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Multitime generalization of Maxwell electrodynamics and gravity

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Abstract

System of equations of motion for interacting electromagnetic field and charged bodies is derived by means of an action principle in six-dimensional space with a three-dimensional time vector. The influence of the additional (hidden) components of time on the body motion is considered. Macroscopic experiments and astrophysical observations which can reveal the time multi-dimensionality are discussed. At present there are no facts which would be in contradiction with the hypothesis of the multi-dimensionality of time.

1. Introduction

From a historical viewpoint one can see three "roots" of the idea of a multitime generalization of present theory: (i) the Poincare and Einstein initiated aspiration for the more and more symmetrization of space and time co-ordinate, (ii) the attempt to bystep the difficulties of superluminal theories and (iii) the investigation of the physical consequences of additional time co-ordinates appearing in string and other generalizations of existing theory (see papers [1 - 5] where one can find detailed bibliography). In such an approach the customary four-vector $\mathbf{x}$ is replaced by the six-dimensional one: [1]:

$$(\hat{\mathbf{x}})_\mu = (-\mathbf{x}, c\hat{t})^T_\mu \quad , \quad (\hat{\mathbf{x}})^\mu = (\mathbf{x}, c\hat{t})^{\mu T}.$$

Positions of any body on its trajectories in x- and t-subspaces are determined by a scalar "proper time" t:

$$\mathbf{x} = \mathbf{x}(t) \quad , \quad \hat{t} = \hat{t}(t),$$

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[1]Here and in what follows vectors in x- and t-subspaces are denoted, respectively, by bold face and by a "hat". Six-dimensional vectors will be denoted at once by bold face and the "hat". In manuscripts it is convenient to use the notations $\bar{x}$, $\hat{x}$ and $\hat{\bar{x}}$. We shall also suppose that the latin and greek indices take values $k = 1, ..., 3, \mu = 1, ..., 6$).

where t is a length along the t-trajectory. This trajectory itself can be determined in this case by a unit vector $\hat{\tau} = d\hat{t}/dt$.

The performed investigations convince us that multitime hypothesis is logically consistent and the following question now arises: how the virtual time multi-dimensionality, if it exists indeed can be revealed in experiment? The discovery of any contradictions would be a proof of the one-dimensionality of time; their lack, on the contrary, attracts the physicist's attention to multitime generalizations.

An important feature of our approach is the requirement of time irreversibility, i. e. the impossibility to reproduce any event in all its details backwards in time as is a direct consequence of the inexhaustibility the inner and outer interconnection of every material object. Namely, this property of Nature, but not a specific time non-invariant process, is a reason for the invariant "time arrow". A repetition of all alterations of any system is possible only in approximate theories taking into account a finite number of parameters. Only such t-invariant theories never come true in the real world, where one can find for any process a proceeding couse having its own time effect. These events can not be displaced. On a philosophical level, it is formulated as the principle of causation [2].

Taking into account the material reason of time irreversibility, one has to admit that this property must be true for any time direction. It means that a preferred ("relic" from the cosmological point of view) time reference frame must exist in multi-dimensional universe. In this frame projections of all time trajectories on axis t_i must always be positive:

$$\hat{\tau} = d\hat{t}/dt \geq 0. \tag{1}$$

. (Particularly, in the two-dimensional case, the t-trajectory of every body must pass on from the third angular quadrant to the first one). Only such trajectories exist in the Nature. The time sequence of all events in the world corresponds to the relic frame. In reference frames turned with respect to this time frame the sign of some intervals Δt_i may be negative. However, the use of such co-ordinate systems makes sense with a formal renumbering of time co-ordinates as an inverse time reading, which we some time use in our everyday practice.

2. Equation of motion in the multitime world

To derive these equations, we use the action principle

$$\delta S = \int \delta \mathcal{L}\, dt = 0$$

with the Lagrangian $\mathcal{L}(\hat{\mathbf{x}}, \hat{\mathbf{u}}, t)$, where the six-dimensional velocity vector is defined as

[2]Several elucidation of the time irreversibility must be added in the case of elementary particle interactions which possesses a high symmetry with respect to a change of time direction. One must not forget, however, that our description of elementary processes demands taking into account macroscopic surroundings. Describing elementary processes, we attract ourself away from a consideration of accompanying macroscopic time irreversibility, which means some idealization. Time arrow is given for our world a priori, i. e. at the first moments of its creation.

follows:

$$\hat{\mathbf{u}} = d\hat{\mathbf{x}}/ds = \gamma c^{-1}(\mathbf{v}, c\hat{\tau})^T = \gamma c^{-1}\hat{\mathbf{V}}, \tag{2}$$

$$\gamma = (1 - v^2/c^2)^{1/2}, \quad ds = cdt/\gamma, \quad \hat{\tau}^2 \equiv (\sum_i dt_i/dt)^2 = 1.$$

