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Resummation of Ir Renormalons in a Single Meson Photoproduction

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Abstract

Single pseudoscalar and vector mesons hard semi-inclusive photoproduction $\gamma h \to MX$ via higher twist mechanism is calculated using the QCD running coupling constant method. It is proved that in the context of this method a higher twist contribution to the photoproduction cross section cannot be normalized in terms of the meson electromagnetic form factor. The structure of infrared renormalon singularities of the higher twist subprocess cross section and the resummed expression (the Borel sum) for it are found. Comparisons are made with earlier results, as well as with leading twist cross section. Phenomenological effects of studied contributions for π, K, ρ-meson photoproduction are discussed.

One of the fundamental achievements of QCD is the prediction of asymptotic scaling laws for large-angle exclusive processes and their calculation in the framework of perturbative QCD (pQCD) [1-3]. In the context of the factorized QCD an expression for an amplitude of an exclusive process can be written as integral over $\mathbf{x}, \mathbf{y}$ of hadron wave functions (w.f.)[1] $\Phi_i(\mathbf{x}, \hat{Q}^2)$ (an initial hadron), $\Phi_f^*(\mathbf{y}, \hat{Q}^2)$ (a final hadron) and amplitude $T_H(\mathbf{x}, \mathbf{y}; \alpha_S(\hat{Q}^2), Q^2)$ of the hard-scattering subprocess [2]. This approach can be applied for investigation, not only exclusive processes but also for the calculation of higher twist (HT) corrections to some inclusive processes, such as large-p_T dilepton production [4], two-jet+meson production in the electron-positron annihilation [5], etc. The HT corrections to a single meson semi-inclusive photoproduction and jet photoproduction cross sections were studied by various authors [6,7]. In these early papers for calculation of integrals over $\mathbf{x}, \mathbf{y}$, which appear in an expression of the amplitude, the frozen coupling constant approximation was used. In our recent work we consider the hard semi-inclusive photoproduction of single pseudoscalar and vector mesons $\gamma h \to MX$ using the running coupling constant method.

The two HT subprocesses, namely $\gamma q_1 \to M q_2$ and $\gamma \overline{q}_2 \to M \overline{q}_1$ contribute to the photoproduction of the single meson M in the reaction $\gamma h \to MX$. The Feynman

[1]Strictly speaking, $\Phi_M(\mathbf{x}, \hat{Q}^2)$ is a hadron distribution amplitude and it differs from a hadron wave function. But in this paper we use these two terms on the same footing.

diagrams for the first subprocess are shown in Figure 1. The momenta and charges of the particles in question are indicated in Figure 1(a). The amplitude for the subprocess $\gamma q_1 \to M q_2$ can be found by means of the Brodsky-Lepage method [2],

$$M = \int_0^1 \int_0^1 dx_1 dx_2 \delta(1 - x_1 - x_2) T_H(x_1, x_2; \alpha_S(\widehat{Q}^2),\ \widehat{s},\ \widehat{u},\ \widehat{t}) \Phi_M(x_1, x_2; \widehat{Q}^2) \qquad (1)$$

In (1), T_H is the sum of graphs contributing to the hard-scattering part of the subprocess, which for the subprocess under consideration is $\gamma + q_1 \to (q_1 \overline{q}_2) + q_2$, where a quark and antiquark from the meson form a color singlet state $(q_1 \overline{q}_2)$.

The important ingredient of our study is the choice of the meson model w.f. Φ_M. In this work we calculate the photoproduction of the pseudoscalar (pion, kaon) and vector (ρ-meson) mesons. For these mesons in the literature [3,8] various w.f. were proposed.

The pion and ρ-meson wave functions have the form

$$\Phi_M(x, \mu_0^2) = \Phi_{asy}^M(x) \left[a + b(2x-1)^2 \right]. \qquad (2)$$

For the model w.f. the coefficients a, b take the following values:
Chernyak-Zhitnitsky w.f. [3];

$$\begin{aligned} a &= 0, b = 5, \quad \textit{for the pion}, \\ a &= 0.7, b = 1.5, \quad \textit{for the longitudinally polarized } \rho_L - \textit{meson}, \\ a &= 1.25, b = -1.25, \quad \textit{for the transversely polarized } \rho_T - \textit{meson}. \end{aligned} \qquad (3)$$

Ball-Braun w.f.[8];

$$a = 0.7, \quad b = 1.5,$$

for both longitudinally and transversely polarized ρ-meson. Here we have denoted by $x \equiv x_1$ the longitudinal fractional momentum carrying by the quark within the meson. Then, $x_2 = 1 - x$ and $x_1 - x_2 = 2x - 1$.

The pion and ρ-meson w.f. are symmetric under replacement $x_1 - x_2 \leftrightarrow x_2 - x_1$. But the kaon w.f. is non-symmetric; $\Phi_K(x_1 - x_2) \neq \Phi_K(x_2 - x_1)$ [3]. Indeed, the kaon w.f. includes a term proportional to odd power of $(2x - 1)$,

$$\begin{aligned} \Phi_K(x, \mu_0^2) &= \Phi_{asy}^K(x) \left[a + b(2x-1)^2 + c(2x-1)^3 \right], \\ a &= 0.4, \quad b = 3, \quad c = 1.25, \end{aligned} \qquad (4)$$

and may be written as the sum of the symmetric $\Phi_s(x, \mu_0^2)$ and antisymmetric $\Phi_a(x, \mu_0^2)$ parts,

$$\Phi_s(x, \mu_0^2) = \Phi_{asy}^K(x) \left[a + b(2x-1)^2 \right], \quad \Phi_a(x, \mu_0^2) = \Phi_{asy}^K(x) c (2x-1)^3. \qquad (5)$$

In (2),(4),(5) $\Phi_{asy}^M(x)$ is the asymptotic w.f.

$$\Phi_{asy}^M(x) = \sqrt{3} f_M x(1-x), \qquad (6)$$

where f_M is the meson decay constant; $f_\pi = 0.093\ GeV, f_K = 0.112\ GeV$. In the case of the ρ-meson we take $f_\rho^L = f_\rho^T = 0.2\ GeV$ for the CZ w.f., and $f_\rho^L = 0.2\ GeV$, $f_\rho^T = 0.16\ GeV$ for BB w.f. The normalization of $\Phi_M(x, \mu_0^2)$ at $\mu_0 = 0.5\ GeV$ is given by the condition

$$\int_0^1 dx \Phi_M(x, \mu_0^2) = \frac{f_M}{2\sqrt{3}}. \tag{7}$$

The factor $\sqrt{2}$ appearing in the normalization of a vector meson is included into the ρ-meson decay constant.

