Domain wall motion in ferromagnetic nanowires driven by arbitrary time-dependent fields: An exact result

Arseni Goussev, JM Robbins, Valeriy Slastikov
School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, UK
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We address the dynamics of magnetic domain walls in ferromagnetic nanowires under the influence of external time-dependent magnetic fields. We report a new exact spatiotemporal solution of the Landau-Lifshitz-Gilbert equation for the case of soft ferromagnetic wires and nanostructures with uniaxial anisotropy. The solution holds for applied fields with arbitrary strength and time dependence. We further extend this solution to applied fields slowly varying in space and to multiple domain walls.

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Introduction.— The motion of magnetic domain walls (DWs) in ferromagnetic nanowires has recently become a subject of intensive research in the condensed matter physics community [1]. Manipulation of DWs by external magnetic fields, and in particular, the question of how the DW propagation velocity depends on the applied field have drawn considerable attention [2–4].

In ferromagnetic nanowires, the dynamics of the orientation of the magnetization distribution, $\mathbf{m}(x,t)$ (normalized so that $|\mathbf{m}| = 1$), is described by the Landau-Lifshitz-Gilbert (LLG) equation [5]

$$\frac{\partial \mathbf{m}}{\partial t} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} = (1+\alpha^2)\mathbf{m} \times \big(\mathbf{H}(\mathbf{m}) + \mathbf{H}_a\big) , \quad (1)$$

where x is the coordinate along the nanowire, t is time, α is the Gilbert damping parameter, $\mathbf{H}_a$ denotes the applied magnetic field, and $\mathbf{H}(\mathbf{m}) = -\delta E/\delta \mathbf{m}$, where

$$E(\mathbf{m}) = \frac{A}{2}\int_{\mathbb{R}} \left|\frac{\partial \mathbf{m}}{\partial x}\right|^2 \mathrm{d}x + \frac{K_1}{2}\int_{\mathbb{R}} \big(1 - (\mathbf{m}\cdot\hat{\mathbf{x}})^2\big)\,\mathrm{d}x + \frac{K_2}{2}\int_{\mathbb{R}} (\mathbf{m}\cdot\hat{\mathbf{y}})^2\,\mathrm{d}x. \quad (2)$$

is the reduced micromagnetic energy. Here, A is the exchange constant of the material, and $K_1, K_2 \geq 0$ are the anisotropy constants along the (easy) x- and (hard) y-axes. The anisotropy constant along the z-axis is taken to be zero by convention.

To date only one *exact* spatiotemporal [7] solution of the LLG equation has been reported in the literature, namely the so-called Walker solution [6]. The analysis of Schryer and Walker [6] applies to the case where $K_2 > 0$, ie where the anisotropy constants in the transverse plane are strictly unequal. This is appropriate for a thin film or thin strip geometry. The applied field is taken to be uniform in space, constant in time, and directed along the nanowire, i.e., $\mathbf{H}_a(x,t) = H_a\hat{\mathbf{x}}$. For $|H_a|$ less than a certain threshold H_W, the so-called Walker breakdown field, a planar domain wall propagates rigidly along the nanostrip with velocity depending nonlinearly on H_a.

In this Letter we present an *exact* spatiotemporal solution of the LLG equation that, to our knowledge, has not been previously reported in the literature. We consider the case of transverse isotropy, ie $K_2 = 0$. This is appropriate for soft ferromagnetic nanowires whose cross-sectional dimensions are comparable, as well as for uniaxial nanowires whose easy axis lies along the wire. We take the applied field to lie along the nanowire, as in the case of the Walker solution, but allow for arbitrary time dependence, i.e., $\mathbf{H}_a(x,t) = H_a(t)\hat{\mathbf{x}}$.