The scalar time t is read along the trajectory $\hat{t}$. The variation is performed around the space-time trajectory $\hat{\mathbf{x}}(t)$ of the considered body taking into account that the scalar time t has to remain constant:

$$\delta S = \int \left[\frac{\partial L}{\partial \hat{\mathbf{x}}}\delta\hat{\mathbf{x}} + \frac{\partial L}{\partial \hat{\mathbf{u}}}\delta\hat{\mathbf{u}}\right] ds = 0.$$

Here L is the covariant Lagrangian $\gamma\mathcal{L}$. The corresponding Lagrange equations

$$\frac{\partial L}{\partial \hat{\mathbf{x}}} - \frac{d}{ds}\left(\frac{\partial L}{\partial \hat{\mathbf{u}}}\right) = 0, \tag{3}$$

where

$$\frac{\partial}{\partial \hat{\mathbf{x}}} = \left(\frac{\partial}{\partial \mathbf{x}}, \frac{\partial}{\partial \hat{x}}\right), \quad \frac{\partial}{\partial \mathbf{x}} = \nabla, \quad \frac{\partial}{\partial \hat{x}} = c^{-1}\hat{\nabla}$$

and the time differentiation is performed, if the velocity $\hat{\mathbf{u}}$ is considered as an independent variable, i. e. $\hat{\boldsymbol{\nabla}}\hat{\mathbf{u}} = \hat{\mathbf{u}}\hat{\boldsymbol{\nabla}}$. The operator d/ds can be replaced by $-(\hat{\mathbf{u}}\hat{\boldsymbol{\nabla}})$.

If the Lagrangian for the field and a charged body with the velocity $\hat{\mathbf{u}}$ and the mass m

$$L = \frac{1}{c^2}\int \hat{\mathbf{A}}\hat{\mathbf{J}}d^3x + \frac{1}{16\pi c}\int \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}d^3x + \frac{m_o c}{2}\hat{\mathbf{u}}^2 - \frac{q}{c}\hat{\mathbf{u}}\hat{\mathbf{A}}, \tag{4}$$

where the current

$$\hat{\mathbf{J}}(\hat{\mathbf{x}})_\mu = (-\mathbf{v}\rho(\hat{\mathbf{x}}), c\hat{\rho}(\hat{\mathbf{x}}),$$

then the equation of motion (3) can be represented in the form

$$(\boldsymbol{\nabla} \times \mathbf{H})_i + \hat{\boldsymbol{\nabla}}_k E_{ik} = 4\pi c^{-1}\rho(\hat{\mathbf{x}})v_i,$$

$$(\hat{\nabla} \times \hat{G})_i + \boldsymbol{\nabla}_k E_{ki} = 4\pi\rho(\hat{\mathbf{x}})\tau_i,$$

$$\hat{\boldsymbol{\nabla}}_i H_k - \epsilon_{kmn}\boldsymbol{\nabla}_m E_{ni} = 0,$$

$$\boldsymbol{\nabla}_i G_k + \epsilon_{kmn}\hat{\boldsymbol{\nabla}}_m E_{in} = 0,$$

$$\boldsymbol{\nabla}\mathbf{H} = \hat{\nabla}\hat{G} = 0,$$

with the additional Lorentz condition

$$\hat{\boldsymbol{\nabla}}\hat{\mathbf{A}} \equiv \hat{\nabla}\hat{A} - \boldsymbol{\nabla}\mathbf{A} = 0$$

and the equations for the charged body,

$$md\mathbf{v}/dt = q\hat{\mathbf{E}}\hat{\tau} + qc^{-1}\mathbf{v} \times \mathbf{H} - \mathbf{v}dm/dt,$$

$$md\hat{\tau}/dt = qc^{-2}\hat{\mathbf{E}}^T\mathbf{v} + qc^{-1}\hat{\tau}\times\hat{G} - \hat{\tau}dm/dt,$$
$$dm/dt = qc^{-2}\mathbf{v}\left(\hat{\mathbf{E}}\hat{\tau}\right).$$

Here,

$$(\hat{\mathbf{A}})_\mu = (-\mathbf{A}, \hat{A})^T_\mu, \quad (\hat{\mathbf{A}})^\mu = (\mathbf{A}, \hat{A})^{\mu T},$$
$$\mathbf{H} = \nabla\times\mathbf{A}, \quad \hat{G} = -\hat{\nabla}\times\hat{A},$$
$$\hat{\mathbf{E}} = \mathbf{A}\hat{\nabla} - \nabla\hat{A}$$

Analysis of the structure of these equations and their solutions allows one to obtain an idea about the properties of the multitime world. That is similar to investigation of the non-Euclidian space by N. Labachevsky one and half century ago.