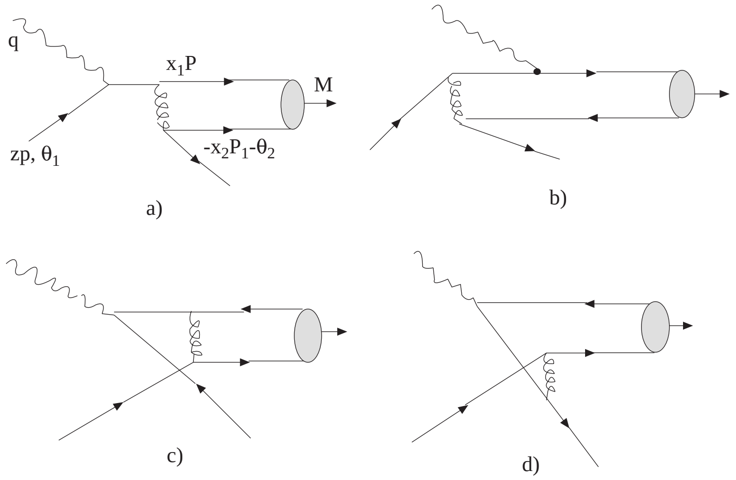


Figure 1. Feynman diagrams contributing to the higher twist subprocess $\gamma q \to Mq$. Here **p** and **P** are the hadron h and meson M four momenta, respectively.

The formalism for calculation of the HT subprocess cross section is well known and described in [6,9]. We omit details of our calculations and write down the final expression for $d\widehat{\sigma}^{HT}/d\widehat{t}$. We find:

for the pseudoscalar and longitudinally polarized vector mesons,

$$\begin{aligned}\frac{d\widehat{\sigma}^{HT}(e_1,e_2)}{d\widehat{t}} &= \frac{32\pi^2 C_F \alpha_E}{9\widehat{s}^2}\left\{-\frac{e_1^2}{\widehat{s}^2}\left[I_1^2\widehat{t} - 2I_1\left(I_1\widehat{s} + I_2\widehat{u}\right)\frac{\widehat{u}}{\widehat{t}} + I_2^2\frac{\widehat{u}^2}{\widehat{t}}\right] - \right. \\ & \frac{e_2^2}{\widehat{u}^2}\left[K_1^2\widehat{t} - 2K_1\left(K_1\widehat{u} + K_2\widehat{s}\right)\frac{\widehat{s}}{\widehat{t}} + K_2^2\frac{\widehat{s}^2}{\widehat{t}}\right] - \\ & \left.\frac{2e_1e_2}{\widehat{s}\widehat{u}\widehat{t}}\left[I_1K_1\widehat{t}^2 - I_1(K_2\widehat{s} + K_1\widehat{u})\widehat{s} - K_1(I_1\widehat{s} + I_2\widehat{u})\widehat{u}\right]\right\}.\end{aligned} \tag{8}$$

for the transversely polarized vector meson,

$$\frac{d\hat{\sigma}^{HT}(e_1,e_2)}{d\hat{t}} = \frac{64\pi^2 C_F \alpha_E}{9\hat{s}^4}\frac{-\hat{t}}{\hat{u}^2}\left[e_1\hat{u}I_2 - e_2\hat{s}K_2\right]^2 \tag{9}$$

In (8),(9), $\alpha_E \simeq 1/137$ is the fine structure constant, $C_F = 4/3$ is the color factor. The Mandelstam invariants for the subprocess are defined as

$$\begin{aligned} \widehat{s} &= (zp+q)^2 = zs, \\ \widehat{t} &= (q-P)^2 = t, \\ \widehat{u} &= (zp-P)^2 = zu, \end{aligned} \tag{10}$$

where s, t, u are the Mandelstam invariants for the process $\gamma h \to MX$, z is the longitudinal fractional momentum of the quark q_1 out of the hadron h.

The main problem in our investigation is the calculation of quantities $I_{1,2}$, $K_{1,2}$,

$$I_1 = \int_0^1 \int_0^1 \frac{dx_1 dx_2 \delta(1-x_1-x_2)\alpha_S(\widehat{Q}_1^2)\Phi_M(x_1,x_2;\widehat{Q}_1^2)}{x_2}, \tag{11}$$

$$I_2 = \int_0^1 \int_0^1 \frac{dx_1 dx_2 \delta(1-x_1-x_2)\alpha_S(\widehat{Q}_1^2)\Phi_M(x_1,x_2;\widehat{Q}_1^2)}{x_1 x_2}, \tag{12}$$

and

$$K_1 = \int_0^1 \int_0^1 \frac{dx_1 dx_2 \delta(1-x_1-x_2)\alpha_S(\widehat{Q}_2^2)\Phi_M(x_1,x_2;\widehat{Q}_2^2)}{x_1}, \tag{13}$$

$$K_2 = \int_0^1 \int_0^1 \frac{dx_1 dx_2 \delta(1-x_1-x_2)\alpha_S(\widehat{Q}_2^2)\Phi_M(x_1,x_2;\widehat{Q}_2^2)}{x_1 x_2}, \tag{14}$$

where for I_1, I_2 the renormalization and factorization scale is $\widehat{Q}_1^2 = x_2 \widehat{s}$, for K_1, K_2 it is given by $\widehat{Q}_2^2 = -x_1 \widehat{u}$.

Here we shall calculate the integrals (11-14) using the running coupling constant method and also discuss the problem of normalization of the higher twist process cross section in terms of the meson electromagnetic form factor obtained in the context of the same approach.

