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A possibility to control the polarization of high-energy photons by means of a laser beam

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Abstract

The elastic light-light scattering below the threshold of the e^+e^- pair production leads to a variation in polarization of hard γ-quanta traversing without loss a region where the laser light is focused. This effect can be used to control the γ-quantum polarization. Equations are obtained which determine the variation of Stokes parameters of γ-quanta in this case, and their solutions are given. It is pointed out that this effect can be observed in the experiment E-144 at SLAC. It should be taken into account and, perhaps, it can be used in experiments at future $\gamma\gamma$ colliders.

1. Introduction

Optics of high-energy γ-quanta ($\hbar\omega \gtrsim 100$ GeV) in matter is determined mainly by process of the e^+e^- pair production. Traversing a crystal, such γ-quanta can essentially vary their polarization accompanying by considerable losses in intensity. These problems including applications in high-energy physics are considered in a number of papers (see, for example, [1], [2] and literature therein).

It is well known that a region with an electromagnetic field can also be regarded as anisotropic medium (see [3] §129-130 and [4]). A possibility to consider a bunch of laser photons as a "crystal" is pointed out, for example, in Ref. [2], but the concrete calculations are not given.

In the present paper we study in detail properties of such a "crystal" considering head-on collision of hard γ-quanta with a bunch of polarized laser photons. For the energy of γ-quanta below the threshold of the e^+e^- pair production

$$\hbar\omega < \hbar\omega_{\rm th} = \frac{m_e^2 c^4}{\hbar\omega_L} = \frac{260 \text{ GeV}}{(\hbar\omega_L/\text{eV})} \tag{1}$$

(here $\hbar\omega_L$ is the laser photon energy and m_e is the electron mass) the main interaction is the elastic light-light scattering $\gamma\gamma_L \to \gamma\gamma_L$. Cross section of this process $\lesssim \alpha^2 r_e^2$

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is approximately by 5 order of magnitude less then a typical cross section for the pair production $\sim \pi r_e^2$, where $\alpha = e^2/(\hbar c) = 1/137$ and $r_e = e^2/(m_e c^2)$ is the classical electron radius. Therefore, the laser bunch is practically transparent for such γ-quanta. On the other hand, the variation in polarization for the γ-quantum traversing the bunch is determined by the interference of the incoming wave and the wave scattered at zero angle. In other words, for such a variation it is responsible not the cross section (which is proportional to **square of the light-light scattering amplitude** of the order of α^4), but **the scattering amplitude itself** $\sim \alpha^2$. As a result, in this case the essential variation in the γ-quantum polarization can occur practically without loss in intensity of γ-quanta.

This effect can be interesting for the following reasons:

(i) It can be used to control the polarization of hard γ-quanta without loss in their intensity. In particular, with its help it is possible to transform the circular polarization into the linear one or the linear polarization into the circular one, and it is possible to rotate the direction of the linear polarization.

(ii) The experimental observation of variation in the γ-quantum polarization at passage through the bunch of polarized laser photons below the threshold of the e^+e^- pair production will be indirect observation the process of the elastic light-light scattering. The conditions close to those necessary for observation of such effect is realized now at SLAC in E-144 experiment [5].

(iii) The discussed problem is also actual in connection with projects of $\gamma\gamma$ colliders which under development now (see Refs. [6], [7], [8]). In these projects it is suggested to obtain the required high-energy γ-quanta by backward Compton scattering of laser light on the electron beam of a linear collider. The planned almost the whole conversion $e \to \gamma$ will take place under condition that an electron travels in a laser "target" an optical thickness t of the order of one:

$$t \sim \pi\, r_e^2\, n_L\, l_L \;\sim\; 1,$$

where n_L is the concentration of the laser photons in the bunch and l_L is its length. The γ-quantum obtained inside the bunch will travel further through the same bunch varying its polarization. We will show that this variation is determined approximately by the same parameter t and it may be quite essential. Therefore, such a variation in polarization should be, generally speaking, taken into account at simulations of the $e \to \gamma$ conversion process performed just now for such colliders (see, for example, [8]). Besides, if one adds in the scheme of the $\gamma\gamma$ collider the laser flash of the defined polarization, one can exert control over the γ-quantum polarization.