Some interesting properties of the electromagnetic field, particularly the appearance of longitudinal waves, were considered at the previous Workshop [3]. Now we attract attention to the body behavior in the world with hidden time axes.

3. Multitime motion of bodies

It is very important to emphasize that the body speed (2) is defined with respect to an increment Δt along the body time trajectory $\hat{t}$. If it is unknown and an observer uses his proper time $\Delta t_p = \Delta t\cos\theta$ where θ is the angle between the body and observer's time trajectories, then the quantity $\mathbf{v_p} \equiv \Delta\mathbf{x}/\Delta t_p = \mathbf{v}/\cos\theta$ defined in this way may turn out to be larger than the light velocity c. In this case the considered body behaves, from the observer's viewpoint, like a tachyon. For example, if $\theta \simeq \pi/2$, it passes any finite distance practically instantaneously and "grows old" straight away. Nevertheless, as it was shown in papers [6,7], Lorentz transformations depend on $\mathbf{v}$ but not on $\mathbf{v_p}$, therefore in the multi-temporal world no acausal effects can be observed by transformations to moving reference frames in contrary to true tachyons which transfer an information in the new frame, as it is judged by the observer, backwards in time [8].

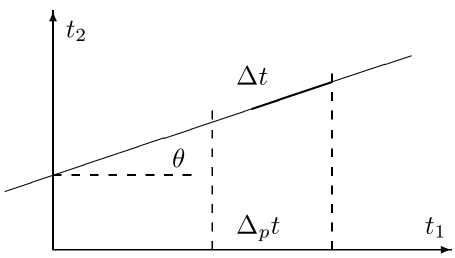

Figure 1. If the time Δt_p is used instead of Δt, one can get an illusive superluminal body speed. To simplify the picture we consider a two-dimensional case and assume that t_1-axis coincides with the observer's time trajectory.

Discovery of any superluminal events would be a serious indication of the multi-dimensionality of world time. As it is known, faster-than-light objects are indeed observed by astronomers. Though up to now they have succeeded in interpreting such phenomena as optical illusions within the limits of one-time notions as optical illusions , one can not exclude that among the observed superluminal objects there are bodies moving along the distinct time directions.We need more experimental information to identify such bodies.

Besides the superluminal illusions, multitime theory predicts another amusing phenomenon which can be observed in experiment. In our world luminous bodies remain visible all the time while they emit light, however, in the multi-temporal case their luminescence is seen, as a rule, only in some restricted time interval. For example,

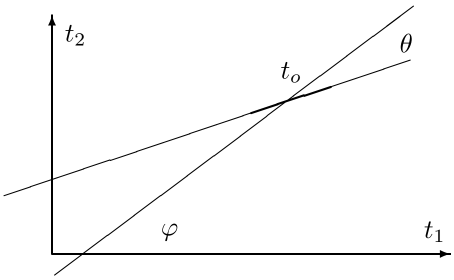

Figure 2. The luminescence is seen only in a restricted interval of the observer's proper time around the intersection point t_o. In order not to complicate the picture, we confine oneself by a case when the luminous body and observer's trajectories are placed on the same plane and the axis t_3 can not be mentioned.

if a motionless in x-subspace luminous body intersects the observer's t-trajectory at an angle θ, one can show [9] that this observer sees its luminescence only in the interval

$$T = \frac{R}{c}\frac{\sin(\varphi+\theta)}{\sin\theta}[1+\cot(\varphi+\theta)] \tag{5}$$
$$\simeq R/c\theta \qquad \text{for} \quad \varphi,\theta \ll 1,$$

where R is the constant distance between the body and the light detector, φ is the inclination of the observers t-trajectory with respect to the axis of the mentioned above preferred ("relic") reference frame.

One should notice that expression (5) differs from the that derived by Cole and Starr [10,11] who did not take the time irreversibility into account. Nevertheless, in both cases the conclusion about the limit of luminescence time is valid. Particularly, because the duration of an interaction of two bodies is proportional to their mutual distance, the interaction time of nearly placed bodies is practically equal to zero, i. e. they "see" each other only an instant, when their trajectories intersect. A subsequent communication of these bodies is possible only with the help of subluminal signals. For angles $\theta \le 1^o$ and distances on the order of a thousand km and less, the interaction duration is equal to a fraction of a second, i. e. the close light sources turn practically at once into invisible "ghostly bodies". Only very remote cosmic objects can shine uninterruptedly for a long time.

