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Time Evolution of Fast Particles During Decay of Hadronic Systems

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Abstract

A phenomenological model is presented based on the formation of a nuclear thermodynamic system during the collision of intermediate and high energy heavy ions. The formulation and the dynamic picture are determined by solving the Vlasov equation. The solution is dressed in the form of a power series the first term of which being the equilibrium distribution in phase space. The rest, are time dependent perturbation terms due to the multiple strong interactions inside the system. The temperature gradient and the derivatives of the phase function are calculated. The time dependence of the angular emission of the produced particles is studied. It is found that particles emitted in the forward direction are produced in the early stage of the reaction, far from the equilibrium. Backward production comes in a later stage when the system constituents undergo multiple cascade collisions.

1. Introduction

Particle production in heavy ion collisions has been well represented by the fireball model [1] at medium energy range, where the concept of global equilibrium may be accepted. The fireball model was developed to fit experimental data at higher energies. A local equilibrium was assumed in the so called fire-streak model [2,3], which treats the variation across the overlap region of the target and projectile in the amount of energy and momentum that it deposited. The expression for calculating any observable takes the form of a sum over a series of terms, each one of which concerns a local equilibrium and consists of geometric, kinematic and statistical factors. As the energy further increases, it is expected that collision time becomes small enough so that particles emitted in the early stage of the reaction possess non-equilibrium characteristics. The density function in phase space should be treated on the time scale to follow up the evolution of the reaction.

Many trials have been done in this concern. The equation of motion can be reformulated to give it the appearance of a classical equation for the phase distribution function. In this approximation, a local one body potential can be defined and the phase distribution function may contain the same information as the one body density matrix. This is the Hartree-Fock approximation [4]. Many body physics enters only through the relation of the potential and the density. One more approximation that reduces the equation to a completely classical form is to make a power series expansion of the one body potential, giving the so called Vlasov equation [5-7]. A situation that can be analyzed with the Vlasov equation is the short time behaviour of the system subjected to an impulsive force. If the potential is sufficiently weak, the solution of the excited system may be treated by the quantum mechanical sum rules introduced first by Fallieros [8] and Noble [9]. While it is not possible to integrate the Vlasov equation in general, some insight may be given by expanding the solution for small intervals of time. The starting point is the equilibrium solution, which is perturbed by the impulsive potential. Another treatment of the Vlasov equation depends on the theory of small oscillations in a finite system [10]. A closed expression has the appearance of Rayleigh‘s variation principle with a certain explicit form for the potential energy function. The solution is represented in the form of a sum of an equilibrium function plus a time dependent one which is assumed small compared with the first. The motion is assumed to have a sinusoidal time dependence with frequency. The variational principle was applied to estimate the frequencies of nuclear vibrations of various multi-polarity.

In this work a method is developed to solve the Vlasov equation with reasonable approximations in a frame of a time-dependent thermodynamic model, which enables the calculation of light and heavy particle spectra on different stages of the reaction. The details of the model are presented in section 2. Results and discussion are displayed in section 3, and finally in section 4, we present conclusive remarks.

2. The Model

Let us consider the collision between a target nucleus T and a projectile P at a given impact parameter $\overline{b}$. The collision goes through sequential stages. The first is a compression of the nuclear matter due to the high energy interaction, forming a fireball with diffusive surface, contrary to standard fireball model assumptions [1] which support the concept of participant and spectator nucleons with pure cylindrical cut in the nuclear matter. The nuclear matter is then treated as a heterogeneous thermodynamic system. Multiple nuclear collisions occur inside the fireball, increasing the energy density and allowing the formation of quark gluon plasma state [11-14]. This leads to creation of new particles and expands the system which gradually approaches the equilibrium state.