Let us clarify our method by calculating the integral (11); the quantities $I_2, K_{1,2}$ can be worked out in the same way. For the mesons with symmetric w.f. Eq.(11) in the framework of the running coupling approach takes the form

$$I_1(\widehat{s}) = \int_0^1 \frac{\alpha_S((1-x)\widehat{s})\Phi_M(x,\mu_0^2)dx}{1-x}. \tag{15}$$

The $\alpha_S((1-x)\widehat{s})$ has the infrared singularity at $x \to 1$ and as a result integral (15) diverges (the pole associated with the denominator of the integrand is fictitious, because $\Phi_M \sim (1-x)$, and therefore, the singularity of the integrand at $x=1$ is caused only by $\alpha_S((1-x)\widehat{s})$). For the regularization of the integral let us relate the running coupling at scaling variable $\alpha_S((1-x)\hat{s})$ with the aid of the renormalization group equation in terms of the fixed one $\alpha_S(\widehat{s})$. The renormalization group equation for the running coupling $\alpha(\widehat{s}) \equiv \alpha_S(\widehat{s})/\pi$

$$\frac{\partial \alpha(\lambda \widehat{s})}{\partial \ln \lambda} \simeq -\frac{\beta_0}{4} \left[\alpha(\lambda \widehat{s})\right]^2, \tag{16}$$

has the solution

$$\alpha(\lambda \widehat{s}) \simeq \frac{\alpha(\widehat{s})}{1 + (\alpha(\widehat{s})\beta_0/4)\ln \lambda}. \tag{17}$$

In (16),(17), the one-loop QCD coupling constant $\alpha_S(\mu^2)$ is defined as

$$\alpha_S(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}$$

with $\beta_0 = 11 - 2n_f/3$ being the QCD beta-function first coefficient.

Having inserted (17) into (15) we get

$$I_1(\widehat{s}) = \alpha_S(\widehat{s}) \int_0^1 \frac{\Phi_M(x, \mu_0^2)dx}{(1-x)(1+(1/t)\ln(1-x))}, \tag{18}$$

where $t = 4\pi/\alpha_S(\widehat{s})\beta_0$.
The integral (18) is, of course, still divergent, but now it is recasted into a form, which is suitable for calculation. Using the method described in details in our work [10] it may be found as a perturbative series in $\alpha_S(\widehat{s})$

$$I_1(\widehat{s}) \sim \sum_{n=1}^{\infty} \left(\frac{\alpha_S(\widehat{s})}{4\pi}\right)^n S_n, \quad S_n = C_n \beta_0^{n-1}. \tag{19}$$

The coefficients C_n of this series demonstrate factorial growth $C_n \sim (n-1)!$, which might indicate an infrared renormalon nature of divergences in the integral (18) and corresponding series (19). The procedure for dealing with such ill-defined series is well known; one has to perform the Borel transform of the series [11]

$$B[I_1](u) = \sum_{n=1}^{\infty} \frac{u^{n-1}}{(n-1)!} C_n, \tag{20}$$

then invert $B[I_1](u)$ to obtain the resummed expression (the Borel sum) for $I_1(\widehat{s})$. This method is straightforward but tedious. Therefore, it is convenient to apply the second method, used in our work [12], which allows us to bypass all these intermediate steps and find directly the resummed expression for $I_1(\widehat{s})$. For these purposes let us introduce the inverse Laplace transform of $1/(t+z)$

$$\frac{1}{t+z} = \int_0^{\infty} \exp[-(t+z)u]du. \tag{21}$$

Then $I_1(\hat{s})$ may be readily carried out by the change of the variable x to $z = \ln(1-x)$ and using (21)

$$\begin{aligned} I_1(\widehat{s}) \quad &= \quad \frac{4\sqrt{3}\pi f_M}{\beta_0} \int_0^\infty \exp\left[-\frac{4\pi u}{\alpha_S(\widehat{s})\beta_0}\right] \left(\frac{a+b}{1-u} - \frac{a+5b}{2-u} + \right. \\ &\left. \frac{8b}{3-u} - \frac{4b}{4-u}\right). \end{aligned} \tag{22}$$

Eq.(22) is nothing more than the Borel sum of the perturbative series (19) and the corresponding Borel transform is

$$B[I_1](u) = \frac{a+b}{1-u} - \frac{a+5b}{2-u} + \frac{8b}{3-u} - \frac{4b}{4-u}. \tag{23}$$

The Borel transform $B[I_1](u)$ has poles on the real u axis at $u = 1; 2; 3; 4$, which confirms our conclusion concerning the infrared renormalon nature of divergences in (19). To remove them from Eq.(22) some regularization methods have to be applied. In this article we adopt the principal value prescription [13]. We obtain

$$\begin{aligned} \left[I_1\left(\widehat{s}\right)\right]^{res} \quad &= \quad \frac{4\sqrt{3}\pi f_M}{\beta_0} \left[(a+b)\frac{Li(\lambda)}{\lambda} - (a+5b)\frac{Li(\lambda^2)}{\lambda^2} + \right. \\ &\left. 8b\frac{Li(\lambda^3)}{\lambda^3} - 4b\frac{Li(\lambda^4)}{\lambda^4}\right], \end{aligned} \tag{24}$$

where $Li(\lambda)$ is the logarithmic integral [14], for $\lambda > 1$ defined in its principal value

$$Li(\lambda) = P.V. \int_0^\lambda \frac{dx}{\ln x}, \qquad \lambda = \widehat{s}/\Lambda^2. \tag{25}$$

For other integrals from (12-14) we find

$$\left[I_2\left(\widehat{s}\right)\right]^{res} = \frac{4\sqrt{3}\pi f_M}{\beta_0} \left[(a+b)\frac{Li(\lambda)}{\lambda} - 4b\frac{Li(\lambda^2)}{\lambda^2} + 4b\frac{Li(\lambda^3)}{\lambda^3}\right], \tag{26}$$

and

$$\left[K_1\left(-\widehat{u}\right)\right]^{res} = \left[I_1\left(-\widehat{u}\right)\right]^{res}, \left[K_2\left(-\widehat{u}\right)\right]^{res} = \left[I_2\left(-\widehat{u}\right)\right]^{res}. \tag{27}$$

