Tr. J. of Physics
22 (1998) , 181 – 191.


# Singlet Axial Current Form Factor in the Chiral Solitonic Bag Model with Massive Quarks

**N. KARAGÖZ**
*Abant İzzet Baysal University,*
*Physics Department, Bolu - TURKEY*

Received 19.09.1997

**Abstract**

Singlet axial current form factor $g_A^0$ is calculated as a function of bag radius in the two flavor chiral solitonic bag model with unequal quark masses. The value of $g_A^0$ near 1 fm is sufficiently small.

## Introduction

The EMC data [1] implies that the quark spin contribution (i. e. the proton form factor of the singlet axial vector current at zero momentum transfer) to the proton angular momentum approximately vanishes. However relatively large experimental uncertainties is consistent with a reasonably small non-zero value.

First of all, this rules out the naive nonrelativistic quark model (NRQM) in which the spin of the proton is completely accounted by the combination of the quark spins. Given this failure of NRQM it was necessary to go to the perturbative QCD framework, to adress the problem and indeed there is sizable literature in this direction [2]. As there are quark orbital contributions, as well as the gluonic ones to the proton angular momentum, the calculations are not easy in this framework. Thus as is usually done, one resorts to effective models of QCD. One of the most popular effective models of QCD is the Skyrme model [3]. This model proved to be quite successful in predicting the low energy properties of the hadrons [4]. In the simplest version of the Skyrme model one gets zero for the singlet axial vector current matrix element [5]. Thus, in the light of the EMC data, The Skyrme model could be taken as a desirable zeroth order model.

In the recent past all possible extensions of the naive Skyrme model have been tried in addressing the problem under consideration [6]. The simplest extension of adding the Wess-Zumino (WZ) term, in the SU(3) case, did not produce any new result different than the naive Skyrme model, as the WZ term does not contribute to the $U_A(1)$ current.

One could think of invoking many additional chiral invariant terms, most of them being rather ad-hoc, however. For instance adding a term of the type $\mathrm{tr}(l_\mu l^\mu l_\nu l^\nu)$ to the lagrangian produces an additional piece proportional to $\mathrm{tr}(l_\nu l^\nu l_\mu)$ in the $U_A(1)$ current. It was shown that [6] after exciting the collective coordinates, this additional term in the $U_A(1)$ current is quadratic in $A^\dagger \dot{A}$, and thus is negligible [ in this semiclassical quantization framework, we keep only the terms in the lagrangian which are quadratic in terms of the angular velocity operators $\mathrm{tr}(T^a A^\dagger \dot{A})$, where $T^a$ are the generation of the SU(N); thus currents must be linear in terms of the angular velocity operator]. Thus another interesting claim [7] that this type of trilinear current should represent the entire $U_A(1)$ current whose coefficient to be determined from an anomaly-like argument, is also ruled out.

It is well known that some of the problems the naive Skyrme model, (and its extensions) face can be remedied by taking into account of short distance effects. That this the case was demonstrated in computing neutron proton mass difference satisfactorily in the framework of chiral bag model, by introducing the up and down quark mass difference explicitly [8]

The failure of the naive Skyrme model [9] in predicting the neutron - proton mass difference made it clear that the predictive power of this model was limited to those problems which do not require the crucial short distance information, that baryons are made of quarks, as the solitonic baryon (expressed as a nonlinearly twisted lump of the Goldstone bosons) do not remember that they are made of quarks. That the neutron is heavier than the proton might be a consequence of the crucial property of the underlying QCD, that the d quark is heavier than the u quark, calls for a more complete effective theory of QCD which contains in addition to the Skyrme lagrangian, explicit quarks as well. This naturally leads one to the so - called chiral solitonic bag model [10], an intuitively appealing framework that the quarks and Skyrmion play complementary roles in the baryon. Namely, the quarks keep the Skyrmion from collapsing, while the Skyrmion keeps the quarks confined. It was shown in ref. [8] that the $n-p$ mass difference is indeed correctly predicted by this model.

