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Magnetic Excitations in Random Anisotropic Magnets

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Abstract

A study is made for the magnetic excitations and their Anderson localization properties for a Heisenberg ferromagnet with random anisotropic fields. The system is assumed to be in a strong applied field so that complete spin alignment is established. The magnetic excitations, i.e., the harmonic spin waves, are computed using coherent-field-anisotropy approximation (a version of coherent potential approximation of the electronic problem). The spin wave shift and damping are calculated using the computed coherent field which is a complex function of energy approximating the random medium as a mean field level. The localizaton of the excitations are calculated by adopting the analogous results of the electronic problem. The model investigated here can be applicable to amorphous magnets in the low energy limit.

1. Introduction

Determining the ground state and excitation spectrum of a random magnetic systems with competing interactions are still interesting and challenging problem. These systems, for instance, are the spin glasses with competing exchange interactions and the amorphous magnets with fluctuating exchange as well as random anisotropic interactions. Their one common feature as a result of competing interactions is the highly degenerate complicated ground state (a degeneracy apart from trivial spin rotations). That is blocks of spins may rotate without coasting any energy. We have lately studied [1] the spin glasses in the strong field limit using effective medium theories. The assumed field was strong enough to align spins completely along itself thus removing the degeneracy of the ground state. Thus the spin glasses in a strong field reduces to a system where small deviations of the spins from the field direction are fluctuating over a ferromagnetic ground state. We have investigated these deviations, harmonic magnons (spin waves), numerically and analytically (using the coherent exchange- approximation (CEA)[2] developed for the bond dilute magnetic systems). Here, a study is made for the Heisenberg model with uniform ferromagnetic exchange interactions with random uniaxial anisotropy in the strong field limit. The ground state of the above system in the zero field is unstable

against formation of overturning spin droplets regardless of the strength of the anisotropy [3]. Here the competition occurs between exchange field forcing spins to rotate along a common axis and anisotropy field forcing spins along the local anisotropy. This model was first suggested for the amorphous ferromagnets containing rare earth atoms with non-zero orbital angular momentum [4]. The random anisotropy arises from the coupling of the spins with the electrostatic (crystal) fields of the local random environments. The Hamiltonian for this model is

$$\mathcal{H} = -\sum_{<i,j>} J_{i,j}\vec{S}_i\vec{S}_j - \sum_i K_i(S_i^z)^2 - \sum_i h_i S_i^z. \tag{1}$$

The first term is the spin-spin interaction, $J_{i,j}$ is the exchange interaction and the summation over the nearest neighbours. The second term is the random anisotropy interaction with a probability distribution law given by

$$P(K_i) = (1-x)\delta(K_i - K) + x\delta(K_i - \lambda K), \tag{2}$$

where x is the concentration of the anisotropy strength K and λ is a parameter: if it is positive, anisotropy favours along the axis (here it is fixed to z direction); otherwise, it favours the plane perpendicular to the axis.

Random anisotropy model is studied in the hydrodynamic limit considering a random axis in a uniform field [5]. They have found three regimes for small anisotropy in comparison with the exchange interaction. If h_{ex}, h_a, and h represent the exchange field, random anisotropy field and applied field respectively, $h/h_{ex} << (h_a/h_{ex})^4$ the system is in correlated spin glass phase while $(h_a/h_{ex})^4 << h/h_{ex} << 1$ it is in a ferromagnetic phase with a wandering axis. The third regime is the mean-field regime where the applied field field is strong. We have studied in this paper the mean-field regime. Ref. [6] also studied the random strength anisotropy model by transforming spin Hamiltonian to the disordered electronic problem whereby they were able to use results of the electronic disorder.

2. The Effect of Random Exchange in a Strong Field Systems

we proceed first to investigate the effect of exchange fluctuations only in a strong field. The exchange interaction is assumed to be $+J$ with probability c and $-J$ with probability 1-c. In Ref [1] and [7], we calculated the density of states and the dynamic structure factor for the entire spectrum using the CEA and numerical simulation techniques. Here in this section we present the results for very low energy excitations. The CEA reduces the fluctuating exchange system to an effective medium characterised by a coherent-exchange given by (for a uniform mode)

$$Jc(E - h = 0)/J = 3(1-2c) + [9(1-2c)^2 - 8]^2/4, \tag{3}$$

for 3 dimension. The real part of the Jc is proportional to the spin wave stiffness and imaginary part is proportional to the damping of the spin wave excitations. The imaginary part of Jc is also interpreted as the appearance of the localized spin waves by Shender [8]

using the analogy between the spin wave stiffness and the conductivity of an equivalent network. From Eq. (3), it is clear that for c as small as 0.029, the spin wave modes are localized (according to Shender's criterion) which is confirmed by numerical calculation [1].