2. Equations for Stokes parameters of the γ-quantum

Let us consider the head-on collision of γ-quanta with the bunch of laser photons. We choose the z axis along the momenta of γ-quanta. The polarization state of γ-quantum is described by Stokes parameters $\xi_{1,2,3}$, among them ξ_2 is the degree of the circular polarization (which is equal to the mean γ-quantum helicity) and $\sqrt{\xi_1^2+\xi_3^2}$ is the degree of the linear polarization. In the helicity basis ($\lambda,\ \lambda'=\pm1$) the density matrix of γ-quantum has the form (see, for example, Ref. [3] §8):

$$\rho^{\gamma}_{\lambda\lambda'}=\frac{1}{2}\begin{pmatrix}1+\xi_2 & -\xi_3+i\xi_1\\ -\xi_3-i\xi_1 & 1-\xi_2\end{pmatrix}. \qquad (2)$$

For the laser photon such a matrix is described by the following parameters: the degree of the circular polarization P_c, the degree of the linear polarization P_l and the direction of the linear polarization. Let us choose the x axis along this direction [1], then

$$\rho^{L}_{\lambda\lambda'}=\frac{1}{2}\begin{pmatrix}1+P_c & -P_l\\ -P_l & 1-P_c\end{pmatrix}. \qquad (3)$$

We will also use a compact expression describing the polarization of both photons

$$\rho_{\Lambda\Lambda'}=\rho^{\gamma}_{\lambda_1\lambda_1'}\,\rho^{L}_{\lambda_2\lambda_2'}\,. \qquad (4)$$

We will obtain further equations for Stokes parameters ξ_i of γ-quantum traversing a laser bunch. As is well known the variations in intensity and polarization of the wave passing through a medium are due to interference it with the wave scattered at zero angle. Let the incoming wave has the form

$$A_\Lambda\,\mathrm{e}^{ikz}\,.$$

Here the amplitude A_Λ describes the polarization state of the γ-quantum and the laser photon, the wave vector $k=\omega/c$ (the frequency of laser photon $\omega_L\ll\omega$). When the wave passes through a "target" layer of a thickness dz it is appeared the forward scattered wave

$$f_{\Lambda\Lambda'}\,A_{\Lambda'}\,2n_L\,dz\int\frac{\mathrm{e}^{ikr}}{r}\,dx\,dy=\frac{2\pi\,i}{k}\,f_{\Lambda\Lambda'}\,A_{\Lambda'}\,2n_L\,dz\,\mathrm{e}^{ikz}\;=\mathrm{e}^{ikz}\,dA_\Lambda\,, \qquad (5)$$

where $f_{\Lambda\Lambda'}$ is the forward amplitude for the process of elastic scattering light by light. The factor 2 in front of n_L is due to relative motion of the γ-quanta and the "target".

The matrix $\rho_{\Lambda\Lambda'}$ from Eq. (4) is expressed trough the product of A_Λ:

$$\rho_{\Lambda\Lambda'}=\frac{\langle A_\Lambda A^*_{\Lambda'}\rangle}{N}\,,\qquad N=\langle A_\Lambda A^*_\Lambda\rangle\,, \qquad (6)$$

where $\langle ...\rangle$ denotes a statistical averaging. The quantity N is proportional to the γ-quantum intensity J. When the wave passes through the layer of a thickness dz its relative variation in intensity is equal to

$$\frac{dJ}{J}=\frac{dN}{N}=\frac{2}{N}\,\mathrm{Re}\,\langle(dA_\Lambda)\,A^*_\Lambda\rangle\;=-\frac{4\pi}{k}\,\mathrm{Im}\,(f_{\Lambda\Lambda'}\,\rho_{\Lambda'\Lambda})\,2n_L\,dz\,. \qquad (7)$$

[1] We restrict ourselves to the case when the direction of the linear polarization is constant inside the laser flash. More general case corresponds only to a little more cumbersome expressions.

If we introduce the total cross section for the light-light scattering

$$\sigma_{\gamma\gamma} = \frac{4\pi}{k}\,\mathrm{Im}\,(f_{\Lambda\Lambda'}\,\rho_{\Lambda'\Lambda})\,, \tag{8}$$

then the Eq. (7) can be presented in the form

$$dJ = -\,\sigma_{\gamma\gamma}\,2n_L dz\,J\,. \tag{9}$$

Analogously,

$$d\rho_{\Lambda\Lambda'} = d\,\frac{\langle A_\Lambda A^*_{\Lambda'}\rangle}{N} = \frac{2\pi\,i}{k}\,(f_{\Lambda\Lambda''}\rho_{\Lambda''\Lambda'} - f^*_{\Lambda'\Lambda''}\rho_{\Lambda\Lambda''})\,2n_L dz - \rho_{\Lambda\Lambda'}\,\frac{dN}{N}\,. \tag{10}$$