From (24),(26),(27), we conclude that in the framework of the running coupling approximation even for mesons with symmetric w.f. we have

$$\left[I_2\left(\widehat{s}\right)\right]^{res} \nsim \left[I_1\left(\widehat{s}\right)\right]^{res}, \quad \left[K_2\left(-\widehat{u}\right)\right]^{res} \nsim \left[K_1\left(-\widehat{u}\right)\right]^{res}.$$

Another question is the normalization of the meson photoproduction cross section in terms of the meson elm form factor. The pion and kaon form factors have been calculated by means of the running coupling approach in our previous papers [10, 12]. Let us write down the pion form factor obtained using the pion's simplest w.f., that is, the asymptotic one ($a = 1, b = 0$ in (3))

$$\left[Q^2 F_\pi(Q^2)\right]^{res}_{asy} = \frac{(16\pi f_\pi)^2}{\beta_0}\left[-\frac{3}{2} + (\ln\lambda - 2)\frac{Li(\lambda)}{\lambda} + (\ln\lambda + 2)\frac{Li(\lambda^2)}{\lambda^2}\right]. \qquad (28)$$

From (24),(26),(28) it follows that in the running coupling approach the HT subprocess cross section (8) cannot be normalized in terms of the meson form factor for mesons with symmetric w.f.

Let us, for completeness, write down $I(\hat{s}), K(-\hat{u})$ calculated for non-symmetric w.f. (5)

$$\begin{aligned}[I_1(\widehat{s})]^{res} &= \frac{4\sqrt{3}\pi f_M}{\beta_0}\left[(a+b+c)\frac{Li(\lambda)}{\lambda} - (a+5b+7c)\frac{Li(\lambda^2)}{\lambda^2} + \right. \\ &\left. 2(4b+9c)\frac{Li(\lambda^3)}{\lambda^3} - 4(b+5c)\frac{Li(\lambda^4)}{\lambda^4} + 8c\frac{Li(\lambda^5)}{\lambda^5}\right], \end{aligned} \qquad (29)$$

$$\begin{aligned}[I_1(\widehat{s})]^{res} &= \frac{4\sqrt{3}\pi f_M}{\beta_0}\left[(a+b+c)\frac{Li(\lambda)}{\lambda} - 2(2b+3c)\frac{Li(\lambda^2)}{\lambda^2} + \right. \\ &\left. 4(b+3c)\frac{Li(\lambda^3)}{\lambda^3} - 8c\frac{Li(\lambda^4)}{\lambda^4}\right]. \end{aligned} \qquad (30)$$

The expressions for $[K_1(-\widehat{u})]^{res}$ and $[K_2(-\widehat{u})]^{res}$ may be obtained from (29),(30) by $c \to -c, \lambda = \widehat{s}/\Lambda^2 \to -\widehat{u}/\Lambda^2$ replacements, respectively. With these explicit expressions and the results of [12] at hand one can check our statements concerning the normalization of the subprocess cross section for kaons.

Let us write down the HT correction to the single meson photoproduction cross section by taking into account both HT subprocesses; $\gamma q_1 \to M q_2$ and $\gamma \bar{q}_2 \to M \bar{q}_1$. It is not difficult to prove that the second subprocess cross section can be obtained from (8),(9) by $e_1 \leftrightarrow e_2$ replacement. Then the HT correction to the single meson photoproduction cross section is given by

$$\begin{aligned}\frac{\sigma^{HT}}{dp_T^2 dy} &= z^* \sum_{q_1,\bar{q}_2}\left\{q_1^h(z^*,-t)\frac{d\hat{\sigma}^{HT}(e_1,e_2)}{d\hat{t}} + \right. \\ &\left. \bar{q}_2^h(z^*,-t)\frac{d\hat{\sigma}^{HT}(e_2,e_1)}{d\hat{t}}\right\}\frac{s}{s+u}. \end{aligned} \qquad (31)$$

where

$$z^* = \frac{p_T e^{-y}}{\sqrt{s} - p_T e^y}.$$

Here the sum runs over the hadron's quark q_1 and antiquark $\overline{q}_2$ flavors. In (31) $q_1^h(z^*,-t)$, $\overline{q}_2^h(z^*,-t)$ are the quark and antiquark distribution functions, respectively. All r.h.s.

quantities are expressed in terms of the process c.m. energy $\sqrt{s}$, the meson transverse momentum p_T and rapidity y using the following expressions

$$\hat{s} = \frac{sp_T e^{-y}}{\sqrt{s} - p_T e^y}, \quad \hat{t} = -p_T\sqrt{s}e^{-y}, \quad \hat{u} = -\frac{p_T^2\sqrt{s}}{\sqrt{s} - p_T e^y}. \tag{32}$$

Eq.(31) is the final result which will be used later in our numerical calculations.

In our study of the single meson photoproduction a crucial point is the comparison of our results with leading twist (LT) ones. This will enable us to find such domains in the phase space in which the higher twist photoproduction mechanism is actually observable.

The LT subprocesses, which contribute to a meson photoproduction are:
a photon-quark (antiquark) scattering

$$\gamma(q) + q_i(p_1) \to q_i(p_2) + g(p_3), \qquad (\gamma\overline{q}_i \to \overline{q}_i g), \tag{33}$$

and photon-gluon fusion reactions

$$\gamma(q) + g(p_1) \to q_i(p_2) + \overline{q}_i(p_3). \tag{34}$$

In this article we consider the inclusive cross section difference in the photon-proton collision, namely

$$\Delta_M = \frac{d\sigma}{dp_T^2 dy}(\gamma p \to M^+ X) - \frac{d\sigma}{dp_T^2 dy}(\gamma p \to M^- X) \equiv \Sigma_{M^+} - \Sigma_{M^-}. \tag{35}$$

The LT subprocess which dominates in this difference is $\gamma q \to gq$ with $q \to M$. Its cross section at the tree level is well known,

$$\frac{d\hat{\sigma}^{LT}}{d\hat{t}} = -\frac{8\pi\alpha_E e_q^2}{3\hat{s}^2}\left[\alpha_S(\hat{s})\frac{\hat{t}}{\hat{s}} + \alpha_S(-\hat{t})\frac{\hat{s}}{\hat{t}}\right], \tag{36}$$

where the Mandelstam invariants of the subprocess are

$$\hat{s} = (q + p_1)^2, \quad \hat{t} = (q - p_2)^2, \quad \hat{u} = (q - p_3)^2.$$