For small wave vector Q, the peak position of the dynamic structure factor is $E - h = ReJc(0)Q^2$ and its half width at half height equal to $\gamma = \Im Jc(0)Q^2$. We can define spin wave mean free path from the ratio of the group velocity $\vec{V_g}$ to the line width

$$\lambda = 2\Re Jc(0)Q/[\Im Jc(0)Q^2] = 6|1-2c|/[Q\sqrt{8-9(1-2c)^2}]. \quad (4)$$

For c=0.5, the most disordered configuration, the mean free path is zero. For c other than 0.5, the mean free path is finite and diverges as Q goes to zero. The $Q = 0$ mode is just uniform precession of the spins along the field direction. This is the mode seemed to be not affected by the disorder nor localized except $c = 0.5$. Using Ioffe-Regel criterion [6], we can study further the localization problem. The rule for the existence of localized mode can be obtained from this restriction $\omega\tau = \frac{\Re Jc}{\Im Jc} \leq 1$ where τ is the lifetime of the spinwaves. This criteria shows that for concentrations $0.167 \leq c \leq 0.833$ spinwaves are completely localized.

3. Random Anisotropic Ferromagnet:the High field limit

The random anisotropy term in Eq (1) can be modelled in two ways. One is to fix the strength of the interaction but direction of the anisotropy changes locally. The other model, adopted in this paper, is the direction is fixed but the strength of anisotropy changes locally. Huber [9] developed a version of the coherent potential approximation (CPA) coherent-anisotropy-field approximation (CAFA) to investigate the magnon spectrum in a random uniaxial magnets. We will utilize his version the CAFA method to calculate the spin wave resonance shifts and damping. Spin wave excitations can be calculated through the equation of motion for the spin operator (assuming harmonic time dependence $S^+ \sim e^{-i\omega t}$ is

$$(\omega - h)S_i^+ = \sum_j J_{i,j}(S_i^+ - S_j^+) + K_i S_i^+, \quad (5)$$

where $< S^z >= 1$. One can see that this is the same equation of motion for tight binding Hamiltonian with diagonal disorder. The analogous random potential here is the random anisotropy term in Eq.(6) so we can directly use disordered electronic results particularly localization properties of the excitations.

The assumption of the CAFA is that the random medium is characterized by a complex energy dependent coherent-anisotropy-field which will be determined self consistently by the following equation

$$\int P(K)\frac{K - Kc}{1 - (K - Kc) < G_0(E - Kc) >}\, dK = 0, \quad (6)$$

where $P(K)$ is the distribution function given by Eq.(2), $Kc(E)$ the coherent-anisotropy

field and $< G_0(E-Kc) >$ is the configurationally averaged green's function with $E = \omega - h$. The efficient use of the CPA like approximations requires the knowledge of the green's function. The green's function can be computed actually after carrying out complicated integral equations. To avoid the complications it is usually employed an approximate form of green's function. For cubic system using an approximate elliptic density of states [1,9], the green's function is obtained

$$< G_0(E) >= \frac{8}{M}[\frac{E}{M} - \frac{1}{2} - \sqrt{\frac{E}{M}(\frac{E}{M} - 1)}], \tag{7}$$

where M is normalizitaion coefficient. The error involved employing above green's function is less than that of computing the integral equations. Another point is that this green's function assumes a cubic symmetry which does not exist in amorphous materials, however for low excitations (the long wavelength limit) the lattice structure is usually irrelevant and for amorphous ferromagnets above equation has the correct density of states in the low energy limit [9]. The Eq.(6) is nonlinear self consistent equation, however using the green's function above one can find the solution by employing the following transformation [10]

