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The Image Reconstruction by Photocounts Triple Correlations

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Abstract

An optoelectronic system is constructed for the recording of the triple correlation function for radiation intensity being detected in a photon counting operation. The results of the investigation show the possibility an image reconstruction for faint objects from the derivation of comprehensive information related to the coherence function.

1. Introduction

Instensity interferometry is used for efficient solution to the image- reconstruction problem [1] by eliminating the effects of turbulence in the atmosphere on the radiation propagating from a faint remote object. In general case, however, the phase problem remains unsolved since only the modulus of the coherence function (CF) of a received field is detected, however, for the complete reconstruction of the incoherent object intensity distribution using the inverse Fourier transform, the CF argument must also be determined.

In particular, radiation statistics of the higher order coherence function contains more comprehensive information and have less sensitivity to additive noise [2,3]. In this paper, the optoelectronic system for registering the intensity triple correlation function of optical radiation detected in photon counting mode is presented. The method of deriving the phase information is based on equations [2] which relate the second and the third order cumulants of intensity and the coherence function with Gaussian statistics are given later:

$$K_{12}^{(2)}(I_1, I_2) = <\Delta I_1 \Delta I_2>= |\Gamma(\Delta x)|^2, \tag{1}$$

$$K_{13}^{(2)}(I_1, I_3) = <\Delta I_1 \Delta I_3>= |\Gamma((n+1)\Delta x)|^2, \tag{2}$$

$$\begin{aligned}
K_{23}^{(2)}(I_2, I_3) &= <\Delta I_2 \Delta I_3>= |\Gamma(n\Delta x)|^2, &(3)\\
K^{(3)}(I_1, I_2 I_3) &= <\Delta I_1 \Delta I_2 \Delta I_3>= 2\sqrt{K_{12}^{(2)} K_{13}^{(2)} K_{23}^{(2)}} \times \\
&\times \cos\{\varphi[(n+1)\Delta x] - \varphi(n\Delta x) - \varphi(\Delta x)\}, &(4)
\end{aligned}$$

where Δx is the step of the detector spatial scanning; $n = 1, 2, \ldots$ -and defines the distance between detectors; and $\varphi(x)$ is the phase of the coherence function $\Gamma(x)$.

Expression (4) shows that the 3d-order cumulant includes information of the differential value of the phase $\varphi(x)$, and expressions (1)-(3) give the CF modulus. The differential phase $\Phi(x)$ may be calculated by using system (1)-(4) [4].

$$\begin{aligned}
&\Phi(n\Delta x) \equiv \varphi[(n+1)\Delta x] - \varphi(n\Delta x) - \varphi(\Delta x) = \\
&= \pm arccos\{K^{(3)}/2[K_{12}^{(2)} K_{13}^{(2)} K_{23}^{(2)}]^{1/2}\}. \qquad (5)
\end{aligned}$$

Thus the phase $\varphi(x)$ is calculated from the recurrent relation

$$\varphi(n\Delta x) = \sum_{m=1}^{n-1} \Phi(m\Delta x) + n\varphi(\Delta x). \qquad (6)$$

At this stage, however, two problems need to be solved. First, the value of $\varphi(\Delta x)$ should be defined. This value linearly effects the phase (5), hence, it specifies only the gravity-center position of the reconstructed image. Therefore, this quantity may be chosen arbitrarily in many cases and, if it is possible, the intrinsic value may be estimated from another priori data. Second, in expression (5) the sign of $\Phi(x)$ must be defined. This should be done at every scanning point by considering the differential phase (5) for the larger interval which is a multiple of (Δx) [4].

