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Solution of Optic Reconstructive Problem Considering Registered Signal Velocity

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Abstract

A solution to the tomographic problem of dynamic reconstruction of a processes, the rate of which is comparable with the velocity of the registered signal is presented. The obtained data can be used both for theoretical and applied tomography.

In optical tomography, until now the registered signal was considered to have spread instantly, and thus signal velocity was not accounted for. However, there are now high-speed photo techniques, which makes the rate of signal registration comparable with the speed of light and the characteristic length of the investigated object. Under such conditions each projection bears information about various times for various points of the object and the inability to account for signal velocity in corresponding reconstructive problems can result in serious distortions. Therefore we have formulated the task as:

$$g(\phi, s, t) = \int_{xa=s} f(x, t + (r - xb)/v) dx, \tag{1}$$

for which we want to find $f(x,t)$; where x is the 2D coordinate of an object's point; (ϕ, s) is the normal coordinates of choosing integral; r is the distance between center and measuring instruments (Fig. 1); v is the speed of measured signal; and $a = (\cos(\phi), \sin(\phi))$, $b = (\sin(\phi), -\cos(\phi))$.

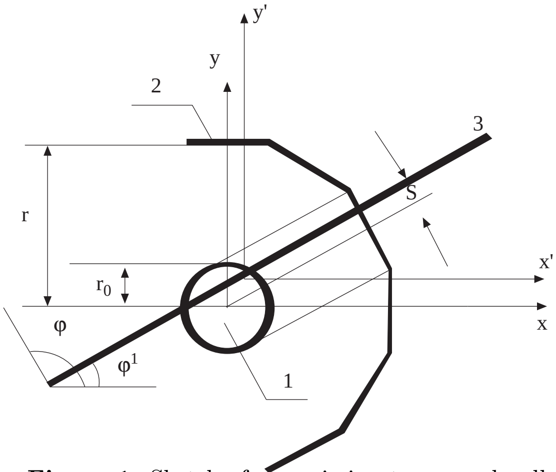

Figure 1. Sketch of an emission tomograph, allowing to receive projections $g(\phi, s, t)$ for the various moments t simultaneously:
1. Field of $f(x, t)$
2. System of measuring devices
3. A line of projection $g(\phi, s, t)$

For simplification we have assumed that $v = 1$ and have moved the scale of time for $f(x, t)$ to the left on as r/v. Thereby we obtain:

$$g(\phi, s, t) = \int_{xa=s} f(x, t - xb)d\dot{x}, \tag{2}$$

to find $f(x, t)$. For a solution to this problem $f'(x, t)$ is determined so that

$$g(\phi, s, t) = \int_{xa=s} f'(x, t)dx. \tag{3}$$

Comparing Eqn. (2) and Eqn. (3) and taking into consideration the simpleness of the Radon transform [1, 2], we can conclude that

$$f((0, 0), t) = f'((0, 0), t) = R^{-1}g(\phi, s, t), \tag{4}$$

where R^{-1} is the inverse Radon transform.

To find $f(x, t)$ for any x, we take the coordinate scale (x', y'), for which point x is the center. For this coordinate scale, an equality similar to (2)

$$g_x(\phi, s', t) = \int_{xa=s} f_x(x', t - x'b')dx', \tag{5}$$

where

$$g_x(\phi, s', t) = g(\phi, s' + xa, t + xb). \tag{6}$$

For Eqn. (5) we have the solution for $x' = (0,0)$ similar to Eqn. (4). Thus we have the final the solution of our problem:

$$f(x,t) = f_x(0,0) = R^{-1} g_x(\phi, s', t)|_{x'=(0,0)}, \tag{7}$$

where $g_x(\phi, s', t)$ is defined by Eqn. (6). Applying the classical Radon formula [1] to Eqn. (7) we find that

$$f(q,t) = (\pi)^{-1} \int_{-\infty}^{\infty} q^{-1} dF_x(q,t), \tag{8}$$

where

$$F_x(q,t) = (2\pi)^{-1} \int_0^{\pi} g_x(\phi, s', t) d\phi. \tag{9}$$

Returning to the first problem, where r and v were uncombined, we can rewrite Eqn. (6) in the form:

$$g_x(\phi, s', t) = g(\phi, s + xa, t + (xb - r)/v). \tag{10}$$

Using formulae (8)-(10) we can use any filtered backprojection algorithm [2] without further ado. It is also important to note that, if any prior information about $f(x,t)$ is known, then, account of velocity of signal can permit other solutions. The case of correlated quanta has been discussed in [3-5]. If there is a multitude of different speed characteristic signals, then it is possible to consider a tomographic task in the time-space plane, [6-8].

The above suggested theory can be used at different fields of investigations (such as nonlinear optic, atomic and nuclear physics, physics of plasma, etc.), in which processes are accompained by radiation. Perhaps, it may be used at acoustic tomography, where velocity of the signal is always quite small.

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