Algebraic implementation of Gauss' law

Andrzej HERDEGEN[*]
Institute of Physics, Jagiellonian University
Reymonta 4, 30059 Kraków-POLAND

Abstract

A C^*-algebraic model may be constructed for the description of asymptotic fields in quantum electrodynamics. In this model charged fields carry the asymptotic Coulomb field of associated charged particles.

In this talk I present the physical motivation and the main ideas of the construction of a C^*-algebraic model of asymptotic (in the causal, "in" or "out" sense) fields in quantum electrodynamics. This model has been developed by the author in a series of papers, the most recent of which is: A. Herdegen, "Semidirect product of CCR and CAR algebras and asymptotic states in quantum electrodynamics", J. Math. Phys. **39** (1998) 1788. See this paper also for references to preceding papers and to the literature on the infrared structure of quantum electrodymnamics.

Investigations of the infrared structure of QED have a long, and in many respects, fruitfull history. The central result is the discovery of a superselection structure in the algebra of local observables which is connected with the distribution of the electric field flux at spatial infinity. Representations of local observables from different sectors are unitarily nonequivalent. In particular, states differing by total charge value are inequivalent in consequence of Gauss' law. Another consequence of this superselection structure is the spontaneous breaking of Lorentz symmetry in charged sectors. This means that although the Lorentz transformations of local observables are defined, they cannot be obtained by the action of a unitary representation of the Lorentz group in the representation space of the charged state.

What has not been achieved up to now is a complete clarification of the physical nature of a charged particle. The theory of superselection sectors of QED tells us that the contribution of a charged particle to the spectrum of squared four-momentum (the mass spectrum squared) is not pointlike. A charged particle, being accompanied by

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[*]e-mail : herdegen@thrisc.if.uj.edu.pl

its Coulomb field, is an object more complex then a "bare", neutral particle (this fact is referred to as "the infraparticle problem"). There exist numerous investgations and proposals in this field, both of general as well as model-like character. What, however, is lacking, is the full understanding of the algebraic structure here involved. Of course, there is no final formulation of quantum electrodynamics at hand (beyond the perturbation calculus), so one has to make guesses on some of the properties of the prospective full theory. In doing this, one has to properly take into account the constraint structure of the electromagnetic interaction. I believe that what the standard wisdom based on locality and/or canonical quantization has to say on the algebraic nature of the long-range degrees of freedom, which are decisive for the understanding of charge structure of quantum electrodynamics, may need extension or even modification.

With this motivation in mind we try to construct a closed algebraic model, which incorporates electromagnetic fields with their infrared part included, and matter fields together with the accompanying Coulomb fields. This turns out to be possible in the (causally) asymptotic regions, where the details of dynamics cease to play role, but the constraint structure of the theory persists. The main idea of the construction is to consider first the classical asymptotic structure (in two different asymptotic regions: timelike for the matter and lightlike for the electromagnetic fields respectively), and only then quantize this structure according to the correspondence principle. In the resulting model Gauss' law is implemented on the algebraic level, and in the positive-energy representations of the algebra it takes on a functional form analogous to the classical one. In a class of physically distinguished representations there is always a low-energy electromagnetic cloud present, preventing the appearence of energy or mass eigenvalues.