Instead of the scattering amplitudes $f_{\Lambda\Lambda'}$ it is convenient to use the invariant scattering amplitudes $M_{\lambda_1\lambda_2\,\lambda_1'\lambda_2'}$ defined in Ref. [3] §127

$$f_{\Lambda\Lambda'} \equiv f_{\lambda_1\lambda_2\,\lambda_1'\lambda_2'} = \frac{k}{4\pi}\,\frac{(\hbar c)^2}{s}\,M_{\lambda_1\lambda_2\,\lambda_1'\lambda_2'}\,,\quad s = 4\,\hbar\omega\,\hbar\omega_L\,, \tag{11}$$

where quantity s is the square of the total energy of the γ-quantum and the laser photon in their centre-of-mass system. Among the five independent helicity amplitude only three ones are not equal to zero for the forward scattering, namely

$$M_{++++} = M_{----}\,,\ \ M_{+-+-} = M_{-+-+}\,,\ \ M_{++--} = M_{--++}\,.$$

We will use further the following real quantities R and I proportional correspondingly to the real and imaginary parts of the scattering amplitudes divided by $\alpha^2 s$:

$$I_{np} = \frac{(\hbar c)^2}{s}\,\frac{\mathrm{Im}\,(M_{++++}\,+\,M_{+-+-})}{2\pi r_e^2},$$

$$R_c + iI_c = \frac{(\hbar c)^2}{s}\,\frac{M_{++++}\,-\,M_{+-+-}}{2\pi r_e^2},\qquad R_l + iI_l = \frac{(\hbar c)^2}{s}\,\frac{M_{++--}}{2\pi r_e^2}\,. \tag{12}$$

By substituting Eqs. (4), (11), (12) into Eqs. (7), (8), (10) we shall obtain the expression for the cross section

$$\sigma_{\gamma\gamma} = \pi r_e^2\,(I_{np} + \xi_2 P_c\,I_c + \xi_3 P_l\,I_l) \tag{13}$$

and equations for Stokes parameters. To write down these equations it is convenient to introduce the quantity

$$dt = 2\pi r_e^2\,n_L dz \tag{14}$$

which we will call **the reduced optical thickness** of the layer dz. Then

$$\frac{d\xi_1}{dt} = (-R_c\xi_3 + I_c\xi_1\xi_2)\,P_c\,+\,(R_l\xi_2 + I_l\xi_1\xi_3)\,P_l\,,$$

$$\frac{d\xi_2}{dt} = -I_c(1-\xi_2^2)\,P_c + (-R_l\xi_1 + I_l\xi_2\xi_3)\,P_l\,, \qquad (15)$$
$$\frac{d\xi_3}{dt} = (R_c\xi_1 + I_c\xi_2\xi_3)\,P_c - I_l(1-\xi_3^2)\,P_l\,.$$

Integrating these equations one can obtain the dependence of Stokes parameters on the reduced optical thickness t. After that the dependencies of cross section (13) and then the intensity (9) on t are determined.

The physical meaning of different items in the cross section (13) can be easily established if one considers the collisions of photons in pure quantum states. Let σ_0 and σ_2 denote the cross sections for collisions of photons with the total angular momentum $J_z = \xi_2 - P_c$ equals 0 and 2, and $\sigma_\parallel$ and $\sigma_\perp$ denote the cross sections for collisions of photons with parallel ($\xi_3 = P_l = 1$) and orthogonal ($\xi_3 = -P_l = 1$) linear polarizations. Then the quantity I_{np} corresponds to the cross section for nonpolarized photons

$$\sigma_{np} = \pi r_e^2\, I_{np} = \frac{1}{2}(\sigma_0 + \sigma_2) = \frac{1}{2}(\sigma_\parallel + \sigma_\perp)\,, \qquad (16)$$

and quantities I_c and I_l correspond to asymmetries for the circular and linear polarization respectively

$$\pi r_e^2\, I_c = \frac{1}{2}(\sigma_0 - \sigma_2)\,, \quad \pi r_e^2\, I_l = \frac{1}{2}(\sigma_\parallel - \sigma_\perp)\,. \qquad (17)$$