The last stage is the fireball decay. Particle emission from the fireball is allowed at diffususeness points on the time scale of the reaction. Light created particles are expected to be emitted on the early stage in a narrow forward cone angle, due to the first few collisions. The higher order collisions draw the system towards the equilibrium state producing particles in isotropic distribution in phase space. It is then convenient to consider the state of equilibrium as a time reference for the reaction. Drawing back, we may follow the historical grow of particle emissions on the time scale. Hadronic matter inside the fireball is partially formed by the fast projectile nucleons and the slow target ones. The relative projectile density in this mixture is a very important parameter. It determines the fireball parameters, the center of mass velocity, the temperature and the temperature gradient inside the fireball matter. We use a Gaussian density distribution [15] for nuclei of mass number $A < 20$, while a Fermi density for $A \succeq 20$. Consider a frame of reference coincides with the center of the target nucleus in the lab. system. Then the relative projectile density $\eta(r, b)$, at a given distance r inside the fireball matter and a given impact parameter b is given by:

$$\eta(r,b) = \frac{\rho_P(r-b)}{\rho_P(r-b) + \rho_T(r-b)}, \tag{1}$$

where for $A < 20$

$$\begin{aligned} \rho_i(\overline{r}) &= A_i(\pi r)^{-3/2}\exp(-r^2/R_i^2) \\ R_i^2 &= (3/2)\frac{< r^2 >}{(1-1/A_T)} \qquad\qquad i = p, T \end{aligned} \tag{2}$$

and for $A \succeq 20$

$$\begin{aligned} \rho_i(\overline{r}) &= A_i\rho_o[1+\exp(\frac{r-c}{d})]^{-1} \\ c_i &= 1.19A_T^{1/3} - 1.61/\ A_T^{1/3} \quad \text{fm} \qquad\qquad i = p, T \\ d &= 0.54 \quad \text{fm} \end{aligned} \tag{3}$$

The local temperature $T(\overline{r})$at a position vector $\overline{r}$ is the solution of the thermodynamic energy conservation Eq.,

$$\zeta_{cm} = 3T + m\frac{K_1(m/T)}{K_2(m/T)},$$

$$[m^2 + 2\eta(1-\eta)mt_i]^{1/2} = 3T + m\frac{K_1(m/T)}{K_2(m/T)} \tag{4}$$

where m is the rest mass of the constituent particle of the nuclear medium under investigation, K_1, K_2 are the McDonalds functions of first and second order [16] and t_i is the incident kinetic energy per nucleon. Equation (4) is valid for each type of particles forming the fireball. The temperature is very sensitive to the form of the nuclear density. The momentum distribution of the fireball nucleons in the center of mass system is given by the following relation:

$$\frac{d^2N}{p^2dpd\Omega} = \frac{N}{4\pi m^3}\frac{\exp(-E/T)}{2(T/m)^2K_1(m/T) + (T/m)K_0(m/T)} \tag{5}$$

The equilibrium energy distribution in the lab system is given by:

$$f_o(E, r) = pE'\frac{d^2N}{p'^2dp'd\Omega} \tag{6}$$

The prime letters are defined in the center of mass system and relativistically transformed as:

$$E' = \gamma_{cm}(E_L - \beta_{cm}P_L \cos\theta_L), \tag{7}$$

where the center of mass velocity β_{cm} is given by:

$$\beta_{cm} = \frac{P_L}{E_L} = \frac{\eta[t_i(t_i + 2m)]^{1/2}}{m + \eta t_i}. \tag{8}$$

Since particles emission is allowed before approaching the equilibrium state, then it is convenient to use the Vlasov equation [4] to deal with the particle energy spectra at any time of the reaction. The Vlasov Eq. has the form.

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\overline{P}}{m} \cdot \nabla_r f - \nabla_r U \cdot \nabla_p f, \tag{9}$$

where $U(r)$ is a scalar potential acting among the particles. Equation (9) may be solved under some approximations. First, we shall consider a pre-equilibrium state where the time derivative $\frac{df}{dt}$ may be approximated as $(f - f_0)/t_c$, where f_o is the equilibrium distribution. Since we are dealing with a state near equilibrium, so it is convenient to consider that the rate of change of the function f is approximately equal to that of f_o. So we replace f by f_o in the RHS of Equation (9). Moreover, let us consider the particles as almost free so that we neglect the potential U in this stage of approximation. Equation (9) then becomes.