At this point, it is worth commenting on the possible modification of the two flavor model [2] by exciting a massive isosinglet $\eta$ field outside the bag to account for the axial anomaly [2]. This is done in such a way, that the classical global axial $U_A(1)$ symmetry of the underlying QCD which is broken explicitly by the continuity equation is restored; that is $\eta$ is coupled in such a way that isosinglet axial current is continuous across the bag surface. This brings a new contribution to $A_\mu^{(0)}$ outside the bag which is a pure gradient proportional to $\partial_\mu \eta$. The nonconservation of $A_\mu^{(0)}$ as generated by nonperturbative effects outside the bag can be represented by choosing $m_\eta \neq 0$. Being a pure gradient, the contribution of this mesonic part to the quark moment of inertia is shown to be small [6]. Thus, we will neglect the anomaly contribution in our discussion, and restrict our attention exclusively to the quark contribution.

We are going to follow the same model here in computing the singlet axial vector current matrix element as well. We will demonstrate that the presence of $u-d$ quark mass difference (i. e. explicit breaking of the isospin symmetry) plays a crucial role in obtaining a reasonably small matrix element for the $U_A(1)$ current consistent with EMC data in the two flavor case.

### Chiral Solitonic Bag Model and its Quantizations

The lagrangian of the two flavor chiral solitonic bag model is defined by [10]

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_q\theta(R-r)+\mathcal{L}_m\theta(r-R)+\mathcal{L}_B\delta_B \\ \mathcal{L}_q &= \overline{\psi}(i\gamma^\mu\partial_\mu - M)\psi \\ \mathcal{L}_m &= \frac{F_\pi^2}{16}\mathrm{tr}(\partial_\mu U^\dagger\partial^\mu U)+\frac{1}{32a^2}\mathrm{tr}\left[U^\dagger\partial_\mu U\,,\,U^\dagger\partial_\nu U\right]^2 \\ &+ \frac{m_\pi^2F_\pi^2}{8(m_u+m_d)}\mathrm{tr}\left(M\left(U+U^\dagger-2I\right)\right) \\ \mathcal{L}_B &= -\frac{1}{2}(\overline{\psi}_L U\psi_R+\overline{\psi}_R U^\dagger\psi_L) \\ M &= \mathrm{diag}(m_u,m_d). \end{aligned} \tag{1}$$

where $F_\pi$ is the pion decay constant, and $a$ is the Skyrme's dimensionless coupling constant.

Following ref. [8], we will describe the the meson phase by the static classical fileld configuration $U=\exp(i\vec{\tau}\cdot\hat{x}F(r))$ where $F(r)$ is determined by minimizing energy and imposing the bag boundary conditions $F(0)=\pi$ and $F(\infty)=0$, and the quark phase by the quantum field operator $\psi(\vec{x},t)$.

The standard method to excite the solitonic baryon degree of freedom, that is, to construct the low-lying quantum states above the semiclassical ground state is to make the substitution

$$\begin{aligned} U(\vec{x},t) &= A(t)U_0(\vec{x})A^\dagger(t) \\ \psi(\vec{x},t) &= A(t)\psi_0(\vec{x},t) \end{aligned} \tag{2}$$

where $U_0(\vec{x})$ is the static soliton solution and $A(t)$ is the arbitrary time dependent SU(2) matrix, the quark field $\psi_0(\vec{x},t)$ is the field in the rotating frame. Substituting these in eq. (1), we get

$$L = L_0+\lambda_m\mathrm{tr}(\dot{A}^\dagger\dot{A})+\frac{i}{2}X_i\int d^3x\bar{\psi}_0\gamma_0\tau_i\psi_0-\frac{1}{2}\Delta m_q R_{3i}\int d^3x\bar{\psi}_0\tau_i\psi_0 \tag{3}$$

where

$$X_i = \mathrm{tr}\left(\tau_i A^\dagger\dot{A}\right) \tag{4}$$

$$R_{ij} \quad = \quad -\frac{1}{2}\mathrm{tr}\left(A^{\dagger}\tau_i A\tau_j\right) \ .$$

and $\lambda_m$ is the moment of inertia of the meson phase, associated with the collective rotation, and is given by

$$\lambda_m = \frac{2\pi F_\pi^2}{3}\int_R^\infty drr^2\sin^2 F(r)\left\{1+\frac{4}{(aF_\pi)^2}\left[\left(\frac{dF(r)}{dr}\right)^2+\frac{\sin^2 F(r)}{r^2}\right]\right\}. \tag{5}$$