Exact solution of the LLG equation.— The boundary conditions appropriate for a domain wall with finite micromagnetic energy $E(\mathbf{m})$ are given by $\mathbf{m}(x,t) \to \pm\hat{\mathbf{x}}$ as $x \to \pm\infty$. For $K_2 = 0$ the magnetization-dependent field $\mathbf{H}$ is given by

$$\mathbf{H}(\mathbf{m}) = A\frac{\partial^2 \mathbf{m}}{\partial x^2} + K_1(\mathbf{m}\cdot\hat{\mathbf{x}})\hat{\mathbf{x}}\,. \quad (3)$$

We now take into account the fact that $\mathbf{m}$ has its values on S^2, and parametrize $\mathbf{m}$ in terms of angles $\theta(x,t)$ and $\phi(x,t)$ according to $\mathbf{m} = (\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi)$. From Eqs. (1) and (3) we obtain the LLG equation in the equivalent form

$$\dot{\theta} - \alpha\dot{\phi}\sin\theta + A(1+\alpha^2)\big(\phi''\sin\theta + 2\theta'\phi'\cos\theta\big) = 0\,, \quad (4a)$$

$$\alpha\dot{\theta} + \dot{\phi}\sin\theta + (1+\alpha^2)\big(-A\theta'' + A(\phi')^2\sin\theta\cos\theta + K_1\cos\theta\sin\theta + H_a(t)\sin\theta\big) = 0\,, \quad (4b)$$

where dot $\dot{}$ denotes $\partial/\partial t$ and prime $'$ denotes $\partial/\partial x$.

We now look for a solution of Eq. (4) in the form

$$\theta_*(x,t) = \theta_0\left(x - x_*(t)\right)\,, \quad \phi_*(x,t) = \phi_*(t)\,, \quad (5)$$

where

$$\theta_0(x) = 2\arctan\exp\left(-x/d_0\right)\,, \quad d_0 = \sqrt{A/K_1}. \quad (6)$$

$\theta_0(x)$ describes the static domain wall in the absence of an applied field. The magnetization density determined by $\theta_0(x)$ and $\phi_0(x) = \pi/2$ minimizes the micromagnetic energy $E(\mathbf{m})$ for the specified boundary conditions. Substituting Eq. (6) into Eq. (4), and taking into account that $\theta_0' = -\sin\theta_0/d_0$ and $\theta_0'' = \sin 2\theta_0/(2d_0^2)$, we find that θ_* and ϕ_* satisfy the LLG equation (4) provided that $x_*(t)$ and $\phi_*(t)$ satisfy

$$\dot{x}_* = -\alpha d_0 H_a(t)\,, \quad \dot{\phi}_* = -H_a(t)\,. \tag{7}$$

(In fact, (6) and (7) provide the only solution of the form (5).)

Equations (5-7) constitute the main result of this Letter. They represent an *exact* solution of the LLG equation, and describe a DW, with profile independent of the applied field, propagating along the nanowire with velocity $\dot{x}_*$ while precessing about the nanowire with angular velocity $\dot{\phi}_*$. *No restrictions have been imposed on the strength of the applied magnetic field and no assumptions have been made about its time dependence.*

We now compare the *precessing solution* Eqs. (5-7) with the Walker solution [6]. The Walker solution is defined only for $K_2 > 0$ (the fully anisotropic case) and time-independent H_a less than the breakdown field

$$H_W = \alpha K_2/2. \tag{8}$$

It is given by

$$\theta_W(x,t) = \theta_0\left(\frac{x - V_W t}{\gamma}\right)\,, \quad \phi_W(x,t) = \phi_W\,, \tag{9}$$

where

$$\sin 2\phi_W = H_a/H_W \tag{10}$$

determines the (fixed) inclination of the DW plane and

$$V_W = \gamma\frac{1+\alpha^2}{\alpha} d_0 H_a,\ \gamma = \left(\frac{K_1}{K_1 + K_2\cos^2\phi_W}\right)^{1/2} \tag{11}$$

gives the DW velocity.

There are several characteristic differences between the Walker solution and the precessing solution which should be distinguishable experimentally. Foremost is the fact that the Walker solution exists only for constant applied fields whose strength does not exceed a certain threshold, so that the DW velocity is bounded. The precessing solution is defined for time-dependent applied fields of arbitrary strength, so that the DW velocity, which for the precessing solution is proportional to the field strength, can be arbitrarily large. Next, while for the Walker solution the plane of the DW remains fixed, for the precessing solution it rotates about the nanowire at a rate proportional to H_a. Finally, we observe that, for the Walker solution, the DW profile contracts ($\gamma > 1$) or expands ($\gamma > 1$) in response to the applied field, whereas for the precessing solution the DW profile propagates without distortion.