$$f_1 = f_0 + t_c\frac{\overline{P}}{m} \cdot \overline{\nabla}_r f_o \tag{10}$$

$$= f_0 + t_c\frac{P}{m}\cos\theta\frac{\partial f_o}{\partial r}.$$

Here, f_1 is the first order approximation of the particle spectrum; t_c is the time interval required by the system to approach the equilibrium state f_o; and θ is the scattering angle, the angle between the direction of particle emission $\overline{P}$ and the radial direction $\overline{r}$. A second order approximation is obtained by using f_1 instead of f in the RHS of Eq. (9), so that.

$$f_2 = f_0 + t_c\frac{P}{m}\cos\theta\frac{\partial f_o}{\partial r} + (t_c\frac{P}{m}\cos\theta)^2\frac{\partial^2 f_o}{\partial r^2}. \tag{11}$$

By the same analogy we get the recursion relation for the n^{th} order approximation as:

$$f_n = f_0 + \sum_{i=1}^{n} (t_c \frac{P}{m} \cos\theta)^i \frac{\partial^i f_o}{\partial r^i},$$

so that the third and fourth order approximations are:

$$f_3 \ = f_0 + t_c \frac{P}{m} \cos\theta \frac{\partial f_o}{\partial r} + (t_c \frac{P}{m} \cos\theta)^2 \frac{\partial^2 f_o}{\partial r^2} + (t_c \frac{P}{m} \cos\theta)^3 \frac{\partial^3 f_o}{\partial r^3} \tag{12}$$

$$f_4 \ = f_0 + t_c \frac{P}{m} \cos\theta \frac{\partial f_o}{\partial r} + (t_c \frac{P}{m} \cos\theta)^2 \frac{\partial^2 f_o}{\partial r^2} + (t_c \frac{P}{m} \cos\theta)^3 \frac{\partial^3 f_o}{\partial r^3} + (t_c \frac{P}{m} \cos\theta)^4 \frac{\partial^4 f_o}{\partial r^4}. \tag{13}$$

3. Results and discussion

The predictions of the pre equilibrium model are applied to the Ne-U collisions at 400 and 2100 A MeV. Assume a frame of reference coincident with the center of the target, and that the projectile is located at a position $\overline{r}$, with an impact parameter $\overline{b}$ as shown in Figure (1). The relative projectile density $\eta(r, b)$ is calculated according to Eq. (1). In Figure (2) we demonstrate $\overline{\eta}(r, b)$ averaged over the whole range of impact parameter. The function $\overline{\eta}(r, b)$ shows a peak value of a height 0.5 at a distance $R_p + R_T = 10.7$fm, where the projectile and the target have equal densities. According to the model assumptions, the nuclei have no sharp surface density but instead, a diffuseness surface which extends the range of the nuclear matter to about twice the sum of the nuclear radii. On the other hand the geometrical factor represented by the size of the nuclear matter has heavy weight near the origin and falls exponentially with $\overline{r}$ toward the surface as may be described by the tail of the Gaussian distribution. The effective range, where the nuclear matter forming the nuclear thermodynamic system has appreciable value, is estimated to about $1.5(R_p + R_T)$. The parameter η has a main role in the evaluation of the temperature and its gradient inside the nuclear matter as seen by Eq. (4). Figure (3) shows the temperature as a function of $\overline{\eta}$ for reactions at 400 and 2100 MeV incident kinetic energy per nucleon. The maximum temperature is found to be 55 and 230 MeV, respectively.

The proton density function in its equilibrium form is calculated according to Eq. (6) over the effective range of the thermodynamic system. The results are shown in Figure (4) for protons emitted with $E_L = 30, 120$ and $180 MeV$ with emission Lab angle 30^o. The protons produced at low energy show anisotropic distribution with peaks near the origin and the surface of the thermodynamic system. The position of the two peaks correspond to the regions characterized by low $\overline{\eta}$ values and consequently low temperature. High energy emission $(120 - 180 MeV)$ shows plateau shape density distribution, the bulk of which corresponds to high temperature zones. The yield from the low temperature zones decreases with increasing the energy of the emitted protons. The spatial variation of the function $f_o(\overline{r}, \overline{p})$ is also studied. Figure (5) exhibit the n^{th} order derivative of f_o. Maximum variation of the derivatives occurs at the origin and near the surface where the temperature and its gradient also change rapidly.