The forward scattering amplitudes (and, therefore, quantities R and I) depend on the single variable

$$r = \frac{s}{4m_e^2c^4} = \frac{\omega}{\omega_{\rm th}}\,.$$

Using for amplitudes $M_{\lambda_1\lambda_2\,\lambda_1'\lambda_2'}$ formulas from Refs. [9] and [3] §127 we obtain the following expressions for functions (12):

$$I_{np} = 0 \ \text{ at } r<1\,; \ \ I_{np} = \frac{1}{r}\left[2\left(1+\frac{1}{r}-\frac{1}{2r^2}\right)\cosh^{-1}\sqrt{r} - \left(1+\frac{1}{r}\right)\sqrt{1-\frac{1}{r}}\,\right] \ \text{ at } r>1\,,$$

$$R_c + iI_c = \frac{2}{\pi r}(-3B_- + T_-),\ \ R_l + iI_l = \frac{1}{\pi r}\left(1+\frac{1}{r}B_- + \frac{1}{2r^2}T_+\right)\,, \qquad (18)$$

where

$$B_- = \begin{cases} \sqrt{\dfrac{1}{r}-1}\,\sin^{-1}\sqrt{r} - \sqrt{\dfrac{1}{r}+1}\,\sinh^{-1}\sqrt{r} & \text{at } r<1 \\ \sqrt{1-\dfrac{1}{r}}\cosh^{-1}\sqrt{r} - \sqrt{\dfrac{1}{r}+1}\,\sinh^{-1}\sqrt{r} - i\dfrac{\pi}{2}\sqrt{1-\dfrac{1}{r}} & \text{at } r>1, \end{cases}$$

$$T_\pm = \begin{cases} -(\sin^{-1}\sqrt{r})^2 \pm (\sinh^{-1}\sqrt{r})^2 & \text{at } r<1 \\ -\dfrac{\pi^2}{4} + (\cosh^{-1}\sqrt{r})^2 \pm (\sinh^{-1}\sqrt{r})^2 - i\pi\cosh^{-1}\sqrt{r} & \text{at } r>1\,. \end{cases}$$

Functions R and I are presented in Figs. 1 and 2. Note that extremums of R_c and R_l are at the threshold of the pair production:

$$R_c = 0.315, \quad R_l = -0.348 \quad \text{at } s = 4m_e^2c^4\,. \tag{19}$$

In the region below the threshold these functions decrease very rapidly with decreasing of s:

$$R_c = \frac{64}{315\pi}\,r^2, \quad R_l = -\frac{4}{15\pi}\,r \quad \text{at} \quad r \ll 1. \tag{20}$$

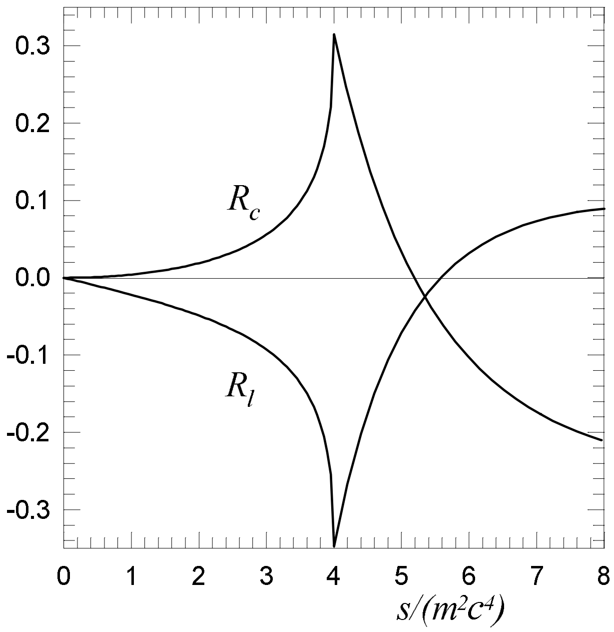

Figure 1. The real parts of the scattering amplitudes for the light-light scattering at zero angle (see Eqs. (12) (18)) in dependence on the parameter $s/(m_e^2c^4) = 4\omega/\omega_{\rm th}$.