Spatially nonuniform applied fields and multiple domain walls.— We now extend our results to applied fields that depend on both position along the nanowire and time, i.e, $\mathbf{H}_a = H_a(x,t)\hat{\mathbf{x}}$. For any (non-singular) applied field, Eq. (4) is satisfied at x outside the DW transition layer $|x - x_*(t)| \gg d_0$ (up to exponentially small terms). Assuming now that the field varies slowly across the transition region,

$$\big|H_a(x,t) - H_a\big(x_*(t),t\big)\big| \ll \big|H_a\big(x_*(t),t\big)\big| \quad \text{for} \quad |x - x_*(t)| \lesssim d_0\,, \tag{12}$$

we obtain an approximate solution of the LLG equation: the magnetization density is given by Eqs. (5) and (6) with

$$\dot{x}_* = -\alpha d_0 H_a\big(x_*(t),t\big)\,, \quad \dot{\phi}_* = -H_a\big(x_*(t),t\big)\,. \tag{13}$$

The physical meaning of Eq. (13) is quite obvious: the DW is only sensitive to the applied field within the transition layer.

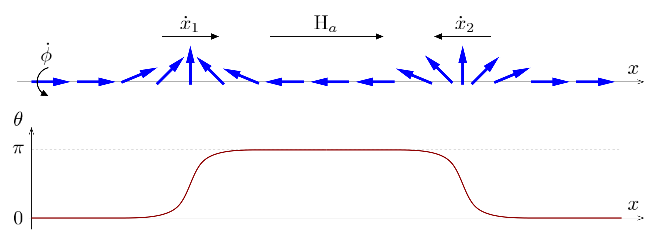

FIG. 1: (Color online) Dynamics of domain walls. See text for discussion.

This approximation can now be extended to the case of N non-overlapping DWs. Indeed,

$$\theta_N(x,t) = \sum_{n=1}^{N}\theta_0\big((-1)^{n+1}(x - x_n(t))\big)\,, \tag{14a}$$

$$\phi_N(x,t) = \phi_{\bar{n}}(t)\,, \quad n = \bar{n} \text{ minimizes } |x - x_n(t)|\,, \tag{14b}$$

with $x_{k+1}(t) - x_k(t) \gg d_0$ for $k = 1,\dots,N-1$, constitutes an approximate solution of the LLG equation given that

$$\dot{x}_n = (-1)^n \alpha d_0 H_a\big(x_n(t),t\big)\,, \tag{15a}$$

$$\dot{\phi}_n = -H_a\big(x_n(t),t\big)\,, \tag{15b}$$

for $n = 1,\dots,N$. For the case of a spatially uniform applied field Eqs. (14) and (15) describe the time evolution of N DWs such that any two adjacent WDs travel in opposite directions while rotating in the same direction (and with the same angular velocity) around the nanowire.

Conclusions.— In this Letter we have presented an exact spatiotemporal solution of the LLG equation that has not been previously reported in the literature. The validity of the new solution requires no assumptions about the time-dependence or strength of the applied field.

We have then provided a natural extension of the solution to physical situations in which the applied field varies slowly in space. An approximate solution of the LLG equation for the case of multiple domain walls has also been obtained.

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[1] see e.g., S. S. P. Parkin, M. Hayashi, and L. Thomas, Science **320**, 190 (2008); R. P. Cowburn, Nature **448**, 544 (2007).
[2] Z. Z. Sun and J. Schliemann, Phys. Rev. Lett. **104**, 037206 (2010).
[3] X. R. Wang, P. Yan, and J. Lu, Europhys. Lett. **86**, 67001 (2009); X. R. Wang, P. Yan, J. Lu, C. He, Ann. Phys. **324**, 1815 (2009).
[4] M. C. Hickey, Phys. Rev. B **78**, 180412(R) (2008).
[5] see e.g., A. Hubert and R. Schäfer, *Magnetic Domains: The Analysis of Magnetic Microstructures* (Springer, Berlin, 1998).
[6] N. L. Schryer and L. R. Walker, J. Appl. Phys. **45**, 5406 (1974).
[7] The only other exact solution of the LLG equation reported in the literature [Z. Z. Sun and X. R. Wang, Phys. Rev. Lett. **97**, 077205 (2006)] appears in the problem of magnetization switching, where the magnetization density is considered to be uniform in space and is a function of time only, i.e., $\mathbf{m} = \mathbf{m}(t)$.