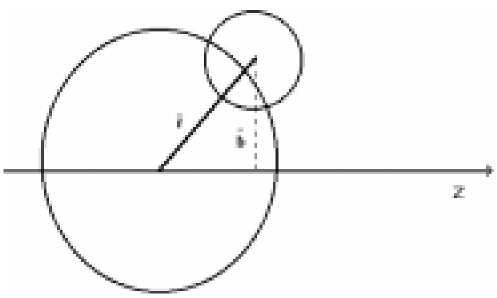

Figure 1. Schematic diagram of the projectile and target collision at a given impact parameter $\bar{b}$.

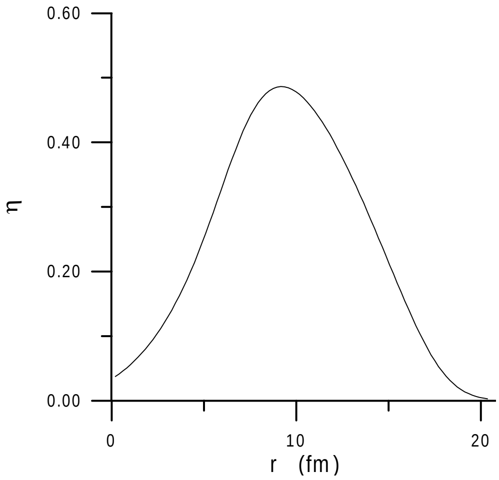

Figure 2. The relative projectile density η for Ne-U collision as measured from the center of the U target.

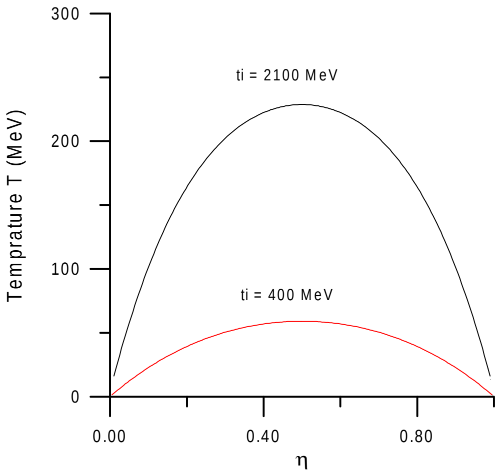

Figure 3. The variation of the temperature as a function of the relative projectile density η, for the Ne-U interactions at 400 and 2100 A MeV incident kinetic energies.

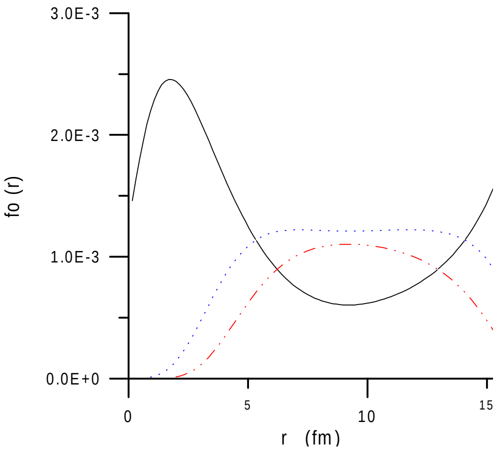

Figure 4. The proton density spectra $f_o(r,p)$ at emission Lab energies E$_L$= 30, 120,180 MeV and Lab angle θ_L= 30^o produced in Ne-U collision at 400 MeV kinetic energy.