$$E - Kc - \frac{1}{2} = \frac{1}{2}cosh\phi \tag{8}$$

and the self consistent equation takes the form

$$z^3 + z^2 - (4K)^2 z + (4K)^2 = 0, \tag{9}$$

where $z = e^{-\phi} - 1$, x=0.5 and $\lambda = -1$. This is the case where fifty percent of the local anisotripies favour along the field and fifty percent favour plane perpendicular to the field. Now the green's function has a very simple form $< G_0(E - Kc) >= 4e^{-\phi}$ (note that energy is normalized with M). This cubic equation either have all three real solutions in which case density of states is zero or one real two complex roots then nonzero density of states. Moreover, the roots can be solved analytically, but the solution is very cumbersome. One can gain more insight if we use rescaling expansion method [12] when $K \to 0$. Substituting $z \to \delta^a y$ in Eq.(8) we get

$$\delta^{3a} y^3 + \delta^{2a} y^2 - \delta^{1+a} y + \delta = 0, \tag{10}$$

where $\delta = (4K)^2$. If $a = \frac{1}{2}$ the quadratic term and constant term can be balanced, we get $\delta(\delta^{\frac{1}{2}}(y^3 - y) + y^2 + 1) = 0$. The first two terms are negligible compare to the rest of the terms. The solution is $z = -i\delta^{\frac{1}{2}} = -i4K$ and using Eq.(8) the coherent-anisotropy Kc can be obtained

$$Kc = -\frac{1}{2} - \frac{1}{4}[-1 - i4K - (1 - i4K + (-i4K)^2 - (-i4K)^3 + \ldots] \ . \tag{11}$$

The shift for the uniform mode is proportional to $\Re Kc = -4K^2$ and the damping is proportional to $\Im Kc = 16K^3$ to the lowest order of K. The shifts obtained by perturbation

expansion and for the random anisotropic axis problem are found to be proportional also to negative K^2 and perturbation expansion also predicts that damping is proportional to higher power of K than 2 [9]. So as the strength of the anisotropy interaction increases, uniform mode shifts downward and damped. The dynamic structure factor is a Lorentzian

$$S(E, Q = 0) = \Im < G_0(E, 0) >= \frac{\Im Kc}{(E - \Re Kc)^2 + \Im Kc^2}, \tag{12}$$

where $\Im Kc$ is the inverse lifetime τ.

As already pointed out, the equation of motion is equivalent to that of tight binding model so we can use the results found for the electronic disorder. It was found for the electronic case that if the strength of the anisotropy strong, greater than the bandwidth 6M, all the states are localized. If the strength is smaller than the bandwidth, the spectrum consists of extended and localized modes. The localization of a mode means that mode loses its phase coherence over a distance. A localized states have a dramatic effect on the physical properties of the system. In electronic case, for example, no electrical conductivity occurs. The important question is that what is the energy level that separates localized states from extended states i.e., so called mobility edge. For weak anisotropy strength (weak disorder limit), the mobility edge obtained when mean free path comparable to wavelength [6] $\frac{V_g}{\gamma} = \lambda(E)$ (another way of stating Ioffe-Regel Rule) that we get

$$E_{mobility} = \Re Kc + \pi \Im Kc. \tag{13}$$

If wavelength is less than the mean free path the states are extended; otherwise, the states are localized.

4. Summary and Discussion

Spinwaves and their localization properties are studied in the low-energy regime for a Heisenberg system with random-strength anisotropic field in the high-field limit. The high-field limit enables us to use analytical tools developed for disordered systems the CPA or for our case the relevant version the CAFA. Low-field or zero-field limits of the system in question has highly degenerate ground state making it difficult to handle analytical means. The system we study is indeed similar to that random alloy problem or electronic disorder thus enabling us to use various results developed for the disordered systems. We have found that fluctuations either in exchange or random anisotropy leads to localized states. We investigated the localization parameters using Ioffe-Regel restriction. As a result of the disorder, the excitations are shifted towards lower energy levels and they are damped. The shift and the damping are proportional to real an imaginary part of the coherent-anisotropy-field. The system we study here contains very rich physics. For example one check the dilute limit where $\lambda = 0$ and for asymmetric concentrations to see the behaviour of the shifts and damping. It would be interesting to compare our results with numerical simulation results which we will deal with next.

Acknowledgements Systems

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