$\psi_0(x,t)$ satisfies the equation of motion

$$\left(i\gamma^\mu\partial_\mu - m_0 + i\gamma_0 A^\dagger\dot{A}+\frac{1}{2}\Delta m_q A^\dagger\tau_3 A\right)\psi_0(\vec{x},t)=0 \tag{6}$$

and the bag boundary condition

$$-i\hat{x}\cdot\vec{\gamma}\psi_0(\vec{x},t)\mid_{Bag}=\exp(i\gamma_5\hat{x}\cdot\vec{\tau}F(r))\psi_0(\vec{x},t)\mid_{Bag} \ . \tag{7}$$

We can formally solve the eq. (5) as [8]

$$\psi_0(\vec{x})=\chi_0(\vec{x})-\int d^3y\, S(\vec{x},\vec{y};w)\left[i\gamma_0 A^\dagger\dot{A}+\frac{1}{2}\Delta m_q A^\dagger\tau_3 A\right]\psi_0(\vec{y}). \tag{8}$$

Hedgehog quark state solutions [10] $\chi_0(\vec{x})$, satisfies the equation

$$\left(w\gamma_0+i\vec{\gamma}\cdot\vec{\nabla}-m_0\right)\chi_0(\vec{x})=0 \tag{9}$$

with

$$\chi_0(\vec{x})=\frac{N}{\sqrt{4\pi}}\begin{pmatrix} i\sqrt{\frac{w+m_0}{w}}j_0(kr)|0> \\ -\sqrt{\frac{w-m_0}{w}}j_1(kr)\vec{\sigma}\cdot\hat{x}|0> \end{pmatrix}. \tag{10}$$

$S(\vec{x},\vec{y};w)$ is the bag propagator defined by [11]

$$\begin{aligned}&\left(w\gamma_0+i\vec{\gamma}\cdot\vec{\nabla}-m_0\right)S(\vec{x},\vec{y};w)=\delta^3(\vec{x}-\vec{y}) \qquad (11)\\ &(\exp(i\gamma_5\vec{\tau}\cdot\hat{x}\theta)+i\vec{\gamma}\cdot\hat{x})\,S_B\mid_{Bag}=0.\end{aligned}$$

where

$$\begin{aligned} S(\vec{x},\vec{y};w) \quad &= \quad S^0(\vec{x},\vec{y};w)+R^2\int d\Omega_\alpha S^0(\vec{x},\vec{\alpha};w)K_\alpha S^0(\vec{\alpha},\vec{y};w) \\ &+ \quad (R^2)^2\int d\Omega_\alpha d\Omega_\beta S^0(\vec{x},\vec{\alpha};w)K_\alpha S^0(\vec{\alpha},\vec{\beta},w)K_\beta S^0(\vec{\beta},\vec{y};w)+\cdots \end{aligned} \tag{12}$$

with

$$K_\alpha = \exp(i\gamma_5 \vec{\tau} \cdot \hat{n}_\alpha \theta) + i\vec{\gamma} \cdot \hat{n}_\alpha \,, \tag{13}$$

$S^o(\vec{x}, \vec{y}; w)$ is the usual Dirac propagator written in terms of two component spherical harmonics $\Phi_{jlm}(\Omega)$

$$S^0(\vec{x}, \vec{y}; w) = \sum_{jll'm} S^0_{jll'}(r, r'; w) \Phi_{jlm}(\Omega) \Phi^\dagger_{jl'm}(\Omega') \tag{14}$$

with

$$\begin{aligned} S^0{}_{jll'}(r, r'; w) &= -ik \left[ (m_0 + w\rho_3)\, \delta_{l\,l'} + k\rho_2 (l' - l) \right] f_l(kr) f_{l'}(kr') \\ f_l(kr) &= j_l(kr)\theta(r' - r) + h_l(kr)\theta(r - r') \end{aligned} \tag{15}$$