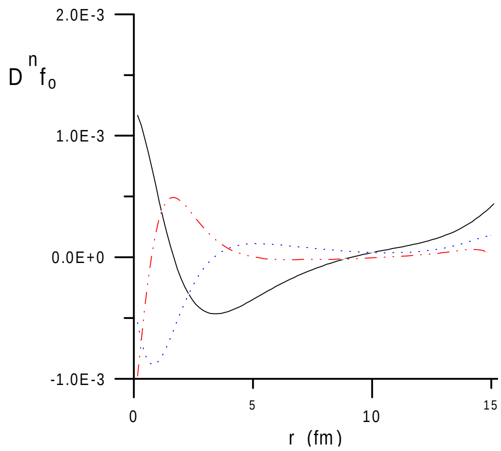

Figure 5. The nth order derivatives of the proton density spectra $f_o(r,p)$. The first three derivative at emission Lab energy E$_L$= 30 MeV and Lab angle θ_L= 30, produced in Ne-U collision at 400 MeV.

The solution of the Vlasov equation is calculated to the fourth order approximation as clarified by Eq.(8). The result is integrated over the effective range with a geometrical weight factor $W(\eta)$, and depends on the size of the nuclear matter. Assuming azimuthal symmetry of the system, then one can finally find the Lab energy spectra, which is calculated at specific values of emission angles $\theta = 30, 60, 90, 120$ and 150^o:

$$f_L^n(E) = \int f^n(E,r)W(\eta(r))4\pi r^2 dr \tag{14}$$

Figures(6-11) shows the lab energy spectra of protons produced in Ne-U at 400 A MeV corrected to the second order. The prediction of the model is compared to the zero order correction (the equilibrium distribution) as well as the experimental data. The emission time parameter is found by fitting method to be $-12, -12, -8, -4.5$ and $-2GeV^{-1}$ corresponding to the emission angles $\theta = 30, 60, 90, 120$ and 150^o respectively. A global fair agreement is obtained by the second order corrected solution of the Vlasov equation. An appreciable improvement is observed in the prediction of the model, particularly in the forward emission spectra (θ = 30 and 60^o). The prediction of the model comes closer to the equilibrium distribution for the wide emission angles (θ = 90, 120 and 150^o). Particles produced at such wide angles are expected to make multiple collisions inside the nuclear matter before emission takes place. In other words, the wide angle production is a signal of the approach to equilibrium state. Such a system is characterized by a large number of inter-nuclear cascade collisions which increases the entropy of the system and leads to equilibrium. It is found that the solution of the Vlasov equation through the present approximation forms a converging series. It is enough to consider only the first two terms in the series on dealing with the Ne-U collisions at 400 A MeV, while terms up to the fourth order are found to have appreciable values for the case of Ne-U collisions at 2100 A MeV. The second correction and the fourth order correction improve the calculation toward the virtual values. Although we considered four terms for the reaction at 2100 MeV, but the agreement with experimental data was not fair enough. The hypothesis of pre-equilibrium includes many approximations that may not fit the systems formed at high energies. The last term in Vlasov equation containing the field potential between the interacting particles should be considered, and the problem then is treated microscopically instead of the macroscopic picture as presented in this article. Despite of the existence of the field theory of strong interactions, the theoretical description of this phenomena is necessarily phenomenological because of the very nature of the problem which involves many degrees of freedom. In a forthcoming article, one may encounter the problem taking into consideration the study of collective properties of hadronic matter, in particular its possible phase transition to the quark-gluon phase. The main merits of this approach lie, in our view, in the fact that the phenomenology is reduced to microscopic concepts like parton- parton cross sections and structure function and different equilibrium properties of the gluon and quark components of hadrons which allow for identification of the coherent parts of the interactions.

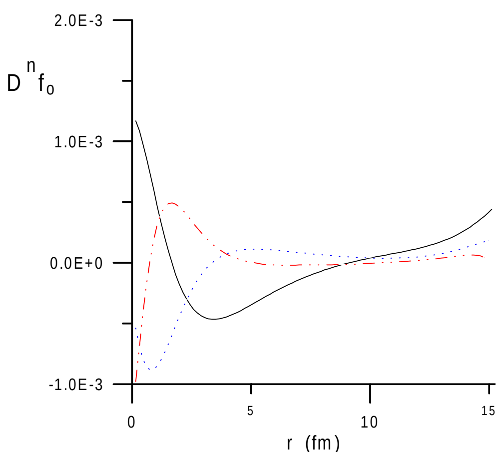

Figure 6. The energy spectra of protons produced in Ne-U interactions at 400 A MeV, at emission angle 30^o. The solid line represents the equilibrium distribution, while the second correction is represented by dashed line, with an emission time parameter $t = -12\ (GeV)^{-1.}$

