Turk J Phy
24 (2000) , 521 – 529.

The Decay of a Black Hole as a Quantum Dissipative Process

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Received 19.09.1997

Abstract

By considering the decay of a black hole as a result of the interaction of a quantum system with a dissipative vacuum, or in terms of string theory, the interaction of localized string modes with non-local string modes, we demonstrate how the single parameter representing dissipation can be calculated in terms of known fundamental constants. In view of the uncertainty embodied in all the present theories of black hole decay it is suggested that the approach of this note might provide fresh insight to the problem of black hole decay.

P.A.C.S.: 98.80-Dr Cosmology

1. Introduction

Classical black hole solutions occurs in general relativity and provide us with convincing evidence for the "cosmic" censorship principle which hides a singularity behind an event horizon [1]. The celebrated theorems of black hole physics essentially prevent us from thinking of the internal structure of a black hole in a classical sense and allow us to describe a black hole in terms of its mass, angular momentum and electric and magnetic charge [2, 3]. Actually, the recent work of Coleman et. al. [4], Krauss et. al. [5] have shown that classical black holes can carry "quantum hair" than can be probed by a "Aharanov Bohm" type effect which has also led to the belief that Planck black holes are truly elementary particles [6]. The stunning discovery of Hawking [7] that a black hole radiates a thermal spectrum of particles due to quantum field theoretic effects in a classical gravitational background has opened up a whole new field of inquiry in black hole physics. The celebrated relation originally proposed by Bekenstein [8] relating the black hole entropy to its area and the black hole temperature to its surface gravity has encouraged both students of string theory and quantum gravity to search for a deeper microscopic origin to the Bekenstein formula for black hole entropy. In this direction

Strominger and Vafa [9] have derived the Bekenstein-Hawking entropy by counting the microstates in string theory, and Das and Mather [10] along with Mukohyama [11] have also derived the Bekenstein-Hawking formula by counting D brane states for the corresponding black hole geometry. The same authors have also shown that the decay rate of a configuration of D branes by the emission of low energy quanta is equivalent to the corresponding decay rate according to the Hawking emission process.

One of the central problems with the Hawking process is that it does not allow for the quantization of the gravitational degrees of freedom of a black hole. In this regard Kastrup [12] has discussed the quantization of the surface area of a black hole in terms of Bohr-Sommerfeld quantization and has arrived at a formula of a black hole mass in terms of the quantum number n according to the formula $E\alpha\sqrt{n}$. Barreira et. al. [13] have argued that this should lead to a discrete emission spectrum (non-thermal) for a black hole as opposed to a continuous thermal spectrum; but if the quantum states are sufficiently close, the continuous spectrum emerges. The discreteness would most likey apply to low quantum transitions. Along with these pioneering works in Ref. [9, 10, 11] Nanopoulos has expounded on the universality of the Procrustean principle wherein all quantum systems must be considered open systems due to the interaction of truncated delocalized string modes with the local model (particles) [14]. This suggests that the decay of a black hole can be viewed as the dissipative evolution of the black-hole geometry (internal string states) interacting with the environment (non-local modes). Raine and Sciama [15] have also discussed that black hole decay can be viewed as the dissipative quantum evolution of the black hole interacting with a dissipative vacuum and the evaporation process is associated with the infalling negative Casimir energy in much the same way that the Lamb shift is represented by a change in the energy of the vacuum due to the presence of the atom.

In the following note, motivated first by the notions of string theory and the Procrustean principle and secondly by the inspiration of Raine and Sciama (Ref. [15]) we discuss the quantum dissipation of a black hole to be represented by a retarded Schrodinger evolution. The original papers of Caldirola and Montaldi [16] and Caldirola [17] discussed a retarded Schrodinger evolution with the result that all excited states decay. In this paper we interpret this mechanism of decay to be either the result of the string interaction (Ref. [14]) or the non-pertubative interaction of the black hole with the vacuum. By considering the black hole state to be represented by the eigenstates derived by Kastrup (Ref. [12]) we show how the "discrete time parameter", being the result of all environmental interactions of the black hole with the environment, can be calculated in terms of the fundamental constants we already know. Original motivations for introducing a discrete time parameter stem from studies of the Schrodinger Langevin equation [18] along with studies of the interaction of a harmonic oscillator with a string in the quantum domain where certain features of the quantum motion can be described by a few parameters after the modes of the string (reservoir) are eliminated [19, 20, 21]. Here, one of the parameters is the discrete time interval. Also in the study of the fission of superheavy nuclei into daughter nuclei there exists a transient time that, in a certain sense, can be viewed as a discrete time parameter in the spirit of the Caldirola dissipative