$j_l$ and $h_l$ are Bessel and first kind Hankel functions, and $\gamma_0 = \rho_3$, $\gamma_i = -i\rho_2 \otimes \sigma_i$, $\gamma_5 = \rho_1 \otimes 1$, where $\rho$ and $\sigma$ are Pauli matrices. Let us decompose

$$S_{jll'}(r, r'; w) = S^0{}_{jll'}(r, r'; w) + S^B{}_{jll'}(r, r'; w) \tag{16}$$

here $S^B_{jll'}(r, r'; w)$ is the boundary term [12]

$$\begin{aligned} S^B_{jll'}(r, r'; w) &= -\frac{(kR)^2 j_l(kr) j_{l'}(kr')}{[(1 - M\cos\theta)^2 - (K\sin\theta + W\cos\theta)^2]} \\ &\times \Bigg\{ \Bigg[ \Big( (m_0 + w\rho_3)^2 h_l^2 - k^2 h_{\bar{l}}^2 \Big) \cos\theta \\ &+ 2k(\bar{l} - l) h_l h_{\bar{l}} (m_0\rho_3 + w) \sin\theta \Bigg] \Big( a + b\rho_3 (\bar{l} - l) \Big) \delta_{ll'} \\ &+ k(l' - l) \Bigg[ \Big( w(h_l^2 - h_{\bar{l}}^2) + m_0\rho_3 (h_l^2 + h_{\bar{l}}^2) \Big) (\bar{l} - l) \cos\theta \\ &+ 2k h_l h_{\bar{l}} \sin\theta \Bigg] \Big( b\rho_2 + ia\rho_1 (l - \bar{l}) \Big) \Bigg\} \end{aligned} \tag{17}$$

with

$$\begin{aligned} \bar{l} &= 2j - l \\ K &= -ik^2 R^2 \Big( j_0(kR) h_1(kR) + j_1(kR) h_0(kR) \Big) \\ M &= -ikm_0 R^2 \Big( j_0(kR) h_0(kR) + j_1(kR) h_1(kR) \Big) \\ W &= -ikw R^2 \Big( j_0(kR) h_0(kR) - j_1(kR) h_1(kR) \Big) \end{aligned} \tag{18}$$

where Eq. (8) can be solved perturbatively. Since $\Delta m_q$ is small, it is consistent to solve it to first order in $\Delta m_q$. Furthermore the collective rotations are adiabatic, thus $\tau^a A^\dagger \dot{A}$

is also small. Therefore these rotations will also be treated to first order. Then eq. (8) can be rewritten as

$$\psi_0(\vec{x}) = \chi_0(\vec{x}) - \int d^3y\, S(\vec{x},\vec{y};w)\left[i\gamma_0 A^\dagger \dot{A} + \frac{1}{2}\Delta m_q A^\dagger \tau_3 A\right]\chi_0(\vec{y}). \tag{19}$$

Substituting eq. (19) in eq. (3) we get the complete Lagrangian

$$L = L_0 - \frac{1}{2}\Lambda_{ij}X_iX_j - i\frac{\Delta m_q}{4}R_{3j}C_{ji}X_i \tag{20}$$

where

$$\Lambda_{ij} = \lambda_m\delta_{ij} + \frac{1}{2}\int d^3xd^3y\,[\bar{\chi}_0(\vec{x})\tau_i\gamma_0 S(\vec{x},\vec{y};w)\gamma_0\tau_j\chi_0(\vec{y}) + \text{h.c.}] \tag{21}$$

and

$$C_{ij} = \int d^3xd^3y\left[\chi_0^\dagger(\vec{x})\left(\tau_i S(\vec{x},\vec{y};w)\tau_j + \tau_i\gamma_0 S(\vec{x},\vec{y};w)\gamma_0\tau_j\right)\chi_0(\vec{y}) + \text{h.c.}\right] \tag{22}$$

A lengtly analysis shows that both $\Lambda$ and $C$ matrices are diagonal in flavor space

$$\Lambda_{ij} = \delta_{ij}(\lambda_m + \lambda_q)\ , \qquad C_{ij} = \delta_{ij}C \tag{23}$$

where

$$\lambda_q = \frac{1}{2}\int d^3xd^3y\,[\bar{\chi}_0(\vec{x})\gamma_0 S(\vec{x},\vec{y};w)\gamma_0\chi(\vec{y}) + \text{h.c}] \tag{24}$$

$$C = \int d^3xd^3y\,\left\{\chi^\dagger(\vec{x})\,[S(\vec{x},\vec{y};w) + \gamma_0 S(\vec{x},\vec{y};w)\gamma_0] + \text{h.c}\right\} \tag{25}$$