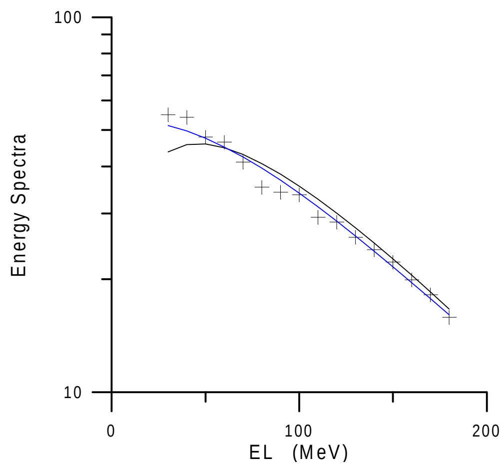

Figure 7. The energy spectra of protons produced in Ne-U interactions at 400 A MeV, at emission angle 60^o. The solid line represents the equilibrium distribution, while the second correction is represented by dashed with an emission time parameter $t = -12\ (GeV)^{-1.}$

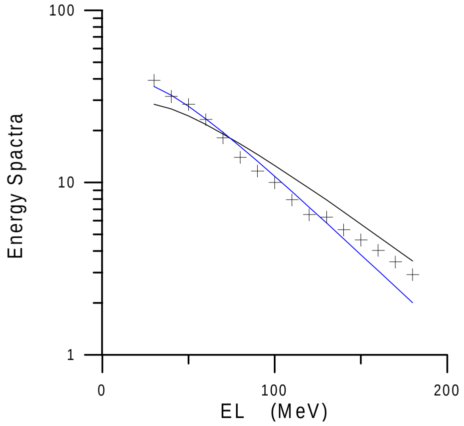

Figure 8. The energy spectra of protons produced in Ne-U interactions at 400 A MeV, at emission angle 90^o. The solid line represents the equilibrium distribution, while the second correction is represented by a dashed line, with an emission time parameter $t = -8\ (GeV)^{-1.}$

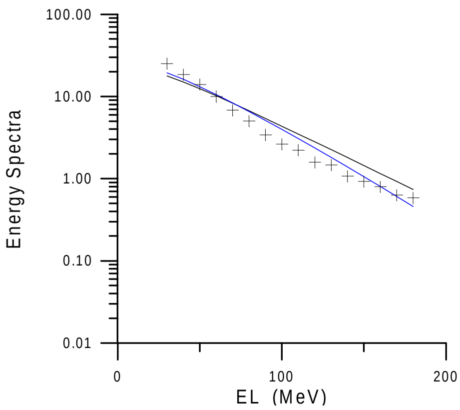

Figure 9 The energy spectra of protons produced in Ne-U interactions at 400 A MeV, at emission angle 120^o. The solid line represents the equilibrium distribution, while the second correction is represented by dashed line with an emission time parameter $t = -4.5$ $(GeV)^{-1.}$

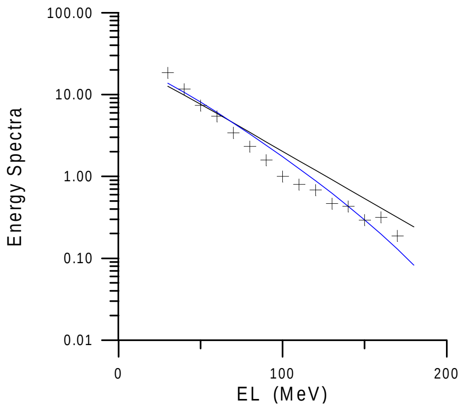

Figure 10. The energy spectra of protons produced in Ne-U interactions at 400 A MeV, at emission angle 150^o. The solid line represents the equilibrium distribution, while the second correction is represented by dashed line with an emission time parameter $t = -2.5$ $(GeV)^{-1.}$