One may wait also for any unusual explosive phenomena at the moment when the time trajectories of bodies located at the same place intersect each other.

As we do not encounter, however, an appearance of material objects "from anywhere" or their disappearance "to nowhere", and do not observe inexplicable explosions, one may be sure that the time trajectories of all surrounding us bodies are extremely close to each other: $\theta = 0$.

4. Astrophysical phenomena

First we consider the Mercury perihelion advance for which the value calculated in paper [2] is 2.3 times larger than the observed one. If this result is correct, it shut down the multitime electrodynamics also.

Planet motion in the gravitation potential

$$\hat{\mathbf{A}} = (0, \hat{\varphi}(\mathbf{x})), \quad \hat{\varphi}(\mathbf{x})\hat{\tau}_{\mathbf{s}} = -\kappa \mathbf{M}/\mathbf{r}$$

obeying the Poisson equation

$$\nabla^2 \hat{\varphi} = 4\pi\kappa M \hat{\tau}_s \delta(r)$$

is described by the last part of Lagrangian (4), where M and $\hat{\tau}_s$ are the prossent Sun mass and the Sun time vector, and m and $\hat{\mathbf{u}}$ are the planet mass and covariant velocity respectively.

In paper [12] it was shown that the cosine of the angle between the constant time vector of the Sun and a rotating planet time vector $\hat{\tau}$ is

$$\chi = \hat{\tau}_s \hat{\tau} = 1 - (1 + 3\chi_o^2/2)/Q,$$

where

$$\begin{aligned} Q(r) &= \kappa c^{-2} M(1/r - \gamma/\gamma_o r_o) \\ &\simeq \kappa c^{-2} M(1/r - 1/r_o). \end{aligned}$$

It is well known that within experimental error of the perihelion advance of $\Delta\theta_{exp} \simeq \pm 0.9''$ per century the observed perihelion advance agrees precisely with the one-time correction θ^o_{gr}. If we suppose that Mercury turns around the Sun in approximately the same time as our Earth, than it follows from expression (5) that $\Delta\theta < 10^{-10}\Delta\theta_{exp}$ and, therefore, the multi-dimensional correction is less than $10^{-10}\Delta\theta_{exp}$.

A principally different result is obtained in paper [2], where angle independent multi-time correction $\Delta\theta = 7\theta^o_{gr}/3$ is obtained. This result could be considered demonstration of the one-dimensionality of our Universe. However, it is a consequence of the use of the vector quantity $\hat{\varphi}(\mathbf{x})$ and the scalar potential $\varphi(r)$, which is independent of the time vector $\hat{\tau}_s$. Such an approach means an equivalent treating of all three time co-ordinates t_i, the coefficient triples in the expression of Ricci tensor determining the value of $\Delta\theta$, equivalently.

An interesting possibility to check the existence the additional time component is provided by the huge gravitation detectors which are now under construction in many countries. Waves and matter with diverse t-trajectories would appear in cosmic cataclysms where enormous amounts of energy are produced. In such very strong gravitational fields the concept of energy itself loses its sense and the energy conservation law becomes inexact. All of this must influence the properties of emitted gravitational waves.

Calculations have shown [5] that there is quantitative difference between one- and multi-time cases: in the latter. there is no correlation of the detector "width" and "height" oscillations in a plane perpendicular to the direction of gravitational wave propagation . In the customary one-time theory the amplitudes of these oscillations are equal.

6. Conclusion

In the region of macroscopic phenomena a time multi-dimensionality hypothesis does not contradict any known experimental fact. It is quite possible that our World is like that indeed. Nevertheless, comparison of theory with experiment proves the high degree of time flow parallelism in the part of the Universe surrounding us.

Energy conservation and time irreversibility laws forbid any change of body time trajectories because, in such a case, bodies with compensating energy components $E_i < 0$, i. e. moving backward in time, must be present. (It will be recalled that time vectors $\hat{\tau} \sim \hat{E}$). Objects whose time trajectories are differ from our's can be found only in microscopic phenomena and in regions with strong gravitation fields where usual (classical) energy conservation law does not act.

For one more condition in which there may be a t-trajectory change, suppose the time trajectories of a decaying body and products of its decay are strongly declined with respect to the axes of the mentioned above preferred reference frame. In such cases all projections of time vectors $\hat{\tau}$ can be positive.

As we do not observe any described above peculiarities associated with time multi-dimensionality, one may conclude that our own t-trajectory is close to the preferred reference frame t-axis.

One can expect the appearance of objects with a "turned time" on the level of microscopic space-time intervals where energy necessary for their creation is about their rest-mass. This aspect demands special investigation, however, one can predict that if the deviation of time trajectories is significant, then the time of an interaction of such particles with their surroundings is very short.

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