**Singlet Axial Current**

The EMC measurement of the proton structure function, and the baryonic weak decay data on $F$ and $D$ ratio yield a value for the singlet axial current matrix element.

$$< p\uparrow|A_\mu^{(0)}|p\uparrow>= 0.00 \pm 0.24. \tag{26}$$

Let us now construct the singlet axial current from eq. (2). As the pionic part does not contribute, it is given purely by the quark contribution

$$\begin{aligned} A_\mu^{(0)} &= \frac{1}{2}\bar{\psi}\gamma_\mu\gamma_5\psi \qquad \text{inside the bag} \\ A_\mu^{(0)} &= 0 \qquad\qquad\quad \text{outside the bag} \end{aligned} \tag{27}$$

The matrix element of the current eq. (27) between nucleon states can be written in terms of two form factor $g_A$ and $h_A$

$$< p', s|A_\mu^{(0)}(0)|p, s>= \frac{1}{2}\bar{u}(p',s)\left[g_A(q^2)\gamma_\mu\gamma_5 + h_A(q^2)q_\mu\gamma_5\right]u(p,s) \tag{28}$$

where $p'$, $p$ and $s$ are the nucleon momenta and spin, and $u$ is the free Dirac spinor of the nucleons and $q = p - p'$. As it is well known that the form factor $g_A(q^2)$ has no pole at $q \simeq 0$, due to $U_A(1)$ anomaly, we can take the limit $q \to 0$ uniformly to obtain

$$< p, s|A_\mu^{(0)}(0)|p, s >= \frac{2}{3} g_A(0) < S_\mu > \tag{29}$$

here $< S_\mu >$ is the nucleon spin. Following the general philosophy in the Skyrme model calculations, we will compute the left hand side of eq.(29) explicitly, by computing the matrix element of $\int d^3x A_i^{(0)}(x)$.

Now we write the singlet axial current for the quark part

$$\int d^3x A_i^{(0)}(\vec{x}) = \frac{1}{2} \int d^3x \bar{\psi}(\vec{x}) \gamma_i \gamma_5 \psi(\vec{x}) \tag{30}$$

Using eq. (19), to first order in $A^\dagger \dot{A}$ and $\Delta m_q$, we obtain

$$\int d^3x A_i^{(0)}(\vec{x}) = -i\Gamma_{ij} X_j + \frac{\Delta m_q}{4} D_{ij} R_{3j} \tag{31}$$

where

$$\begin{aligned} \Gamma_{ij} &= \frac{1}{2} \int d^3x d^3y \left[ \chi_0^\dagger(\vec{x}) \gamma_0 \gamma_i \gamma_5 S(\vec{x}, \vec{y}; w) \gamma_0 \tau_j \chi_0(\vec{y}) + \text{h.c} \right] \\ D_{ij} &= \int d^3x d^3y \left[ \chi_0^\dagger(\vec{x}) \gamma_0 \gamma_i \gamma_5 S(\vec{x}, \vec{y}; w) \tau_j \chi_0(\vec{y}) + \text{h.c.} \right] \end{aligned} \tag{32}$$

Using $\gamma_0 = \rho_3$, $\gamma_i = -i\rho_2 \otimes \sigma_i$, $\gamma_5 = \rho_1 \otimes 1$, and the identity $\tau_i \chi_0 = -\sigma_i \chi_0$, again after a lengtly calculation one can show that $\Gamma_{ij}$, $D_{ij}$ are diagonal in flavor space, namely, $\Gamma_{ij} = \delta_{ij} \lambda_q$, $D_{ij} = \delta_{ij} D$ with

$$D = \int d^3x d^3y \left[ \chi_0^\dagger(\vec{x}) S(\vec{x}, \vec{y}; w) \chi_0(\vec{y}) + \text{h.c} \right] \tag{33}$$

eq. (31) can be rewritten as

$$\int d^3x A_i^{(0)}(\vec{x}) = -i\lambda_q X_i + \frac{\Delta m_q}{4} D R_{3i} \tag{34}$$