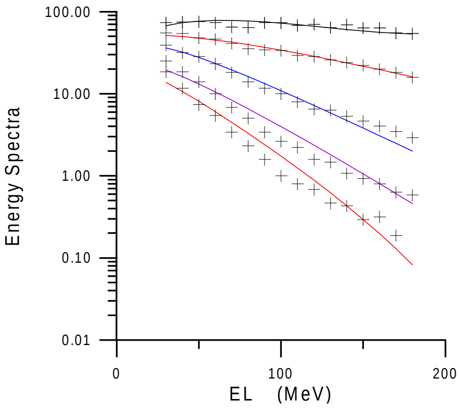

Figure 11. The second order corrected pre-equilibrium energy spectra of protons produced in Ne-U interactions at 400 A MeV, at emission angles of 30, 60, 90, 120 and 150^o. The solid lines represent the calculated distributions and the experimental data are represented by (+) signs.

4. Conclusive remarks

i. The pre-equilibrium model with reasonable approximations may fit the experimental data of heavy ion collisions within the regime of few hundreds A MeV. In this case, a power series is presented to describe the nuclear density function in the frame of a thermodynamic picture.

ii. The relative projectile density plays an important role in determination of the hadronic matter temperature.

iii. The temperature gradient in the system comes due to the assumption that the nuclear matter has a diffuseness surface density with Gaussian distribution.

iv. The temperature has minimum values around the center and near the end of the effective range of the nuclear matter. These regions are responsible of the emission of low energy particles, while fast particles are produced in the bulk region characterized by $\eta = 0.5$ and high temperature.

v. The power series solution is converging in nature. The n^{th} order term depends on the n^{th} derivative of the phase space distribution function.

vi. A series up to the second order correction is sufficient to describe the reaction at 400 A MeV. While the fourth order term has appreciable importance in the reactions at 2100 A MeV.

vii. Particles emitted in the forward direction are produced in the early stage of the reaction, far from the equilibrium. Backward production comes in a later stage when the system constituents undergo multiple cascade collisions.

References

[1] J. Gosset, H. H. Gutbrod, W. G. Meyer, A. M. Pokanger, A. Sandoval, R. Stock and G. D. Westfall, *Phys. Rev.,* **C16,** (1977) 629.

[2] J. Gosset, J. I. Kapusta and G. D. Westafall,*Phys. Rev.*, **C18** (1978) 844.

[3] W. D. Myers, *Nucl. Phys.,* **A 296**, (1978) 177.

[4] G. F. Bertsch and S. Das Gupta, *Phys. Rep.,* **160,** (1988) 189.

[5] H. Kruse, B. V. Jacak and H. Stocker, *Phys. Rev. Lett.,* **54,** (1985) 289.

[6] C. Gregorie, B. Renaud, F. Sebille, L. Vinet and Y. affray, *Nucl. Phys.*, **A465**, (1987)317.

[7] J. Aichelin and G. F. Bertsch, *Phys. Rev.,* **C31,** (1985)1730.

[8] T. Deal and S. Falierosyy, Phys. Rev., C7, (1973) 1709.

[9] J. Noble. *Ann. Phys.*, **67**, (1971) 98.

[10] J. Martorell et. al. *Phys. Lett.,* **60B,** (1976) 313.

[11] M. T. Hussein, A. Rabea, A. El-Naghy and N. M. Hassan, *Prog.Theor.Phys.*, *93,* (1995) 585.

[12] G. N. Fowler, F. S. Navarra, M. Plumer, A. Vourdas, A. M. Weiner and G. Wilk, *Phys. Lett.*, **B214,** (1988) 567.

[13] M. Wellner, *Phys. Rev. Lett.,* **68,** (1992) 1811.

[14] B. Muller and A. Tragonov, *Phys. Rev. Lett.*, **68,** (1992) 3387.

[15] M.K.Hegab, M.T.Hussein and N.M.Hassan, Z.Phys. A 336 (1990) 345.

[16] W. H. Press, Lectures in College on Computational Physics. Trieste, May 17- June11 (1993).