The correlations measuring by means of registering photocounts coincidences by using fast enough photomultipliers (PMT) in photocounting mode gives the following correlation functions:

$$\begin{aligned}
G_{ij}^{(2)} &= <n_i' n_j'>= \alpha_i \alpha_j < I_i(x_0) I_j(x_0 + m\Delta x)> &(7)\\
G^{(3)} &= <n_1' n_2' n_3'>= \\
&= \alpha_1 \alpha_2 \alpha_3 < I_1(x_0 - \Delta x) I_2(x_0) I_3(x_0 + m\Delta x)>, &(8)
\end{aligned}$$

where n_i', α_i are the photocounting rate and the quantum sensitivity of i-th PMT, respectively; m specifies the location point of a detector. These functions are related to cumulants in the following way:

$$K_{ij}^{(2)} = \frac{1}{\alpha_i \alpha_j} \left[G_{ij}^{(2)} - <n'><n_j'> \right], \qquad (9)$$

$$
\begin{aligned}
K^{(3)} \quad &= \quad \frac{1}{\alpha_1\alpha_2\alpha_3}\Big[G^{(3)} + 2 < n_1' >< n_2' >< n_3' > - \\
&\quad - G_{12}^{(2)} < n_3' > - G_{13}^{(2)} < n_2' > - G_{23}^{(2)} < n_3' > \Big], \qquad (10)
\end{aligned}
$$

or proceeding with the values which are measured in experiment:

$$
\begin{aligned}
K_{ij}^{(2)} \quad &= \quad \frac{1}{\alpha_i\alpha_j}\left(\frac{N_{ij}}{2\tau_p T_t} - \frac{N_i N_j}{T_t^2}\right) \qquad (11) \\
K^{(3)} \quad &= \quad \frac{1}{\alpha_1\alpha_2\alpha_3}\left(\frac{N^{(3)}}{4\tau_p^2 T_t} + \frac{2N_1N_2N_3}{T_t^3} - \frac{N_{12}}{2\tau_p T_t}\frac{N_3}{T_t} - \right. \\
&\quad \left. - \frac{N_{13}}{2\tau_p T_t}\frac{N_2}{T_t} - \frac{N_{23}}{2\tau_p T_t}\frac{N_1}{T_t}\right)
\end{aligned}
$$

where N_i and N_j are the photocounts from each PMT and their double coincidences accumulated within time T_t; $N^{(3)}$ is the value of triple coincidence with a similar period; and τ_p is the a resolution time for the coincidence circuits.

Thus, by recording the combinations of photocounting coincidences the required cumulants of optical radiation in the detection plane may be found after the corresponding calculation.

Practical embodiment of the above principles for the needed cumulants to be completely recorded has been realized by a the system shown in Figure 1. A receiving part of such device comprises three PMTs which perform field scanning following the position of the triple correlation algorithm. PMT1 and PMT2 are fixed and PMT3 moves giving a two-coordinate micrometric feed. Global control of this device, data input, and mathematical processing are performed by a computer. The method of measurement is as follows. Depending on the PMTs operation with the initial tune-up and on the level of the received irradiance, the discriminators levels and the optimal sampling time in a control block are set. Moreover, using control outputs one achieves an equality in the avarge photocounting currents which decrease the statistical errors in these measurements. The one cycle of measurement is executed under computer control. For the required statistics to be stored these cycles were repeated at every measuring point.

Figure 2 shows the optical arrangement of the experimental setup used. A He- Ne laser beam passes through lens L_1 and focused onto a pair of ground glass plates, one of which is fixed while the other is rotated by the motor M. A quasi-incoherent source is thus formed and illuminates the object S. The light waves diffracted by object S undergo Fourier transform due to lens L_2 having 1m-focal length, and at the back focal plane an optical field assumes a far-zone approximation. In terms of the nature of pseudothermal sources, the field amplitude distribution may be considered as the Gaussian distribution.

