Tr. J. of Physics
22 (1998) , 667 – 673.

International Workshop on Linac-Ring
Type *ep* and γp Colliders

Tune Shift Limitations for Linac-Ring Type Colliders

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Abstract

Limitations on incoherent and beam-beam tune shift values are calculated by using gaussian beam distribution for linac-ring type *ep* colliders. The advantage of this machine is achievement of multi TeV center of mass energies in lepton-hadron interactions at sufficiently high luminosities. Tune shift puts a limit on the number of electrons per effective area for the stability of proton beam and, therefore, gives an upper bound for the luminosity.

1. Introduction

Recently, new accelerator complexes, to provide collisions of different beams with TeV scale energy have been constructed and new ones have been in process of planning in order to conduct necessary experiments to investigate the properties of known and predicted new particles in elementary particle physics. One of the four known ways to TeV scale is the linac-ring type *ep* collider. In this collider, electrons coming from linear accelerator collide with protons accelerated in a circular orbit. The advantages of linac-ring type collider are to achieve multi TeV center of mass energy ($\sqrt{s}$), high luminosity (L) and better kinematics. However in this collider the stability of proton beam, the energy loss of electron beam (δ) and the transverse betatron oscillations must be kept under control. It is another important fact that these colliders will form the basis of a new type high energy gamma-proton (as well as e-nucleus and γ-nucleus) colliders.

In the literature there are some important studies dealing with parameters and physics goals of linac-ring type *ep* colliders [1-8]. High energy linac-ring type e^+e^- colliders has been studied in detail for some particle factories [7-9] to make precision measurements.

2. Machines and Parameters

In this study, I have investigated tune shift values for different combinations of some existing and proposed ring type proton accelerators with proposed linear electron accelerators. As proton accelerators, HERA (Hadron Elektronen Ring Anlage at DESY), LHC

(Large Hadron Collider at CERN) and FNAL (Fermi National Accelerator Laboratory, USA) have been considered. For linear (Linac) electron accelerator proposals TESLA (TeV Energy Superconducting Linear Accelerator), CLIC (Compact LInear Collider), SBLC (S-Band Linear Collider), NLC (Next Linear Collider) and special e^--Linac have been considered. Table 1 and 2 show some important parameters of ring type colliders and linear accelerators, respectively. In Table 2, $\epsilon = \sqrt{\epsilon_x \epsilon_y}$ is used for emittance. Table 3 presents the values of three important machine parameters (namely $\sqrt{s}, L$ and δ) for some linac-ring type colliders. One see that center of mass energies ranges from 1 to 7 TeV. Luminosity values are sufficient to investigate related physics [2,3]. Energy loss of electron beam is also very low for this type colliders.

The luminosity of ep collisions is given by

$$L = \frac{n_e n_p}{s_{eff}} f_e \tag{1}$$

where s_{eff} is bigger of s_p and s_e. Center of mass energy is given by

$$\sqrt{s} = 2\sqrt{E_e E_p} \tag{2}$$

Electrons oscillate around the beam axis because of the electromagnetic field of the proton bunch. This causes to the bremsstrahlung of electrons. The fractional energy loss is given by [5]

Using the values of corresponding parameters from Tables 1,2 and Eqns. (1),(3) one can easly obtain the results presented in Table 3.

$$\delta = \frac{\Delta E_e}{E_e} = (\frac{n_p}{10^{12}})^2 \frac{cm}{\sigma_z} \frac{(\mu m)^2}{\sigma_x \sigma_y} \frac{E_e}{TeV} \tag{3}$$

Table 1. Parameters of Ring Type Colliders.

Machines	HERA	LHC	FNAL
Max. beam energy, E_p (TeV)	0.82	7.7	1
Particles per bunch, n_p (10^{12})	1	1	1
Bunches per ring, k_p	172	800	10
Emittance, ϵ_t $(10^{-9}\ \pi\ rad\ m)$	1.14	0.12	0.94
γ factor, $\gamma = E_p\ /m_p c^2$	875	8210	1066
Circumference, L (km)	6.336	26.654	6.28
Transverse area, s_p $(10^{-6} cm^2)$	14.3	1.5	11.8
Beam diameter, $\sigma_{x,y} (\mu m)$	10.7	3.5	9.7

Table 2. Parameters of Linac Type Accelerators.

Machines ($E_{c.m.}(GeV)$)	$E_e(GeV)$	$n_e(10^{10})$	ϵ_N^e ($10^{-6} rad\ m$)	$f_{rep}(Hz)$	n_b/pulse
TESLA (500)	250	3.63	1.87	5	1130
TESLA(800)	400	3.63	1.45	3	1130
TESLA(1600)	800	1.8	0.54	3	2260
SBLC(500)	250	1.1	1.1	50	333
SBLC(725)	362.5	1.2	0.71	50	125
SBLC(1000)	500	1.69	0.71	50	125
CLIC(500)	250	0.8	0.67	2530	1-10
CLIC(1000)	500	0.8	0.88	4000	1-10
NLC(1000)	500	0.65	1.0	180	90
e^--Linac	300	2	$2.2 \cdot 10^2$	10	6410

Table 3. Center of mass energies, luminosities and energy losses for e^- beams for some linac-ring type ep colliders.