Then the leading twist contribution to the single meson photoproduction in $\gamma p \to MX$ is given by the expression,

$$\frac{d\sigma^{LT}}{dp_T^2 dy} = \sum_q \int_{x_{min}}^1 \frac{dx q_p(x, -\hat{t}) D_{M/q}(z, -\hat{t})}{z} \frac{d\hat{\sigma}^{LT}}{d\hat{t}}, \tag{37}$$

where

$$z = \frac{p_T e^{-y}}{x\sqrt{s}} + \frac{p_T e^y}{\sqrt{s}}, \quad x_{min} = \frac{p_T e^{-y}}{\sqrt{s} - p_T e^y}. \tag{38}$$

In (37), $q_p(x, -\hat{t})$ and $D_{M/q}(z, -\hat{t})$ are a quark q distribution and fragmentation functions, respectively. The subprocess invariants $\hat{s}$, $\hat{t}$, $\hat{u}$ in (37) are functions of s, p_T, y,

$$\hat{s} = xs, \quad \hat{t} = -\frac{p_T\sqrt{s}e^{-y}}{z}, \quad \hat{u} = -\frac{xp_T\sqrt{s}e^y}{z}. \tag{39}$$

Eq.(37) together with (31) for the HT contributions will be applied for numerical calculations.

In numerical calculations for quark distribution functions, we borrow the leading order parametrization of Owens [15]. Our calculations are performed for $M = \pi\ , K,\ \rho$ at $\sqrt{s} = 14.1\ GeV,\ 25\ GeV$. The quark fragmentation functions are taken from [16]. Recently, in [17], a new set of fragmentation functions for charged pions and kaons, both at leading and next-to-leading order, have been presented. These functions give $D_q^{M^+ + M^-}(x, Q^2)$, but not $D_q^{M^\pm}(x, Q^2)$. Therefore, we cannot apply them in our calculations.

The other problem is a choice of the QCD scale parameter Λ and number of active quark flavors n_f. The HT subprocesses probe the meson w.f. over a large range of Q^2, Q^2 being equal to $\hat{s}$ or $-\hat{u}$. It is easy to find that $-\hat{u}_{min} > 4.04\, GeV^2$, while $\hat{s}_{min} > 16\, GeV^2$. For momentum scales $\hat{s}, -\hat{t}$ used in (36) as arguments of α_S in the LT cross section we get

$$-\hat{t}_{min} > 6\ GeV^2, \quad \hat{s}_{min} > 16\ GeV^2.$$

In other kinematic domains these scales take essentially larger values. Taking into account these facts we find it reasonable to assign $\Lambda = 0.1\ GeV, n_f = 5$ throughout in this section.

Results of our numerical calculations are plotted in $Figs.2\!-\!6$. First of all, it is interesting to compare the resummed HT cross sections with the ones obtained in the framework of the frozen coupling approximation. In $Fig.2$, the ratio $r_M = (\Sigma_M^{HT})^{res}/(\Sigma_M^{HT})^o$ for negatively charged particles (π^-, K^-) is shown. In the computing of $(\Sigma_M^{HT})^o$ we have neglected the meson's w.f. dependence on the scale $\hat{Q}^2$. Let us emphasize that for the kaon we have used the frozen coupling version of our expression (8), but not the Bagger-Gunion formula from [6], which is incorrect in that case.

In all of the following figures we have used the resummed expression for the HT cross section. In $Fig.3$ the ratio $R_M =\mid \Delta_M^{HT}/\Delta_M^{LT} \mid$ is depicted. For all particles the LT cross section difference is positive $\Delta_M^{LT} > 0$, since $\Sigma_{M^+}^{LT} \sim u_p(x, -\hat{t})e_u^2$, while $\Sigma_{M^-}^{LT} \sim d_p(x, -\hat{t})e_d^2$. The smaller quark charge e_d and the smaller distribution function d_p both suppress $\Sigma_{M^-}^{LT}$ [6]. The HT cross section difference may change sign at small p_T and become negative $\Delta_M^{HT} < 0$. For example, $\Delta_{\pi^-}^{HT} < 0$ at $2\ GeV/c \leq p_T \leq 11\ GeV/c$ for $\sqrt{s} = 25\ GeV,\ y = 0$ and at $2\ GeV/c \leq p_T \leq 9\ GeV/c$ for $\sqrt{s} = 25\ GeV,\ y = 0.5$. Only at the phase-space boundary $p_T > 11\ GeV/c$ in the first case or at $p_T > 9\ GeV/c$ in the second one $\Sigma_{\pi^+}^{HT} > \Sigma_{\pi^-}^{HT}$. Therefore, we plot the absolute value of R_M. The similar picture has been also found for other mesons.

The rapidity dependence of R_M at $\sqrt{s} = 25\ GeV, p_T = 3\ GeV/c$ plotted in $Fig.3(b)$ illustrates not only the tendency of the HT contributions to be enhanced in the region of negative rapidity, but also reveals an interesting feature of the HT terms; as is seen from $Fig.3(b)$ the ratio R_M is an oscillating function of the rapidity. This property of the HT terms may have important phenomenological consequences. In fact, in $Fig.4$ we have depicted Δ_M^{tot} and Δ_M^{LT} versus rapidity. In both cases, owing to observed property of $\Delta_M^{HT}(y)$, in certain domains of the rapidity interval $-2 \leq y \leq 2.105$ the total cross section difference is more than Δ_M^{LT} and in some ones less than Δ_M^{LT}. In the case of the kaon photoproduction

$$\Delta_K^{tot} > \Delta_K^{LT}, \quad for \quad -2 \leq y \leq 0.3 \quad and \quad 1.8 \leq y \leq 2.105,$$
$$\Delta_K^{tot} < \Delta_K^{LT}, \quad for \quad 0.3 \leq y \leq 1.8.$$