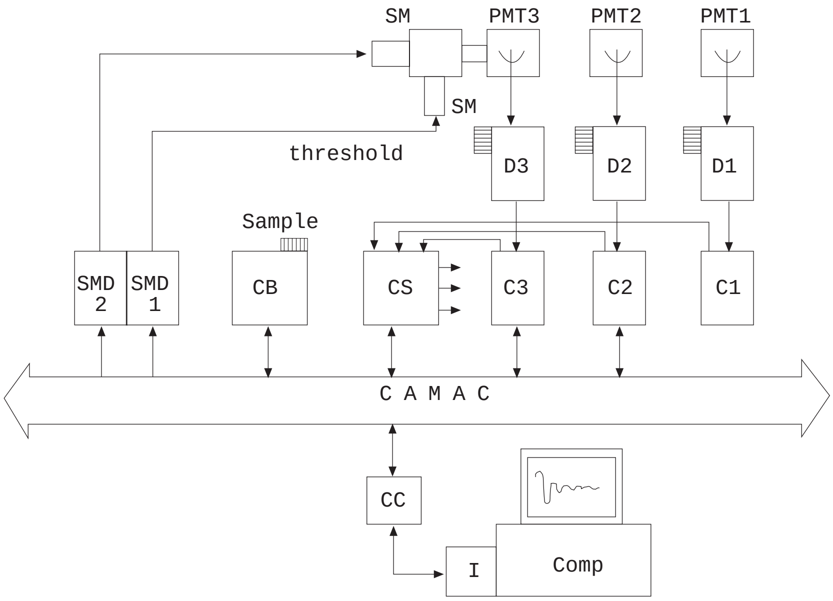

Figure 1. Block-scheme of the photocounts triple- correlator: PMT photomultipliers; D-discriminators; CS-coincidence scheme; CB-control block; I-interface; CC-CAMAC-controller, C-counters

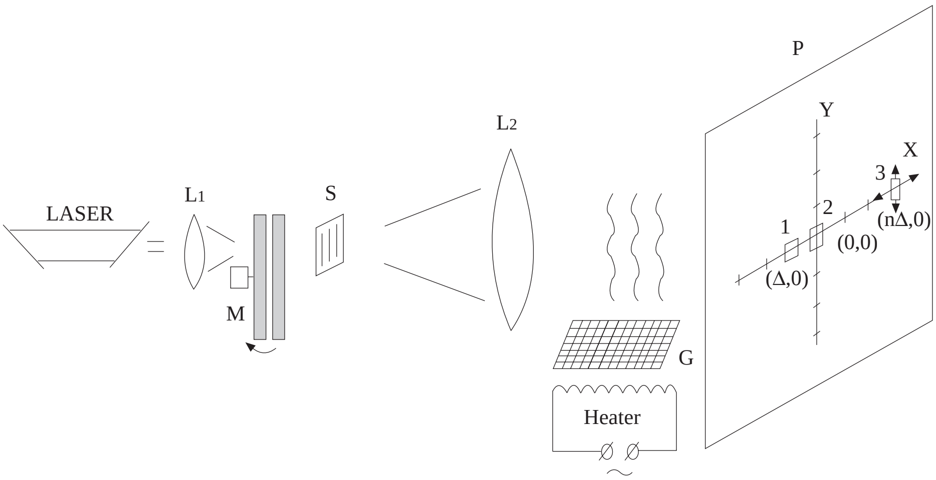

Figure 2. Schematic arrangement of the experiment

To simulate turbulence within a beam propagation line a heater H is used. Over the heater a metallic grid G is mounted the mesh dimensions of which specify the scale of turbulence vortexes. Additional investigations have shown that phase fluctuations of this propagating beam thus increase by π over a frequency range up to 30 Hz.

In the recording plane P a received field is divided by semitransparent plates and is recorded by three photodetectors. The distance between both fixed PMT1 and PMT2 is set equal to a minimal spatial period of correlogram discretization Δx. The scanned PMT3 passes the points which are at $n\Delta x$-distance from PMT2 (see Figure 2).

To form a two-dimensional image, a transparency stencil was used. It was cut from a foil in the form of the letter "T" with maximum dimension 600μm and the slot width was approximately equal to 200μm.