Schrodinger Equation [22]. Whenever a quantum system with a discrete set of quantum states interacts with an environment with a continuous spectrum dissipation will occur which can be described by a few parameters which in principle can be calculated from the temperature, composition and other properties of the environment. As stated above, one of the parameters is the discrete time interval. The analysis which follows enables us to think of black hole decay not as the result of perturbative quantum gravity but rather as a result of a universal interaction of the black hole with the environment.

2. Black Hole Decay as a Quantum Dissipative Process

We begin by writing the retarded dissipative Schrodinger equation discussed in Ref. [16]:

$$H\psi = i\hbar \left(\frac{\psi(t) - \psi(t-\tau)}{\tau} \right) \tag{2.1}$$

($\tau=$ discrete time parameter).
To solve Eq. (2.1) we write

$$H\psi = \frac{i\hbar}{\tau} \left(1 - e^{-\tau \frac{\partial}{\partial t}} \right) \psi$$

Setting

$$\psi(t) = e^{-\alpha t}\psi(0). \tag{2.2}$$

Eq. (2.2) is the result of assuming a discrete time difference for the wave function that is slanted backward in time. Such a backward discrete time difference will always lead to a decreasing exponential in time.
We now find

$$H(e^{-\alpha t}\psi(0)) = \frac{i\hbar}{\tau}(1 - e^{\tau\alpha})e^{-\alpha t}\psi(0),$$

thus

$$e^{\tau\alpha} = \left(1 + \frac{\tau i H}{\hbar} \right)$$

$$\alpha = \frac{1}{\tau} \ln_e \left(1 + \frac{\tau i H}{\hbar} \right) \tag{2.3}$$

In order to stabilize the ground state we modify Eq. (2.1) to read

$$(H - H_0)\psi(t) = i\hbar \left(\frac{\psi(t) - (\psi t - \tau)}{\tau} \right),$$

where $H_0=$ ground state eigenvalue $= \bar{E}_0$.

When this is done Eq. (2.3) becomes

$$\alpha = \frac{1}{\tau} \ln_e \left(1 + \frac{\tau i}{\hbar}(H - H_0)\right) \tag{2.4}$$

We now draw from the work of Kastrup (Ref. [12] where the surface of a black hole is quantized in the spirit of the Bohr Sommerfeld quantization rules, for which the result of his calculation give for the eigenstates

$$E_n = \frac{1}{4}\sqrt{\frac{\bar{\alpha}}{\pi}} m_p c^2 \sqrt{n} \tag{2.5}$$

$m_p =$ Planck mass, $\bar{\alpha} = 4 \ln_e 2$, and the degeneracy of each quantum state is g(n) $= 2^{n-1}$. (Here g(n) results by requiring each eigenstate to have the Bekenstein-Hawking value for the entropy.) The ground state energy would be (n=1)

$$\bar{E}_o = \frac{1}{4}\sqrt{\frac{\bar{\alpha}}{\pi}} m_p c^2.$$

If a quantum black hole is in a superposition of states, we have

$$\psi(0) = \sum_{n,i} C_n^i U_{n_i}(0),$$

where $U_{n_i}(0)$ is the eigenstate of energy E_n and i includes the degenerate states (that is, there are 2^{n-1} different values of C_n for each n). For the wave function at t we have from Eq. (2.2) and Eq. (2.4)

$$\psi(t) = e^{-\frac{t}{\tau}\ln_e\left(1+\frac{i\tau}{\hbar}(H-H_0)\right)} \Sigma C_n^i U_{n_i}(0). \tag{2.6}$$

We now expand the natural log in Eq. (2.6) to give

$$\ln_e \left(1 + \frac{\tau i}{\hbar}(H - H_0)\right) = \frac{\tau i}{\hbar}(H - H_0) + \frac{\tau^2}{2\hbar^2}(H - H_0)^2.$$

The above approximation will be valid provided the excitation energy of the black hole doe not exceed the limit $\tau\frac{(E_n - \bar{E}_o)}{\hbar} < 1$. If the discrete time interval is small enough the approximation will be valid.