The spin and isospin operators can be obtained by applying the Noether's theorem to the Lagrangian in eq. (20), with the transformations respectively being $\delta_r A = iAr$ and $\delta_l A = ilA$ where $(r, l = i\epsilon^i \tau^i/2)$.

$$\begin{aligned} S_i &= -i\lambda_T X_i + \frac{\Delta m_q}{4} C R_{3i} \\ I_i &= R_{ij} S_j \ . \end{aligned} \tag{35}$$

Solving $X_i$ in terms of $S_i$ in eq. (35), and substituting in eq. (34), we get

$$\int d^3x A_i^{(0)} = \frac{\lambda_q}{\lambda_m + \lambda_q} S_i - \frac{\Delta m_q}{4}\left( C \frac{\lambda_q}{\lambda_m + \lambda_q} - D \right) R_{3i}\ , \tag{36}$$

Next we substitute the explicit expression for $\chi_0$ given eq. (10) in these expressions and get

$$\begin{aligned}
\lambda_q &= -\frac{\tilde{R}}{aF_\pi \nu^4 j_0^2(\nu)[\xi[\nu(1+w_1^2) - 2w_1] + \mu w_1]} \int_0^\nu dy y^2 \Big\{ \\
& \int_0^y dx x^2 \Big[ \ (\xi+\mu)^2 j_0^2(x) j_0(y) n_0(y) + (\xi-\mu)^2 j_1^2(x) j_1(y) n_1(y) \\
&+ \nu^2 \Big( \ j_1^2(x) j_0(y) n_0(y) + j_0^2(x) j_1(y) n_1(y) \Big) \ \Big] \\
&+ \int_y^\nu dx x^2 \Big[ \ (\xi+\mu)^2 j_0^2(y) j_0(x) n_0(x) + (\xi-\mu)^2 j_1^2(y) j_1(x) n_1(x) \\
&+ \nu^2 \ \Big( \ j_0^2(y) j_1(x) n_1(x) + j_1^2(y) j_0(x) n_0(x) \Big) \ \Big] \\
&- \ \Big[ \frac{\nu}{2(a-b)} \int_0^\nu dx x^2 \Big[ \cos F \ \Big[ \ (\xi+\mu) j_0^2(x) j_0^2(y) \ \Big( \ (\xi+\mu)^2 h_0^2(\nu) - \nu^2 h_1^2(\nu) \Big) \\
&+ \ \ (\xi-\mu) j_1^2(x) j_1^2(y) \ \Big( \ (\xi-\mu)^2 h_1^2(\nu) - \nu^2 h_0^2(\nu) \Big) \\
&- \ \ 2\nu^2 j_0^2(x) j_1^2(y) \ \Big( \ (\xi-\mu) h_1^2(\nu) - (\xi+\mu) h_0^2(\nu) \Big) \ \Big] \\
&+ \ \ \nu \sin F h_0(\nu) h_1(\nu) \ \Big( \ (\xi+\mu)^2 j_0^2(x) j_0^2(y) - (\xi-\mu)^2 j_1^2(x) j_1^2(y) \Big) \Big] + \text{h.c.} \ \Big] \Big\}
\end{aligned} \tag{37}$$

and

$$\begin{aligned}
C &= -\frac{4\tilde{R}}{aF_\pi \nu^4 j_0^2(\nu)[\xi[\nu(1+w_1^2) - 2w_1] + \mu w_1]} \int_0^\nu dy y^2 \Big\{ \\
& \int_0^y dx x^2 \Big( \ \ (\xi+\mu)^2 j_0^2(x) j_0(y) n_0(y) - (\xi-\mu)^2 j_1^2(x) j_1(y) n_1(y) \ \Big) \\
&+ \int_y^\nu dx x^2 \Big( \ \ (\xi+\mu)^2 j_0^2(y) j_0(x) n_0(x) - (\xi-\mu)^2 j_1^2(y) j_1(x) n_1(x) \ \Big) \\
&- \ \Big[ \frac{\nu}{2(a-b)} \int_0^\nu dx x^2 \Big[ \ \ (\xi+\mu) j_0^2(x) j_0^2(y) \ \big[ \cos F \ \big( (\xi+\mu)^2 h_0^2(\nu) - \nu^2 h_1^2(\nu) \big) \\
&+ \ \ 2\nu(\xi+\mu) h_0(\nu) h_1(\nu) \sin F \Big] \\
&+ \ (\xi-\mu) j_1^2(x) j_1^2(y) \ \Big[ \ \cos F \ \big( (\xi-\mu)^2 h_1^2(\nu) - \nu^2 h_0^2(\nu) \big) \\
&- \ \ \ 2\nu \sin F (\xi-\mu) h_0(\nu) h_1(\nu) \Big] \Big] \ \ + \text{h.c.} \Big] \Big\}
\end{aligned} \tag{38}$$