3. A laser bunch as a transparent anisotropic medium

A laser bunch becomes transparent for γ-quanta with the energy below the threshold of the pair production $\omega < \omega_{\rm th}$ (see Eq. (1)):

$$I_{np} = I_c = I_l = \sigma_{\gamma\gamma} = 0 \quad \text{at} \quad s < 4m_e^2c^4.$$

If the laser photons are linearly polarized ($P_l \neq 0,\ P_c = 0$) the solution of Eqs. (15) has the form

$$\xi_1 = \ \xi_1^0 \cos\varphi_l + \xi_2^0 \sin\varphi_l\,, \quad \xi_2 = -\xi_1^0 \sin\varphi_l + \xi_2^0 \cos\varphi_l\,, \quad \xi_3 = \ \xi_3^0\,, \tag{21}$$

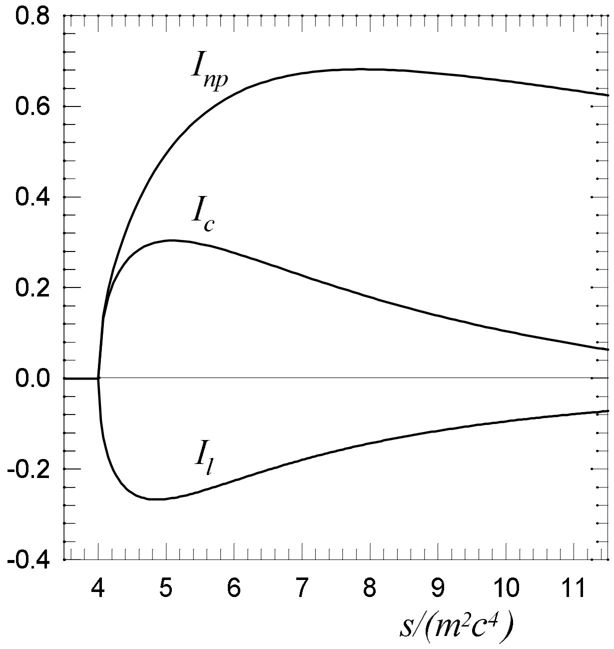

Figure 2. The same for the imaginary parts of amplitudes. The quantity $\pi r_e^2 I_{np}$ is equal to the cross section of the $\gamma\gamma_L \to e^+e^-$ process for nonpolarized particles.

where the phase $\varphi_l = P_l R_l t$ and ξ_i^0 are the initial Stokes parameters. It is seen from this solution that in this case the laser bunch is an anisotropic medium with different refraction indices n_x and n_y along the x and y axes:

$$n_x - n_y = \frac{c}{\omega} 2\pi r_e^2 n_L P_l R_l. \tag{22}$$

Such a medium transforms the circular polarization of γ-quanta into the linear one and vice versa. If, for example, the initial γ-quantum is circularly polarized, $\xi_2^0 \neq 0$, $\xi_1^0 = \xi_3^0 = 0$, its polarization transforms to the linear one, $\xi_1 = -\xi_2^0$, $\xi_2 = \xi_3 = 0$, when the phase φ_l becomes equal to $-\pi/2$.

If the laser photons are circularly polarized ($P_c \neq 0,\ P_l = 0$), the solution of Eqs. (15) has another form

$$\xi_1 = \xi_1^0 \cos\varphi - \xi_3^0 \sin\varphi\,, \quad \xi_2 = \xi_2^0\,, \quad \xi_3 = \xi_1^0 \sin\varphi + \xi_3^0 \cos\varphi\,, \tag{23}$$

where the phase $\varphi = P_c R_c t$. From this solution it is seen that in this case the laser bunch is an gyrotropic medium with different refraction indices n_+ and n_- for the right and left polarized γ-quanta, and

$$n_+ - n_- = \frac{c}{\omega}\, 2\pi r_e^2\, n_L\, P_c\, R_c .$$

Such a medium rotates the direction of the γ-quantum linear polarization on the angle $(-\varphi/2)$.

In a general case

$$\xi_1 = A\cos(\varphi+\varphi_0)\,, \quad \xi_2 = -\frac{P_l\,R_l}{R}\,A\sin(\varphi+\varphi_0) + \frac{P_c R_c}{R}\,B\,, \tag{24}$$

$$\xi_3 = \frac{P_c\,R_c}{R}\,A\sin(\varphi+\varphi_0) + \frac{P_l R_l}{R}\,B\,, \quad R = \sqrt{(P_c\,R_c)^2 + (P_l\,R_l)^2}\,,$$

where the phase $\varphi = Rt$ and constants A, B and φ_0 are determined by the initial conditions. Note that the total degree of the γ-quantum polarization is not changed:

$$\sqrt{\xi_1^2+\xi_2^2+\xi_3^2} = \sqrt{A^2+B^2} = \text{const}\,.$$

4. Variation in polarization above the threshold of the pair production

Above the threshold of the e^+e^- pair production ($\omega > \omega_{\rm th}$) the variation in the γ-quantum polarization is accompanied by a reduction in their intensity in accordance with Eq. (9). In this case the total γ-quantum degree of polarization are not conserved. We give the solutions of Eqs. (15) for two particular cases discussed in the above section.