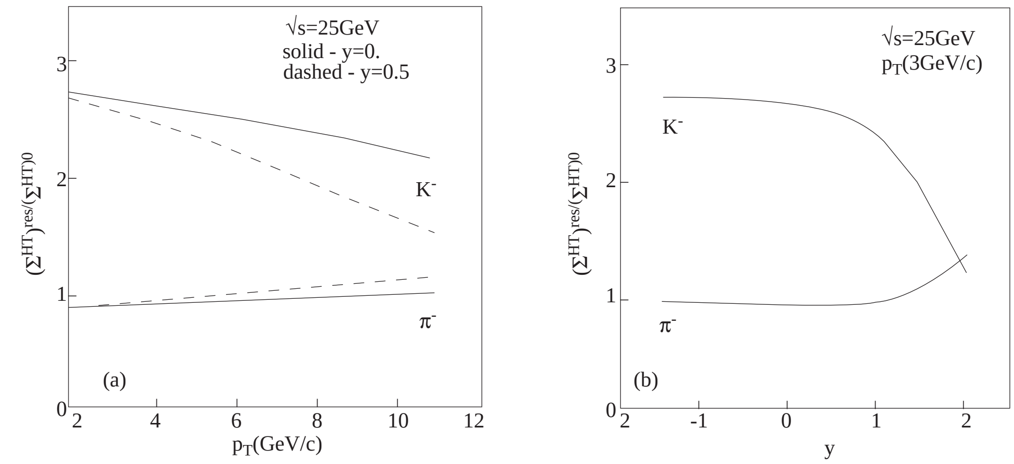
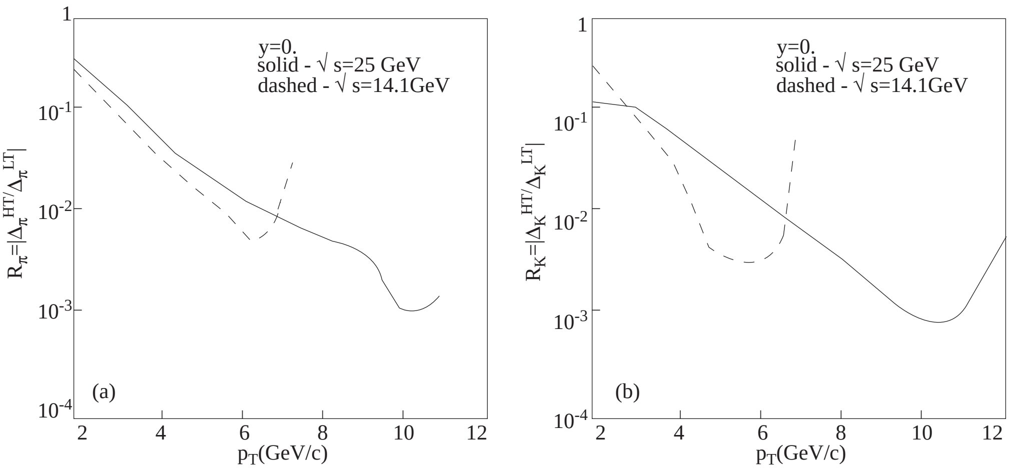


Figure 2. Ratio $r_M = (\Sigma_M^{HT})^{res}/(\Sigma_M^{HT})^o$, where $(\Sigma_M^{HT})^{res}$ and $(\Sigma_M^{HT})^o$ are HT contributions to the photoproduction cross section calculated using the running and frozen coupling approximations, respectively. The ratio is depicted as a function of p_T.



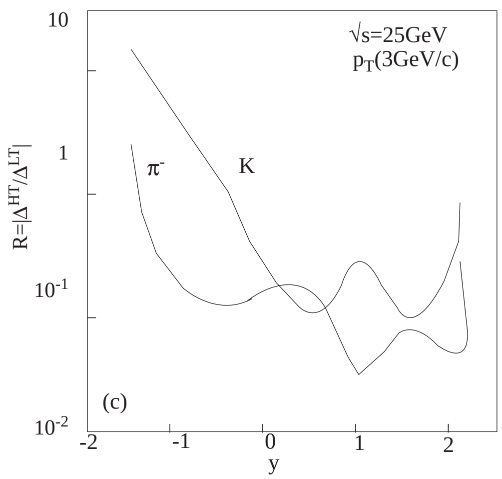


Figure 3. Ratio $R_M =\mid \Delta_M^{HT}/\Delta_M^{LT} \mid$ for the kaon a) at fixed rapidity y=0. In b) R_M is plotted as a function of y for the pion (dashed curve) and for the kaon (solid curve).

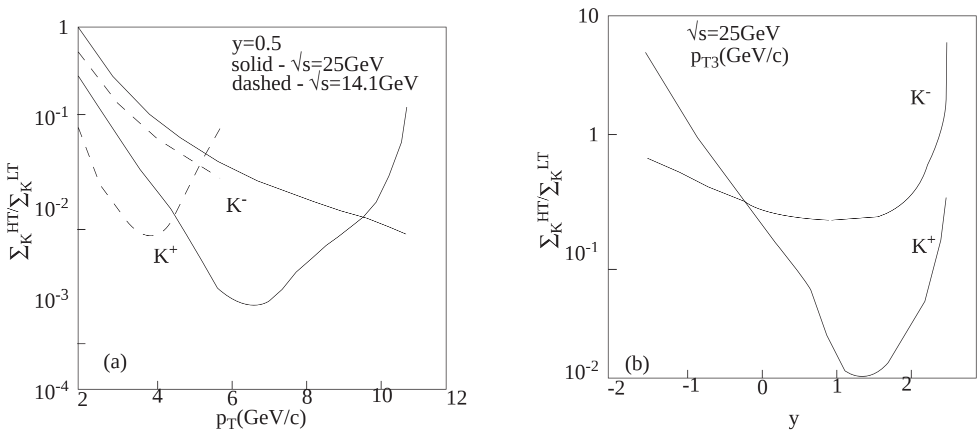


Figure 4. The cross section difference Δ_M as a function of the rapidity for kaons.