and

$$
\begin{aligned}
D &= -\frac{2\tilde{R}}{aF_\pi \nu^4 j_0^2(\nu)[\xi[\nu(1+w_1^2)-2w_1]+\mu w_1]}\int_0^\nu dyy^2 \Big\{ \\
& \int_0^y dxx^2 \Big[\ (\xi+\mu)^2 j_0^2(x) j_0(y) n_0(y) - (\xi-\mu)^2 j_1^2(x) j_1(y) n_1(y) \\
&+ \nu^2 \Big(\ j_1^2(x) j_0(y) n_0(y) - j_0^2(x) j_1(y) n_1(y)\Big)\ \Big] \\
&+ \int_y^\nu dxx^2 \Big[\ (\xi+\mu)^2 j_0^2(y) j_0(x) n_0(x) - (\xi-\mu)^2 j_1^2(y) j_1(x) n_1(x) \\
&+ \nu^2 \Big(\ j_0^2(y) j_1(x) n_1(x) - j_1^2(y) j_0(x) n_0(x)\Big)\ \Big] \\
&- \Big[\frac{\nu}{2(a-b)}\int_0^\nu dxx^2 \Big[\ (\xi+\mu) j_0^2(x) j_0^2(y)\left[\cos F\left((\xi+\mu)^2 h_0^2(\nu) - \nu^2 h_1^2(\nu)\right)\right. \\
&+ \left. 2\nu(\xi+\mu) h_0(\nu) h_1(\nu)\sin F\right] \\
&+ (\xi-\mu) j_1^2(x) j_1^2(y)\Big[\ \cos F\left((\xi-\mu)^2 h_1^2(\nu) - \nu^2 h_0^2(\nu)\right) \\
&- 2\nu \sin F(\xi-\mu) h_0(\nu) h_1(\nu)\Big]\Big]\ + \text{h.c.}\Big]\Big\}
\end{aligned} \tag{39}
$$

where

$$
\begin{aligned}
&\xi = ER\,, \quad w_1 = \frac{j_1(\nu)}{j_0(\nu)}\,, \quad \mu = \frac{m_0\tilde{R}}{aF_\pi}\,, \quad \nu = kR\,, \quad \text{and} \\
&a-b = 1-(M+W)\cos F - K\sin F
\end{aligned} \tag{40}
$$

The $1/(a-b)$ term is to be computed by carrying out the Wick rotations $k \to i\kappa$, $w \to i\eta$ which is necessary for the convergence of MRE [12]. Using eq. (29), The left hand side of eq. (35) between two nucleon state is to be identified with $2g_A^0/3 < N|S_i|N >$. Thus we find for $g_A^0$

$$
g_A^0 = \frac{3}{2}\frac{\lambda_q}{\lambda_T} - \frac{3\Delta m_q}{8}\left(C\frac{\lambda_q}{\lambda_T} - D\right)\frac{< N|R_{33}|N >}{< N|S_3|N >}\,. \tag{41}
$$

Next, using $< p\uparrow|R_{33}|p\uparrow >= 1/3$, we get

$$
g_A^0(p\uparrow) \quad = \quad \frac{3}{2}\frac{\lambda_q}{\lambda_T} - \frac{\Delta m_q}{4}\left(\frac{\lambda_q}{\lambda_T}C - D\right) \tag{42}
$$

and Notice that for $\Delta m_q \to 0$, our result is identical to that of the third reference in Ref. [5], where they have used the cranking formalism. The new contribution proportional to $\Delta m_q$, generated by the isospin breaking, differs for proton, as expected. We have calculated eq. (41) by determining the radial integrals in eqs. (37,38,39) where the numerical solutions of the equation satisfied by the Skyrme profile $F(r)$

$$(\frac{1}{4}\tilde{r}^2 + 2\sin^2 F)F^{''} + \frac{1}{2}\tilde{r}F^{'} + \sin 2F F^{'2} - \frac{1}{4}\sin 2F - \frac{\sin^2 F \sin 2F}{\tilde{r}^2} = 0 \qquad (43)$$

with the boundary conditions $F(r) = \pi$ at $r = 0$, $F(r) \to 0$ as $r \to \infty$