Substituting in Eq. (2.6) gives

$$\psi(t) = \sum_{n,i} C_n^i U_{n_i}(0) e^{-\frac{i}{\hbar}(E_n - \bar{E}_0)t} e^{-\frac{\tau}{2\hbar^2}(E_n - \bar{E}_0)^2 t}.$$

We now assume C_n^i is the same for each degenerate state of n

$$C_n^i = C_n.$$

Because each principal quantum state has a degeneracy of g(n) there must be another internal quantum number in the Bohr-Sommerfield quantization of the black hole that discriminates between different internal states with the same n. The assumption $C_n^i = C_n$ is equivalent to equal probabilities of the internal states.
We write for the energy at time t (mass of black hole)

$$< E >= \psi^* H\psi = \sum_1^{N_0} |C_n|^2 g(n) E_n e^{-\frac{\tau}{\hbar^2}(E_n - \bar{E}_0)^2 t} \tag{2.7}$$

$(g(n))$ = digeneracy function, $E_1 = \bar{E}_0$).
Here we introduce the cut-off for n of N_0. Introducing a cut-off for n is equivalent to saying that there is some natural mechanism that forbids black holes of arbitrarily large size since the radius will be proportional to $n^{1/2}$. This is certainly true in an observational sense, since we don't observe them. We also normalize the initial state such that

$$\sum_1^{N_0} |C_n|^2 g(n) = 1.$$

Suppose for example n is large and $|C_n|^2 = \alpha n$. This relation implies that the higher the quantum number n, the greater the probability the black hole will be in that state. Such a situation is most likely to occur at low temparatures since M varies inversely as T. Then

$$\sum_1^{N_0} an2^{n-1} dn = \int_1^{N_0} an2^{n-1} dn = 1$$

giving

$$a\left[e^{N_0 \ln_e 2}\left(\frac{N_0}{2\ln_e 2} - \frac{1}{2(\ln_e 2)^2}\right) - e^{\ln_e 2}\left(\frac{1}{2\ln_e 2} - \frac{1}{(\ln_e 2)^2}\right)\right] = 1. \tag{2.8}$$

Thus Eq. (2.8) can be s olved for a.

From Eq. (2.7) we have for small t for the energy of the Black Hole

$$E(t) = MC^2 = \sum_1^{N_0} |C_n|^2 g(n) E_n - \sum_{n>1}^{N_0} \frac{|C_n|^2 g(n) E_n}{\hbar^2} \tau (E_n - \bar{E}_0)^2 t \tag{2.9}$$

calling

$$c^2 M_0 = \sum_1^{N_0} |C_n|^2 g(n) E_n,$$

and

$$c^2\beta = \sum_{n>1}^{N_0} |C_n|^2 g(n) E_n \frac{(E_n - \bar{E}_0)^2}{\hbar^2}\tau. \tag{2.10}$$

Then from Eq. (2.9) we have

$$M(t) = M_0 - \beta t. \tag{2.11}$$

Thus for small time the mass decays in a linear fashion.

To compare the results with the results based on the Hawking effect we note that primordial black holes can play a vital role in the reionization of the universe for $Z \leq 60$. Primordial black holes can form from "primordial density perturbations", cosmological phase transitions involving buble collisions and from the collapse of cosmic strings [23]. Their existence can be inferred from the high fraction of anti-protons in cosmic rays, the annihilation line radiation coming from the center of the galaxy as well as the spectrum of the positron background (Ref. [23]). When the Hawking process is applied to a black hole it turns out that the entire spectrum of elementary particles can be emitted from a black hole with higher mass black holes favoring light particle emissions, the equation for the emission rate is of the following form (Ref. [23]).