If the laser photons are linearly polarized ($P_l \neq 0,\ P_c = 0$) then

$$\xi_1 = \left(\xi_1^0 \cos\varphi_l + \xi_2^0 \sin\varphi_l\right)/D_l\,, \quad \xi_2 = \left(-\xi_1^0 \sin\varphi_l + \xi_2^0 \cos\varphi_l\right)/D_l\,, \tag{25}$$

$$\xi_3 = \left(\xi_3^0 {\rm ch}\tau_l - {\rm sh}\tau_l\right)/D_l\,, \quad D_l = {\rm ch}\tau_l - \xi_3^0 {\rm sh}\tau_l\,, \quad \tau_l = P_l\, I_l\, t\,.$$

The similar case (in connection with the problem of passage of γ-quanta through a monocrystal) was considered in detail in Ref. [2]. Note that $\xi_3 \to 1$ at $\tau_l \to -\infty$.

If the laser photons are circularly polarized ($P_c \neq 0,\ P_l = 0$) then

$$\xi_1 = \left(\xi_1^0 \cos\varphi - \xi_3^0 \sin\varphi\right)/D_c\,, \quad \xi_2 = \left(\xi_2^0 {\rm ch}\tau_c - {\rm sh}\tau_c\right)/D_c\,, \tag{26}$$

$$\xi_3 = \left(\xi_1^0 \sin\varphi + \xi_3^0 \cos\varphi\right)/D_c\,, \quad D_c = {\rm ch}\tau_c - \xi_2^0 {\rm sh}\tau_c\,, \quad \tau_c = P_c\, I_c\, t\,.$$

In this case $\xi_2 \to \pm 1$ at $\tau \to \mp\infty$.

5. Discussion

1. As a result, we have shown that below the threshold of the e^+e^- pair production the laser bunch is similar to the transparent anisotropic medium. In particular, the linearly polarized bunch corresponds to the uniaxial crystal and the circularly polarized bunch corresponds to the gyrotropic medium. Let us illustrate the magnitude of the discussed effects using as an example the parameters of the laser bunch given in Ref. [8] (they are close to the parameters which are realized in the experiment E-144 at SLAC [5]): $\hbar\omega_L = 1.18$ eV, the energy of the laser flash is 1 J, the laser bunch length is 1.8 ps, and the peak intensity is about 10^{18} W/cm^2. The reduced optical thickness (14) for this flash is equal to $t = 1.4$. As is seen from Fig. 1 phases $\varphi_l = P_l\, R_l\, t$ and $\varphi = P R\, t$ which determine the magnitude of the effect can reach values $\approx 0.3t \sim 1$. According to Eqs. (21) and (23) it means that the variation in the γ-quantum polarization may be very large. It is also seen from Fig. 1 that the effect depends strongly on the energy and it becomes very small at $\omega \ll \omega_{\rm th}$.

2. With the growth of the intensity of laser flash it is necessary to take into account the effects of intense electromagnetic fields (see Ref. [10] and literature therein) which we neglect in the present paper.

3. In the scheme of the $e \to \gamma$ conversion adopted for the $\gamma\gamma$ colliders, γ-quanta are produced inside the laser bunch. When such γ-quanta travel further in the laser bunch they can essentially vary their polarization. It should be noted, however, that for the optimal conversion the laser photons and the hardiest γ-quanta are circularly polarized [6], [8]. These γ-quanta conserve their polarization on the rest way through the bunch. But the γ-quanta with a lower energy have a linear polarization, and the rotation of the direction of this linear polarization should be, generally speaking, taken into account.

4. The linear polarization of γ-quanta is needed for a number of interesting experiments. For example, in Ref. [11] it is stressed that the best way to determine the CP value of the neutral Higgs boson with the intermediate mass is to use $\gamma\gamma$ collisions with the parallel or perpendicular linear polarizations. However, the $\gamma\gamma$ luminosity for such polarization is considerable less then the luminosity for the circular one. Using the additional laser bunches to transform the circular polarization into the linear one may lead to the luminosity of the $\gamma\gamma$ collision with the linear polarizations close to that for the optimal conditions.

5. We apply the same method to calculate the variation in the polarization of electrons traversing through a bunch of polarized laser photons. The corresponding results will be given in a separate paper.

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