Q_2$$\simeq$ 0.020. The Q value given by RW [9] for 131.0 μHz is 0.035 which seems to be a little bit outside the above range, but the uncertainity in Q value calculations is quoted as around 10%.

Table 2. The chosen nonradial frequencies for model number 2 at harmonic degrees l= 1, 2, 3 and for some selected orders. $\triangle\nu$= $(\nu_o - \nu_c)$ is the magnitude of the difference between the observed and calculated frequencies.

n	l	ν (μHz)	n	l	ν (μHz)	$\triangle\nu$ (μHz)	n	l	ν (μHz)	$\triangle\nu$ (μHz)
-6	1	52.2	-11	2	46.4	**0.1**	-16	3	46.0	0.3
-5	1	59.7	-10	2	50.9		-14	3	52.3	
-4	1	76.9	-9	2	57.7	**1.6**	-13	3	56.9	0.8
-3	1	107.6	-8	2	66.6		-12	3	62.0	
-2	1	140.8	-7	2	78.3		-10	3	79.6	
-1	1	154.7	-6	2	89.0	**0.3**	-8	3	91.3	
0	1	196.5	-5	2	99.3		-5	3	132.2	1.2
1	1	235.2	-4	2	123.3		-2	3	169.7	
2	1	271.2	-3	2	149.7	**0.1**	-1	3	192.4	
			-1	2	201.7		0	3	226.9	
			0	2	227.4		1	3	248.1	
			1	2	246.5		2	3	291.9	
			2	2	284.4					

It is worth noting that in Table 2 (model 2) we found for l=2 case four frequencies that agree well with observed ones. They are, in a sequence of n/l combination, 46.4 μHz (-11/2), 57.7 μHz (-9/2), 89.0 μHz (-6/2) and 149.7 μHz (-3/2). But it must be emphasized that the model parameters do not exactly agree with the observed quantities.

In Table 5, for model 5, the ratios of the fundamental frequency to the first and second harmonics are presented. For comparison some ratios in the observed frequencies also are given. Theoretical models of radially pulsating Population I stars predict (Petersen [25]) frequency ratios of $\nu_{00}/\ \nu_{10} \approx 0.76$, $\nu_{00}\ /\ \nu_{20} \approx 0.60$. Our results are not very different from these values. It is also interesting to note that there are some observed frequency ratios which are not far from our results.

Model number 5 has stellar parameters that fit best the observations of 63 Her. It has M= 1.90 $M_\odot$, L= 16.7 $L_\odot$, R= 2.34 $R_\odot$, T_{eff}= 7636 K with hydrogen fraction in the core X_H= 0.251. Some results in Table 3 deserves attention, especially frequencies at l= 3. Six of the eigth observed frequencies are obtained in this model star. They all are nonradial. As seen in the Table the differences between the observed and calculated frequencies all are less than 1.0 μHz. Only the most persistant frequencies 131.0 μHz, and 232.3

μHz are missing in this model. However it is worth indicating that it may be possible to obtain the observed frequency 131.0 μHz as the fundamental frequency but then we are out of range of the observed effective temperature. The above finding of frequencies as nonradial may be significant because Belmonte et al. [10], concerning 149.7 and 232.3 μHz, argue that their ratio to the fundamental frequency differs significantly from the usual quoted values for the frequency ratios in the radial pulsation modes which weakly depend on the mass and age of the star. Thus they conclude that these modes also are nonradial modes, which agree with our results. For the sake of information, it should be noted that 232.3 μHz frequency is found as a radial second harmonic in model 6 as 231.9 μHz.

Table 3. The nonradial frequencies for model number 5 at harmonic degrees l= 1, 2, 3 and for some selected orders. $\triangle\nu$= $(\nu_o - \nu_c)$ is the magnitude of the difference between the observed and calculated frequencies.

n	l	ν (μHz)	n	l	ν (μHz)	$\triangle\nu$ (μHz)	n	l	ν (μHz)	$\triangle\nu$ (μHz)
-5	1	55.3	-11	2	46.0	0.3	-15	3	**46.6**	0.3
-4	1	68.7	-9	2	54.1	2.0	-12	3	**56.9**	0.8
-3	1	78.9	-6	2	76.2	2.5	-9	3	**73.0**	0.7
-2	1	108.1	-5	2	91.7	2.4	-7	3	**89.5**	0.2
-1	1	141.2	-2	2	141.2		-3	3	**149.1**	0.7
0	1	179.4	0	2	190.9		0	3	**220.7**	0.0
1	1	211.3	1	2	223.9	3.2	1	3	241.9	
2	1	243.5	3	2	263.2		2	3	286.5	

Table 4. A summary of the observed and calculated frequencies for model 5 are presented. Frequencies are in μHz and the periods and Q values are in days. 'o' stands for observation and 'c' stands for calculated.

frequency	ν_o (B)	ν_o (RW)	ν_o (Belm.)	ν_o (Br et al.)	ν_c	n/l	P_c^d	Q^d
ν_1			46.3		46.6	-15/3	0.248	0.095
ν_2			56.1		56.9	-12/3	0.203	0.078
ν_3			73.7		73.0	-9/3	0.158	0.061
ν_4		89.4	89.3	89.3	89.5	-7/3	0.129	0.049
ν_5		131.5	131.0	131.0	131.5*	-3/2	0.088	0.034
ν_6	150.0		149.8		149.1	-3/3	0.077	0.029
ν_7		220.7			220.7	2/0; 0/3	0.052	0.020
ν_8			232.3	232.3	231.9**	2/0	0.050	0.019

B: Breger [5]; RW: Reed&Welch [8]; Belm: Belmonte et al. [9]; Br et al.: Breger et al. [10].
$*$: A result obtained in model 7.
$**$: A result obtained in model 6.

In terms of stellar parameters, among computed models, the model number 5 in Table 1 with 1.90 solar mass is found to best represent 63 Her. In this model, six nonradial frequencies and one radial frequency have been found to match with the observed frequencies.

Table 5. Frequency ratios in model 5 for radial oscillations are given in the first row and some ratios of the observed frequencies are also given for the sake of numerology. μ_{nl} values are given in Table 1. The ratios of the calculated frequencies are fundamental to the first and second harmonics, respectively.

	ν_{10}	ν_{20}
ν_{00}	0.772	0.623
Observed ratios	56.1/73.7= 0.761	56.1/89.3= 0.628

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