$Machine$	$\sqrt{s}(TeV)$	$L(cm^{-2}s^{-1})$	δ
HERA+SBLC(500)	1.28	$1.3.10^{31}$	$5.1.10^{-4}$
HERA+SBLC(725)	1.09	$5.2.10^{30}$	$3.7.10^{-4}$
HERA+SBLC(1000)	1.81	$7.4.10^{30}$	$1.0.10^{-3}$
HERA+TESLA(500)	1.15	$1.4.10^{31}$	$4.1.10^{-4}$
HERA+TESLA(800)	1.62	$8.6.10^{30}$	$8.2.10^{-4}$
HERA+TESLA(1600)	2.30	$8.5.10^{30}$	$1.6.10^{-3}$
HERA+e-Linac(300)	0.99	$9.0.10^{31}$	$3.0.10^{-4}$
LHC+CLIC(500)	3.74	$1.3.10^{32}$	$5.4.10^{-3}$
LHC+CLIC(1000)	5.29	$2.1.10^{32}$	$1.1.10^{-2}$
LHC+TESLA(500)	3.35	$1.4.10^{32}$	$4.3.10^{-3}$
LHC+TESLA(800)	4.73	$8.2.10^{31}$	$8.5.10^{-3}$
LHC+TESLA(1600)	6.69	$8.1.10^{31}$	$1.7.10^{-2}$
LHC+e-Linac(300)	2.90	$8.5.10^{32}$	$3.2.10^{-3}$

3. Tune Shift Calculations

As it is well known, tune is the number of synchrotron oscillations per revolution. For circular accelerators tune values for each revolution must remain practically unchanged for maximum energy gain in each passage from resonant cavity. Tune shift is the shift value of tune number for each revolution. The main goal of this study will be calculation of tune shift values for new type (Linac-Ring) colliders in TeV energy scale. There are two main sources of these tune shift: density of a beam (intra-beam tune shift) and interaction of beam with the fields of other beam (beam-beam tune shift). The tune shift due to density is also known as space charge or incoherent tune shift. The tune shift due to beam-beam interaction is known as beam-beam tune shift. In this section, I investigate these two types of tune shifts for linac-ring type *ep* collider proposals.

3.1. Intra beam dynamics and space charge (incoherent) tune shift

In the case of a beam which is Gaussian in both transverse coordinates with standart deviations $\sigma_x = \sigma_y = \sigma$ and the density is independent of the longitudinal coordinate, the electric and magnetic fields are [10]

$$E = \frac{eN}{2\pi\epsilon_o r}(1 - e^{-r^2/2\sigma^2}), \qquad B = \frac{eNv}{2\pi\epsilon_o r c^2}(1 - e^{-r^2/2\sigma^2}) \tag{4}$$

where v is the speed of the particles, r is the transverse displacement, c is the velocity of light, ϵ_o is the electrical conductivity of empty space and e is the charge of electron. Because the Coulomb repulsion is in the opposite direction to the magnetic attraction, the net outward force is

$$F = \frac{e^2N}{2\pi\epsilon_o\gamma^2 r}(1 - e^{-r^2/2\sigma^2}). \tag{5}$$

where γ is the Lorentz factor. Obviously, for large r (large compared with σ), the force varies inversely with r as one would expect. On the other hand, the force at values of r small compared with σ is

$$F = \frac{e^2N}{4\pi\epsilon_o\gamma^2\sigma^2}r \tag{6}$$

and so varies linearly with transverse displacement. This suggest that particles with small oscillation amplitudes will experience a force similar to the focusing forces of beam optics while particles with larger amplitudes will see less of this effect.

Now let us look in more detail at the tune shift in a circular accelerator. The change in the betatron oscillation tune due to a distrubition of gradient errors is [10],

$$\Delta\nu_s = \frac{1}{4\pi}\oint\frac{\beta(s)\Delta B^{'}(s)}{(B\rho)}\beta(s)\Delta K ds. \tag{7}$$

where, $\beta(s)$ is amplitude function for betatron oscillations, $B\rho$ is the rigidity for beam trajectory and $\Delta B^{'}$ is gradient of magnetic force. In our case

$$\Delta K = \frac{\Delta B^{'}}{(B\rho)} \rightarrow \frac{F^{'}}{ev(B\rho)} = \frac{F^{'}}{pv}. \tag{8}$$