$$\frac{dM}{dt} = -\frac{k}{M^2} f(M) = -\frac{5.34x10^{25} f(M)}{M^2}, \tag{2.12}$$

where

$$f(M) = 1.569 + \Sigma a_i e^{-\frac{M}{C_i}},$$

(C.G.S.) and a_i, C_i refer to the different heavy particle types. The constant 1.567 refers to the emission of $e^-, e^+, \gamma, \nu_e, \nu_\mu, \nu_\tau$ (Ref. 18). Eq. (2.12) results after integrating over the entire thermal spectrum for each particle species. If we study the simple case of the emission of photons we have

$$\frac{dM}{dt} = -\frac{1}{c^2}(4\pi R^2)(\sigma T^4), \tag{2.13}$$

where $R = \frac{2GM}{c^2}$ = radius of black hole, σ= Stefan Boltzman constant $= \frac{2\pi^5 k^4}{15 h^3 c^2}, k =$ Boltzmann constant, $T = \frac{\hbar c^3}{8\pi G k M}$ (Bekenstein-Hawking formula for $T = T(M)$, Eq. (2.13) gives

$$\frac{dM}{dt} = -\frac{\gamma}{M^2}\left(\gamma = \frac{c^6 h \pi^2}{G^2(21120)}\right). \tag{2.14}$$

Integrating Eq. (2.14) gives

$$\frac{M^3}{3} - \frac{M_0^3}{3} = -\gamma t$$

or

$$M = M_0 \left(1 - \frac{3\gamma t}{M_0^3}\right)^{\frac{1}{3}}. \tag{2.15}$$

For small t

$$M = M_0 - \frac{\gamma}{M_0^2} t. \tag{2.16}$$

Comparing Eq. (2.9) and Eq. (2.16) we find

$$\frac{\gamma}{M_0^2} = \frac{1}{c^2} \frac{\sum_{n>1}^{N_0} |c_n|^2 g(n) E_n (E_n - E_0)^2 \tau}{\hbar^2}$$

or

$$\tau = \frac{c^2 \hbar^2 \gamma}{M_0^2 \sum_{n>1}^{N_0} |C_n|^2 g(n) E_n (E_n - E_0)^2} \tag{2.17}$$

where

$$M_0 = \frac{\sum_1^{N_0} |C_n|^2 g(n) E_n}{c^2}.$$

We note in Eq. (2.17) that we have calculated the "discrete time interval" in terms of the fundamental constants, c, h, G and the quantum states of the black hole E_n. If the states of a black hole were specified by the string or D-brane excitations then τ would be calculated in terms of c, h, G and the excited states of the string or D brane. We also note that Eq. (2.17) is just valid for small time intervals and if 1 excitation dominates, that is, very energetic compared to $n = 1$, we have

$$\tau \simeq \frac{c^2 \hbar^2 \gamma}{M_0^5 c^6} \simeq \frac{\hbar^2 \gamma^2}{M_0^5 c^4}.$$

If M_0 is initially in equilibrium with the environment then $M_0 \propto \frac{1}{T_0}$, and $\tau \propto T_0^5 (T_0 =$ absolute temperature of environment). This would lead us to conclude that the discrete time interval depends on the fifth power of the temperature of the environment. Actually, Marsh et. al. [24] have suggested that in a symmetric discrete lattice quantum theory the discrete time interval will vary inversely as the temperature; however, factors in the environment such as its composition could play a vital role in determining the dependence of τ on the temperature for a specific composition [25,26].

3. Conclusion

We have constructed a model for black hole decay in terms of a retarded version of the Schrodinger equation and have given a provisional way of determining the discrete time interval in the theory. Although our calculation of τ holds only for a short time it still demonstrates that environmental quantum dissipation can be described by the fundamental constants h, c, G, and the parameters of the system. Because much of the gravitational physics and quantum physics at the horizon is not understood our approach offers a fresh approach to the decay of quantum systems wherein all the uncertainty at the horizon can be lumped into dissipative environmental interactions specified by discrete time parameter τ. What is needed is a knowledge of the quantum state of the system which, in our case, was given to us by the work of Kastrup (Ref. [12]) through the method of Bohr Sommerfeld quantization. It's a curious fact that the continuum of space time, which normally plays the sterile role of the "arena", can act in a retarded manner to generate the decay of systems not understood using the conventional laws of physics. It once more lends evidence to the fact that space, time, and matter are inseparable and questions such as CP and T violation might only be understandable when we understand how matter is truly intertwined with geometry through perhaps string theory [27] or perhaps a theory of gravitation beyond the structure of general relativity.

Acknowledgments

I'd like to thank the physics departments at Williams College and Harvard University for the use of their facilities.

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