$\tilde{r} = aF_\pi r$ is used as an input. We have taken $\mu = 0.5$, $\Delta m_q = 3.8$ MeV, and $a = 5.45$. The results of the numerical analysis for proton of spin up state is presented in fig.1.

The author would like to thank Professor N.K. Pak for most stimulating discussion and also to thank Tuğrul Yılmaz for his assistance in carrying out the numerical analysis.

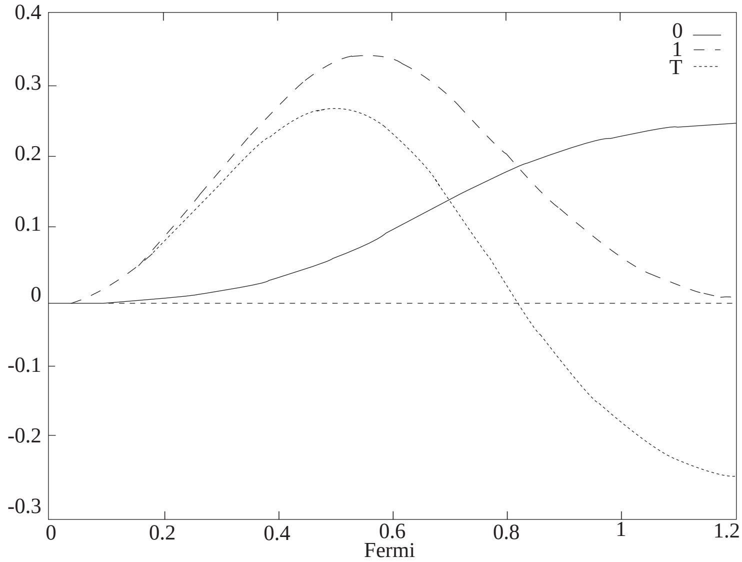


**Figure 1.** $g_A^0$ plotted as a function of the bag radius. T shows the total, 1 is the $\Delta m_q$ contribution and 0 is equal mass case

## References

[1] EMC Collab., J. Ashman et al., Phys. Lett. B **206** 364 (1988).

[2] For a general review of the EMC effect and for further references, see R. L. Jaffe, A. Manohar, Nucl. Phys. B **337** 509 (1993).

[3] T.H.R. Skyrme, Proc. Roy. Soc. **A260**, 127 (1961). N. K. Pak, C. H. Tze, Ann. Phys. (N.Y.) **117**, 169 (1979); R. Balachandran, et al., Phys. Rev. **D27**, 1133 (1983).

[4] G. Adkins, C. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).

[5] S. J. Brodsky, J. Ellis, M. Karliner, Phys. Lett. **B206** 309 (1988); J. Ellis, M. Karliner, Phys. Lett. **B213** 73 (1988). H. Dreiner, J. Ellis, R. A. Flores, Phys. Lett. **B221** 167 (1989).

[6] For a review, see R. Johnson, N. W. Park, J. Schechter, V. Soni, H. Weigel, Phys. Rev. D **42** 2998 (1990);

[7] Z. Ryzak, Phys. Lett. **B217** 325 (1989).

[8] N. K. Pak, T. Yılmaz, Phys. Rev. **D41**, 315 (1990).

[9] M. Durgut, N. K. Pak, Phys. Lett.**159B**, 357 (1985); ibid. (E) **162B**, 405 (1985).

[10] P.J. Mulders, Phys. Rev. **D30** 1073 (1984); E. Witten, Nucl. Phys. **B160**, 57 (1979).

[11] T. H. Hannson, R. L. Jaffe, Phys. Rev. **D28** 882 (1983); Ann. of Phys. **151** 204 (1983).

[12] N. Karagöz, N. K. Pak, Prog. of Theor. Phys. **90**, 369 (1993).