So, for a round Gaussian beam, and for small displacement compared with σ

$$\Delta\nu_s = \oint\frac{1}{4\pi}\beta(s)\frac{F^{'}(s)ds}{pv} = \frac{e^2}{4\pi\epsilon_o mc^2}\frac{N}{(v/c)^2\gamma^3}\frac{1}{4}\oint\frac{\beta}{\pi\sigma^2}ds = \frac{\pi N r_o R}{2\epsilon_N(v/c)\gamma^2} \tag{9}$$

where R is the average radius of the accelerator and ϵ_N is the normalized emittance. The tune decreases due to the defocusing character of the space charge force. For the linac-ring type colliders space charge tune shift relation takes the form

$$\Delta\nu_s = \frac{\pi r_o n_p k_p}{2\epsilon_N^p \gamma^2} \tag{10}$$

Here, r_o is the classical radius of proton and equals to $1.525.10^{-18}$m. n_p is the number of protons in each bunch for proton beams. k_p is the number of bunches, $\epsilon_N^p = \gamma\epsilon_t$ is the normalized emittance of proton beam. Calculated space charge (incoherent) tune shift values for proton beams are given in the first column of Table 4. The values are sufficently small.

3.2. The beam-beam tune shift

Each time the beams cross each other, the particles in one beam feel the electric and magnetic forces due to the particles in the other beam.

Table 4. Incoherent and beam-beam tune shifts of linac-ring type ep colliders.

Colliders	$\Delta\nu_s$	$\Delta\nu_b$
HERA+TESLA (500)	$1.7\cdot10^{-4}$	$7.4\cdot10^{-3}$
HERA+TESLA (800)	$1.7\cdot10^{-4}$	$9.6\cdot10^{-3}$
HERA+TESLA (1600)	$1.7\cdot10^{-4}$	$1.3\cdot10^{-2}$
HERA+SBLC(500)	$1.7\cdot10^{-4}$	$3.8\cdot10^{-3}$
HERA+SBLC(725)	$1.7\cdot10^{-4}$	$6.5\cdot10^{-3}$
HERA+SBLC(1000)	$1.7\cdot10^{-4}$	$2.6\cdot10^{-3}$
HERA+e^--Linac	$1.7\cdot10^{-4}$	$3.5\cdot10^{-5}$
LHC+TESLA(500)	$9.2\cdot10^{-6}$	$7.4\cdot10^{-3}$
LHC+TESLA(800)	$9.2\cdot10^{-6}$	$9.6\cdot10^{-3}$
LHC+TESLA(1600)	$9.2\cdot10^{-6}$	$1.3\cdot10^{-2}$
LHC+CLIC(500)	$9.2\cdot10^{-6}$	$4.6\cdot10^{-3}$
LHC+CLIC (1000)	$9.2\cdot10^{-6}$	$3.5\cdot10^{-3}$
LHC+e^--Linac	$9.2\cdot10^{-6}$	$2.6\cdot10^{-3}$
FNAL+NLC(1000)	$6.8\cdot10^{-6}$	$2.5\cdot10^{-3}$

In our case we are interested only in tune shift of the proton beam, because electron bunches are used only one time and are removed after collision. For particles undergoing infinitesmall betatron oscillations in a highly relativistic Gaussian beam, the net force is,

$$F = \frac{e^2 N}{2\pi\epsilon_o\sigma^2} r, \tag{11}$$

and the tune shift experienced by the proton is

$$\Delta\nu_b = \frac{1}{4\pi}\frac{1}{pc}\frac{e^2}{2\pi\epsilon_o}\oint\frac{N\beta(s)}{\sigma^2(s)}ds = \frac{r_o}{2\epsilon_N}\cdot\frac{1}{2}\int Nds \tag{12}$$

In this case, the force is felt only while that two bunches are colliding. Only half of the integral over the presumed symmetric distribution is necessary, because the two beams are traveling in opposite directions. In terms of the total number of particles in a bunch (n_e), the beam-beam tune shift per collision is then given by

$$\Delta\nu_b = \frac{n_e r_o}{4\epsilon_N^e}. \tag{13}$$

The calculated $\Delta\nu_b$ values for proposed linac-ring type colliders are given in the second column of Table 4. The values are for one crossing. A tune shift value for lepton-lepton or lepton-hadron collision can be reach to big values (for example 0.06) but for hadron-hadron collisions tune shifts must be smaller than 0.003. Our results for beam-beam tune shift are acceptible in respect to these upper limits.

4. Results and Conclusion

In this paper, tune shift values are calculated per revolution for some new TeV energy linac-ring type collider proposals. The space charge tune shifts have been found to be $\leq 10^{-4}$ which seems quite acceptable. It is known that in hadron colliders beam-beam tune shift should be less than 0.003, but in our case each proton bunch collides with electron bunches one time per revolution. Therefore, larger values of $\Delta\nu_b$ may be allowed. Moreover, normalized emittances of electron beams in *ep* regime can be choosen larger than for e^+e^- colliders, because of the transverse sizes of proton beam. This will result essentially in the decreasing of $\Delta\nu_b$ in respect to those given in last column of Table 4.

Finally the main restrictions for new machines will come from emittance growth due to intrabeam scattering [11].

Author is grateful to R. Brinkmann, A.K. Ciftci, D.A. Edwards, S. Sultansoy, D. Trines, B. Wiik and F. Willeke for fruitful discussions.

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