In the recording plane, the minimal period of the correlogram was approximately 1mm. The scanning step was chosen to be 0.5mm. The correlogram was scanned over the two-dimensional map with 32×32 points covering the region of 16×16mm^2. The experimental results plotted in Figure 3 represent the modulus of two-dimensional coherence function. Upon calculating the modulus and the phase for the coherence function at every point, a two-dimensional Fourier transform has been performed for image reconstruction. In Figure 4a the image thus obtained is plotted. Figure 4b shows the intensity-profile contour of the image exceeding the 0.5-level from which the initial object configuration is well-defined.

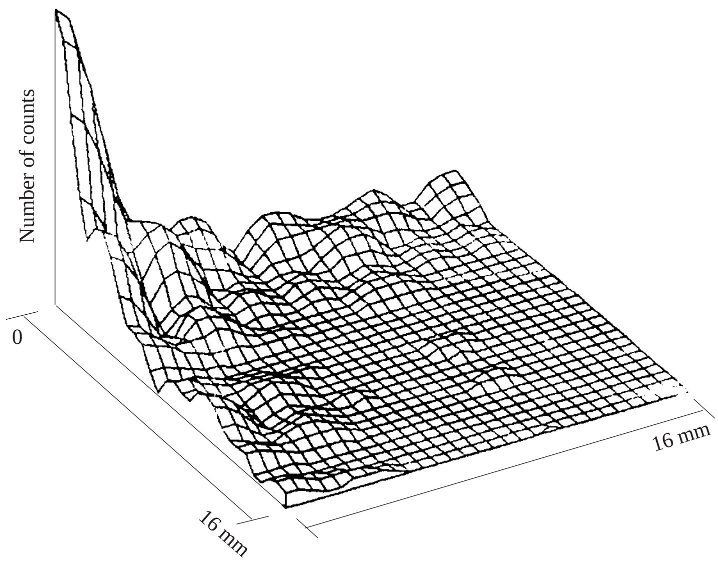

Figure 3. The modulus of correlogramm

Thus, the investigation we carried out have shown that it is possible to use intensity triple correlation for the image reconstruction of faint objects which are registered in photon counting mode. Phase information which is necessary for the complete image restoration is registered by means of additional receiving system resources only, and is analytically derived in accordance with the applied algorithm. Moreover, this method needs not increase the spatial frequency of the receiving system as in the case of using a reference source.

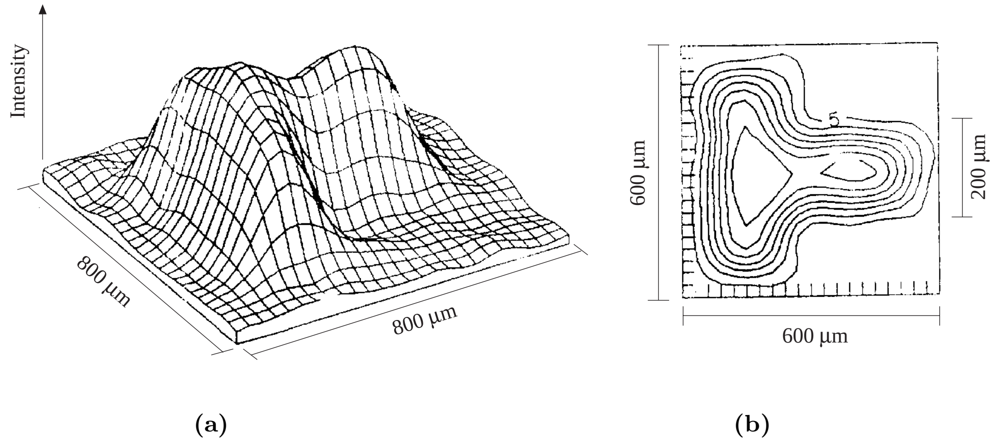

Figure 4. Reconstructed object: (a) orthographic and (b) countour projections

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