Tr. J. of Physics
22 (1998) , 811 – 813.

Delay Time Fluctuations of the Tunnel Josephson Junction in the Case of Linearly Growing Current

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Received 11.04.1997

Abstract

Delay time thermal fluctuations intensity of the tunnel Josephson junction for the case of high current rate is calculated.

1. Introduction

In [1], the influence of thermal fluctuations on $S \to R$ switching (switching from supercurrent branch of the I-V curve to the resistive branch) dynamics of the tunnel Josephson junctions (JJ) for the case of linearly growing current is studied. In [1], the fluctuation intensity of the delay time is calculated in detail for the case of low current rate. However, the case of high current rate has only been semiqualitatively investigated in [2]. More detailed calculation of the intensity of fluctuations of the delay time for $S \to R$ switching of junctions with $\beta >> 1$ (β is the so-called McCumber parameter, $\beta = 2\pi I_c R_N^2 C/\Phi_0$, where I_c is critical current of JJ, C is the capacity of the JJ, R_N is the resistivity of JJ and Φ_0 is the quantum of the magnetic flux) is necessary for understanding the influence of delay time fluctuations on fundamental limits of the time resolution of data samplers on tunnel JJ [3]. Taking into consideration these factors, it becomes also highly important to calculate the intensity of fluctuations of the delay time for the case of high current rate.

2. Fluctuations of the Delay Time

The dynamics of the tunnel JJ is described by the equation [2]

$$\beta\ddot{\phi} + \dot{\phi} + \sin\phi = i + i_j, \tag{1}$$

where we have dimensionless variables: ϕ is the phase of JJ in units Φ_0; β is the McCumber parameter; i is the current in units I_C and dots over ϕ stand for the differentiation of ϕ

over time τ, where τ is the time in units $\Phi_0/2\pi I_c R_N$. Fluctuation of current i_f obeys the following equations:

$$\langle i_f \rangle = 0, \quad \langle i_f i_{f\tau} \rangle = 2\gamma\, \delta(\tau), \tag{2}$$

where $\langle \cdots \rangle$ is the averaging over the statical ensemble, and γ is the relative intensity of the thermal fluctuations $\gamma = 2ekT/\hbar I_C$.

Asymptotic solutions to Eq. (1) without fluctuation for the case of linearly growing current $i = \alpha\tau$ (where $\alpha = (dI/dt)\Phi_0/2\pi I_c R_N$ is the dimensionless rate of current increase via JJ) are presented in [2]:

$$\tilde{\phi} = \begin{cases} -(-2\alpha\tilde{\tau})^{1/2} & \text{at} \quad \tilde{\phi} \to -\infty & (3a) \\ u_0(\tilde{\tau} - \tau_0) & \text{at} \quad \tilde{\phi} \cong 0 & (3b) \\ 12\beta/(\tilde{\tau} - \tau_D)^2 & \text{at} \quad \tilde{\phi} \to \infty & (3c) \end{cases}$$

where $\tilde{\phi} = \phi - \pi/2$, $\tilde{\tau} = \tau - \alpha^{-1}$, $u_0 = 1.64\alpha^{3/5}/\beta^{1/5}$, and $\tau_0 = 0.95/\alpha\beta^{1/2}$.

Mean delay time is given by the formula

$$\tau_D = 4.64\beta^{2/5}/\alpha^{1/5}. \tag{4}$$

If the current growing rate satisfies the condition

$$\alpha >> \gamma^{2/3}/\beta, \tag{5}$$

then in order to find the delay time fluctuations intensity, we linearize equation (1) with respect to small fluctuations $\delta\tilde{\phi}$:

$$\beta\delta\ddot{\phi} + \delta\dot{\phi} - \tilde{\phi}(\tilde{\tau})\delta\tilde{\phi} = i_f. \tag{6}$$

Solution of this equation is of the form:

$$\delta\tilde{\phi}(\tilde{\tau}) = \frac{1}{\beta}\int_{-\infty}^{\tilde{\tau}} K(\tilde{\tau}, \tilde{\tau}')\, i_f(\tilde{\tau}')\, d\tilde{\tau}', \tag{7}$$

where kernel $K(\tilde{\tau}, \tilde{\tau}')$ is defined from equation (6) with $\delta(\tilde{\tau}-\tilde{\tau}')$ ($\delta(\cdot)$ is the Dirac function) on the right side. The variation of the delay time τ_D is proportional to the $\delta\tilde{\phi}$ as $\tilde{\tau} \to \tau_D$. Using this argument we can write for the intensity of fluctuations of the delay time the following expression:

$$\sigma^2(\tau_D) = B^2\beta^{-2}\int_{-\infty}^{\tilde{\tau}} d\tilde{\tau}' \int_{-\infty}^{\tilde{\tau}'} d\tilde{\tau}'' K(\tilde{\tau}, \tilde{\tau}') K(\tilde{\tau}, \tilde{\tau}'') \langle i_f i_{f\tau} \rangle \tag{8}$$

where B is the coefficient of proportionality between $\delta\tilde{\phi}$ and $\delta\tau_D$, which is defined from (3c) by differentiation over $\tilde{\tau}$ at $\tilde{\tau} \to \tau_D$. In the stage of inertial motion (3b) kernel $K(\tilde{\tau}, \tilde{\tau}')$ has a structure:

$$K(\tilde{\tau}, \tilde{\tau}') = \pi Bi(G^{1/3}\tilde{\tau}) Ai(G^{1/3}\tilde{\tau}) \tag{9}$$

where $Ai(\cdot), Bi(\cdot)$ are the Airy functions; $G = 1.14\alpha^{3/5}\beta^{-6/5}$. Substituting (9) into (8) and also using asymptotic solution (3a) in quasistatical stage, we have the following formula for the $\sigma^2(\tau_D)$:

$$\sigma^2(\tau_D) = D_0\gamma\ \alpha^{-5/4}\beta^{-3/4}, \tag{10}$$

where coefficient D_0 is defined from (3b) and from the asymptotical expression of the function Airy $Bi(G^{1/3}\tilde{\tau})$ [4] at $\tilde{\tau} \to \tau_D$ and equals 0.03. Formula (10) differs from the formula presented in [2] and is more accurate.

3. Conclusion

Thus, in this paper a quantitative analysis of the influence of the thermal fluctuations on tunnel JJ delay time for the case of high current rate has been conducted.

References

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