The properties of the HT terms found in the pion and kaon photoproduction processes persist also in the ρ-meson photoproduction. But now the HT contributions change the whole picture of the process arising from the ordinary LT calculations. Thus, as in the case of the pion photoproduction, the HT terms are enhanced relative to the leading ones and $\Delta_\rho^{HT} < 0$ almost for all p_T. But now $\mid \Delta_\rho^{HT} \mid$ takes such large values that it even changes the sign of the total cross section difference. That is, if in accordance with the LT estimations $\Sigma_{\rho^+}^{tot} > \Sigma_{\rho^-}^{tot}$ must be valid for all p_T, for $p_T < p_T^c$ we find $\Sigma_{\rho^+}^{tot} < \Sigma_{\rho^-}^{tot}$. The value of p_T^c depends on the process parameters, as well as on the ρ-meson w.f. used in calculations. At $p_t \approx p_T^c$ we have $\Sigma_{\rho^+}^{tot} \approx \Sigma_{\rho^-}^{tot}$.

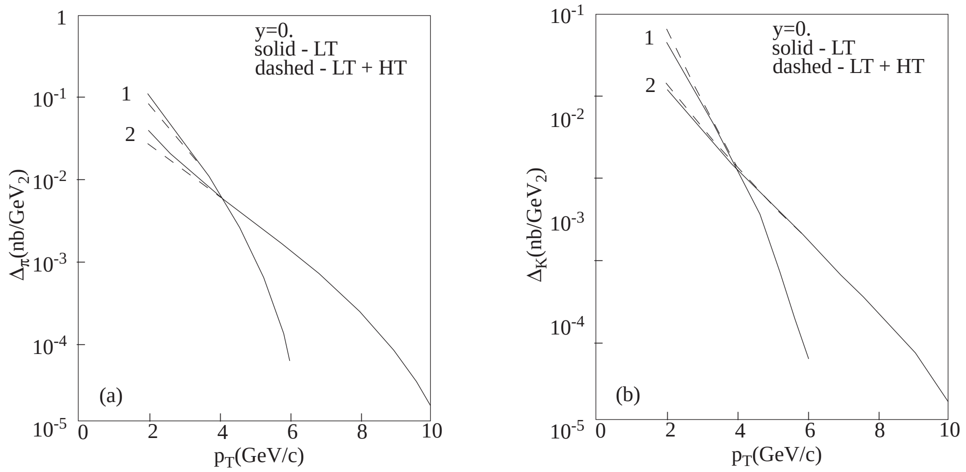


Figure 5. Δ_ρ for ρ-meson. The solid curve describes Δ_ρ^{LT}, whereas the dashed curves correspond to Δ_ρ^{tot}. The long-dashed curve has been obtained using the CZ w.f., the short-dashed one- BB w.f. In the domains $I(BB\ w.f.)$ and $I'(CZ\ w.f.)$ the absolute value of $\mid \Sigma_\rho^{tot} \mid$ or $\Sigma_{\rho^-}^{tot} - \Sigma_{\rho^+}^{tot}$ is plotted.

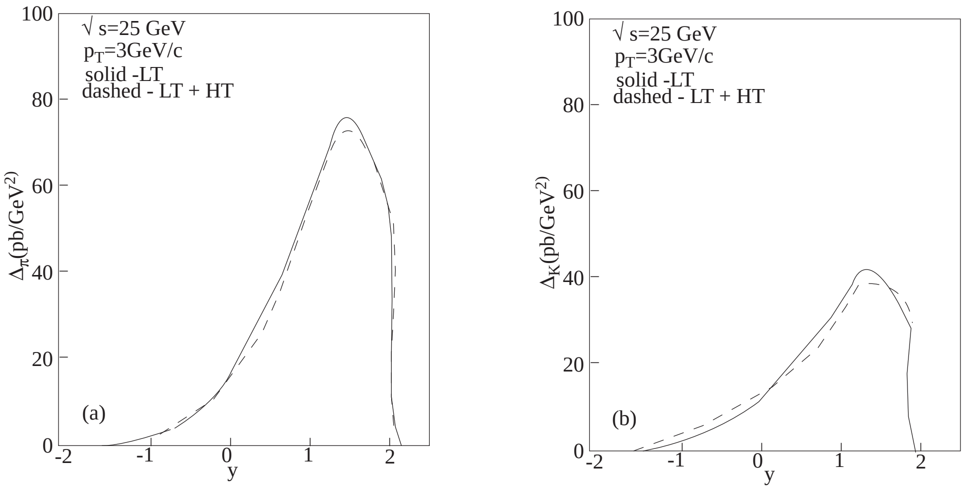


Figure 6. Δ_ρ dependence on the rapidity at $\sqrt{s} = 25\ GeV, p_T = 3\ GeV/c$. The solid curve corresponds to Δ_ρ^{LT}, the long-dashed and short-dashed curves describe Δ_ρ^{tot} obtained using CZ and BB w.f., respectively. In regions $I(BB\ w.f.)$ and $I'(CZ\ w.f.)$ the cross section difference $\Sigma_{\rho^-}^{tot} - \Sigma_{\rho^+}^{tot}$ is shown.

Our results are shown in Figure 5. For the parameters indicated in the figure a critical value of p_T is: $p_{T1}^c \simeq 5.05\ GeV/c$ for CZ w.f., and $p_{T2}^c \simeq 6.25\ GeV/c$ for BB w.f. In all kinematic domains the HT contributions found using BB w.f. exceed the ones obtained by applying CZ w.f., that is, $\mid \Delta_\rho^{HT}(BB) \mid > \mid \Delta_\rho^{HT}(CZ) \mid$. For example, the ratio $\mid \Delta_\rho^{HT}(BB)/\Delta_\rho^{HT}(CZ) \mid$ equals to 2.39 at $\sqrt{s} = 25\,GeV,\ p_T = 5\,GeV/c,\ y = 0$, or to 2.63 at $\sqrt{s} = 25\ GeV,\ p_T = 3\ GeV/c,\ y = -1$. Similar pictures persist in $Fig.6$, where Δ_ρ^{LT} and Δ_ρ^{tot} are depicted as functions of the rapidity y. In $Fig.6$, for the process parameters $\sqrt{s} = 25\ GeV, p_T = 3\ GeV/c$ we have: in domain I' $(-1.74 \leq y \leq 1.3)$ the total cross section difference for CZ w.f. is negative, in I $(-1.5 \leq y \leq 1.5)$ - $\Delta_\rho^{tot}(BB) < 0$.

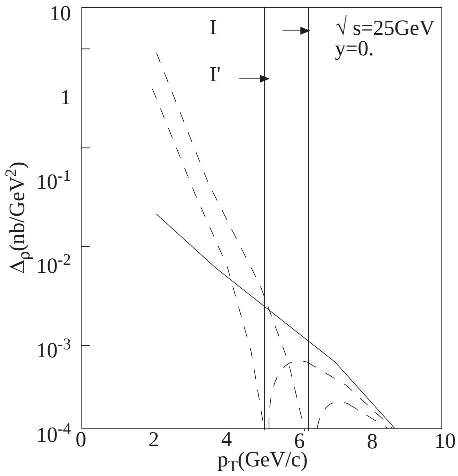


Figure 7.

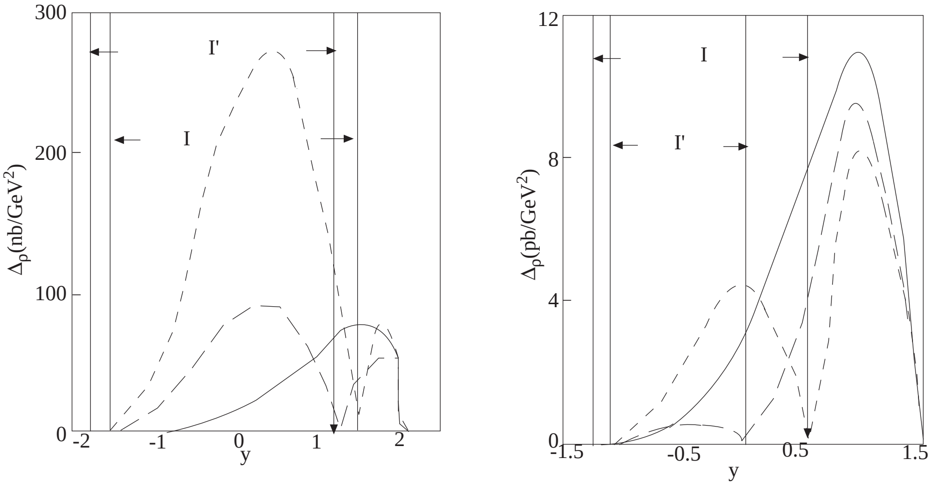


Figure 8.

It is worth noticing that in [6] the authors considered the ρ-meson photoproduction at the same process's parameters and predicted $\Sigma_\rho^{tot} < 0$ at $p_T \leq 3\ GeV/c$, but could not find similar effects for Σ_ρ^{tot} in dependence on the rapidity. Our investigations prove that $\Sigma_\rho^{tot} < 0$ at $p_T < p_T^c$ and p_T^c well into deep perturbative domain. We have also demonstrated that the same phenomenon exists for $y_1^c \leq y \leq y_2^c$.

In this work we have calculated the single meson hard semi-inclusive photoproduction via higher twist mechanism and obtained the expressions for the subprocess $\gamma q \to Mq$ cross section for mesons with both symmetric and non-symmetric wave functions. Summing up we can state that:
i) in the context of the running coupling constant method the HT subprocess cross section cannot be normalized in terms of meson's elm form factor neither for mesons with symmetric w.f. nor for non-symmetric ones;
ii) the resummed HT cross section differs from that found using the frozen coupling approximation, in some cases, considerably;
iii) HT contributions to the single meson photoproduction cross section have important phenomenological consequences, specially in the case of ρ-meson photoproduction. In this process the HT contributions wash the LT results off, changing the LT predictions.

References

[1] S.J.Brodsky, R.Blankenbecler and J.F.Gunion, *Phys.Rev.* **D6**, 2651 (1972); S.J.Brodsky and G.R.Farrar, *Phys.Rev.Lett.* **31**, 1153 (1973).

[2] G.P.Lepage and S.J.Brodsky, *Phys.Rev.* **D22,** 2157 (1980).

[3] V.L.Chernyak and A.R.Zhitnitsky, *Phys.Rep.* **112,** 173 (1980).

[4] S.S.Agaev, *Phys.Lett.* **B283,** 125(1992); *Z.Phys.* **C57**, 403 (1993).

[5] V.N.Baier and A.G.Grozin, *Phys.Lett.* **B96**, 181 (1980); S.Gupta, *Phys.Rev.* **D24**, 1169 (1981).

[6] J.A.Bagger and J.F.Gunion, *Phys.Rev.* **D25**, 2287 (1982).

[7] J.A.Hassan and J.K.Storrow, *Z.Phys.* **C14**, 65 (1982).

[8] P.Ball and V.M.Braun, *Phys.Rev.* **D54**, 2182 (1996).

[9] S.S.Agaev, *Int.J.Mod.Phys.* **A8**, 2605 (1993); **A9**, 5077 (1994).

[10] S.S.Agaev, *Phys.Lett.* **B360**, 117 (1995); *E. Phys.Lett.* **B369**, 379 (1996); S.S.Agaev, *Mod.Phys.Lett.* **A10**, 2009 (1995).

[11] G.'t Hooft, In: The Whys of Subnuclear Physics, Proc.Int.School, Erice, 1977, ed. A.Zichichi, Plenum, New York, 1978; V.I.Zakharov, *Nucl.Phys.* **B385**, 452 (1992).

[12] S.S.Agaev, *Mod.Phys.Lett.* **A11**, 957 (1996); ICTP preprint IC/95/291, September 1995, hep-ph/9611215.

[13] A.H.Mueller, *Nucl.Phys.* **B250**, 327 (1985); H.Contopanagos and G.Sterman, *Nucl.Phys.* **B419**, 77 (1994).

[14] A.Erdelyi, Higher transcendental functions, v.2, McGrow-Hill Book Company, New York, 1953.

[15] J.F.Owens, *Phys.Lett.* **B266**, 126 (1991).

[16] J.F.Owens, *Phys.Rev.* **D19**, 3279 (1979); J.F.Owens, E.Reya and M.Glück, *Phys.Rev.* **D18**, 1501 (1978).

[17] J.Binnewies, G.Kramer and B.A.Kniehl, DESY preprint DESY-95-048, March 1995.